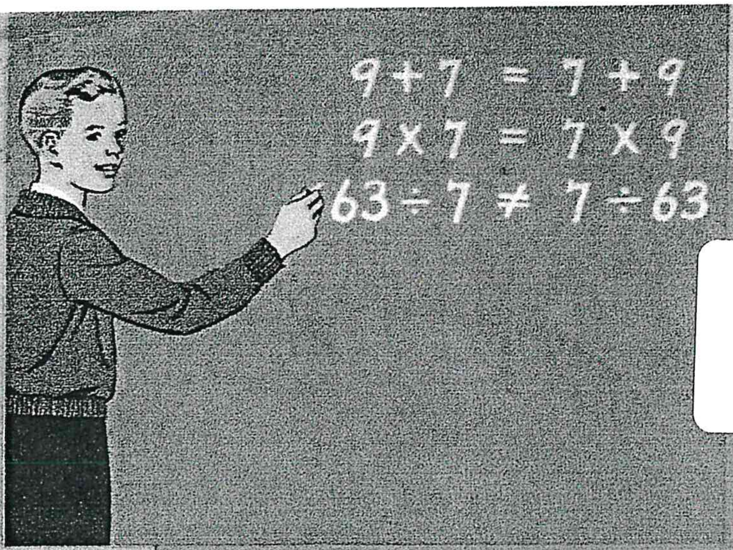

Customized Student Edition

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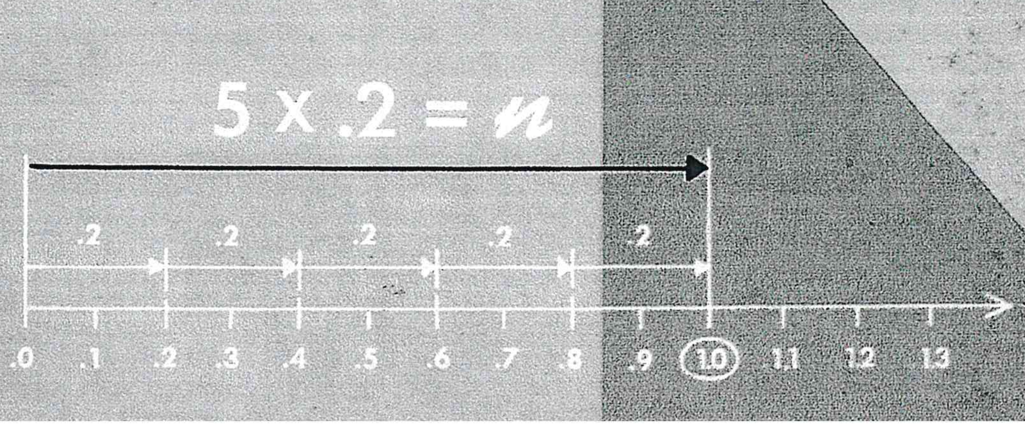

ARITHMETIC

6

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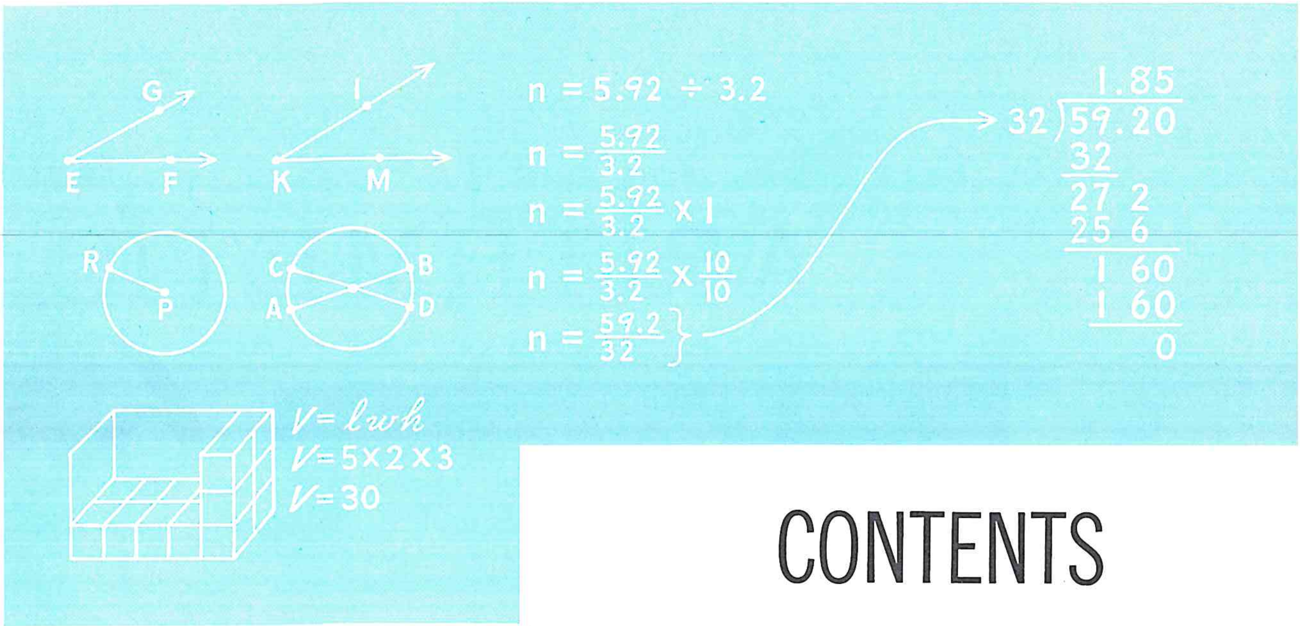
LAIDLAW BROTHERS • PUBLISHERS
A Division of Doubleday & Company, Inc.
RIVER FOREST, ILLINOIS
SUMMIT, NEW JERSEY PALO ALTO, CALIFORNIA DALLAS, TEXAS ATLANTA, GEORGIA



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3456789 210987654

A 0 0 0 0 0 0
B X X X X X X X X

$\frac{1 \text{ set of } 6}{1 \text{ set of } 8} = \frac{1 \times 6}{1 \times 8} = 1 \times \frac{6}{8} = \frac{6}{8}$

A 0 0 0   0 0 0
B X X X X   X X X X

$\frac{2 \text{ sets of } 3}{2 \text{ sets of } 4} = \frac{2 \times 3}{2 \times 4} = 1 \times \frac{3}{4} = \frac{3}{4}$

$\frac{a}{12} = \frac{7}{21}$

$a \times 21 = 7 \times 12$
$a \times 21 = 84$
$a = 84 \div 21$
$a = 4$

Therefore $\frac{4}{12} = \frac{7}{21}$

$\frac{19}{40} = \frac{b}{100}$

$40 \times b = 100 \times 19$
$40 \times b = 1900$
$b = 1900 \div 40$
$b = 47\frac{1}{2}$

$\frac{19}{40} = \frac{47\frac{1}{2}}{100}$ or $47\frac{1}{2}\%$ or 47.5%

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1

123
1230
12,300
123,000
1,230,000
12,300,000

1,230,000,000,000

Our Numeration System

Can you name some of the wonderful inventions that make the boy's telephone call possible? Did you think of naming our **numeration system** as the most wonderful invention of all?

In a numeration system, numerals are used to represent numbers, and the numerals are grouped in a special way. The numerals used in our numeration system are called **digits.** They are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

These numerals are used over and over again in various combinations. Thus 1, 2, and 3 are used to write 123, 132, 231, 213, 312, and 321. Does the position in which each digit is written affect its value?

Read and think about the numeral 7086 to help you understand our numeration system, which is called the Hindu-Arabic system. How many digits are in the numeral 7086? How many **place-value positions** does it have?

The digit 7 in 7086 is in thousands place. The 7 has a value of thousands. The digit 0 in hundreds place tells you that there are no hundreds. What does the 8 in tens place and the 6 in ones place tell you?

Each place-value position has a value ten times greater than the position to its right, as shown.

Hundreds	Tens	Ones
(10×10×1)	(10×1)	(1×1)

Does each place-value position have a value $\frac{1}{10}$ of the value of the position to its left?

Oral Answer each question below.

1. What is the value of 3 in 34? In 345? In 3507? 3 tens; 3 hundreds; 3 thousands

2. Which digit in 3057 represents units with the greatest value? Which represents units having the smallest value? 3; 7

3. Which is the largest possible four-digit number you can name using 3, 4, 5, and 6 only once? 6543

MORE PRACTICE PAGE 295

Reading and Writing Numerals

The number names through billions are given below. See if you can discover any pattern in them.

ones
tens
hundreds
thousands
ten-thousands
hundred-thousands
millions
ten-millions
hundred-millions
billions
ten-billions
hundred-billions

Did you notice the repetition of the words thousands, millions, and billions? Which other words were repeated?

After you discover the system for reading large numerals, you should have little difficulty in reading them. The diagram below may prove helpful.

Billions period			Millions period			Thousands period			Ones period		
Hundred-billions	Ten-billions	One-billion	Hundred-millions	Ten-millions	One-million	Hundred-thousands	Ten-thousands	One-thousand	Hundreds	Tens	Ones
5	2	9,	2	4	8,	6	5	0,	3	9	6

MORE PRACTICE PAGE 295

Beginning from the right, what is the name of each group, or **period?** Observe that a comma separates the periods. Would a comma be necessary in a four-digit numeral? Why or why not?

To read 529,248,650,396, say: *five hundred twenty-nine billion, two hundred forty-eight million, six hundred fifty thousand, three hundred ninety-six.*

In reading any large numeral, decide upon the name of its highest period. Read the numeral in that period and say the name of the period. Then read the numeral in each of the other periods in a similar way. You do not name a period when each of its digits is 0.

Oral Tell where you would place the comma in each numeral below. Be ready to read each numeral.

	a	*b*
1.	5,780,076	248,650,396
2.	39,500,401	39,500,072,100
3.	48,953,216,721	583,692,150,000
4.	497,568,721,000	696,400,500,900
5.	727,005,000,100	800,007,105,000

Written Write each numeral in 1 through 5 above, placing commas correctly. Then write the words you would use in reading each numeral.
See diagram in column 1.

Using Tenths

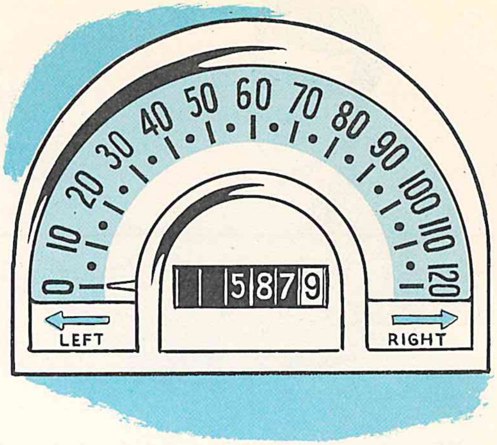
Look at the speedometer in the picture. How many miles has the car been driven? Did you say *five hundred eighty-seven and nine-tenths* miles? If so, your answer was correct.

The numeral 587$\boxed{9}$ on the speedometer has digits in four place-value positions: hundreds, tens, ones, and tenths. You write the numeral like this: 587.9. In which place-value position is the 5? The 8? The 7? The numeral 587 names a *whole number.*

Look at the sign (.) after the 7 in 587.9. This sign is called a **decimal point.** You can show which digit is in ones place by putting a decimal point right after it. All digits to the left of the decimal point represent whole numbers. All digits to the right of the decimal point represent fractional parts of 1.

The place-value position at the right of the ones place is called **tenths.** You obtain a tenth by dividing 1 by 10.

Look at the numeral 587.9 again. Numerals of this kind are called **decimal numerals** or **decimals.** Would you also expect such numerals as .1, .2, .3, and so on to be called decimals? In some books such numerals as 7.9, 87.9, and 587.9 are called mixed decimals. Do you see why?



Read decimals as you read other numerals, but then name the place-value position in which you see the final digit at the right. Read .2 as *two tenths,* .34 as *thirty-four hundredths,* and so on. To read the decimal numeral 587.9, read the numeral 587, use *and* to indicate the decimal point, and then read the decimal fraction. Say: *five hundred eighty-seven and nine tenths.*

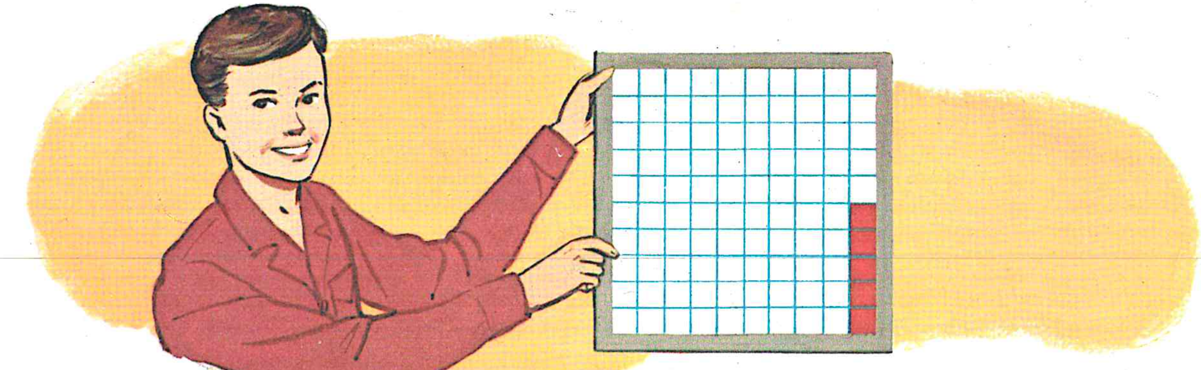
Oral Read each numeral below.
See below.

	a	*b*	*c*
1.	.3	.8	.6
2.	4.7	5.9	7.4
3.	15.9	27.2	61.7
4.	219.8	315.6	500.9

Written Write words to express each decimal numeral in rows 1 through 4 above.

Can you do this? Count by tenths from 1 to 2. 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0

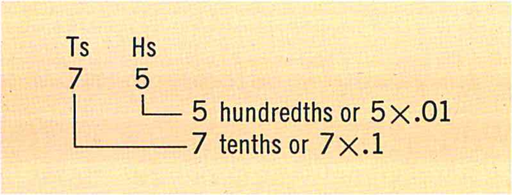
1. Three tenths; eight tenths; six tenths
2. Four and seven tenths; five and nine tenths; seven and four tenths
3. Fifteen and nine tenths; twenty-seven and two tenths; sixty-one and seven tenths
4. Two hundred nineteen and eight tenths; three hundred fifteen and six tenths; five hundred and nine tenths



Using Tenths and Hundredths

The large square which John is holding was cut from a piece of graph paper. Think of the whole square as 1. Look at the blue lines drawn horizontally across the paper. How many of them do you see? Do they divide 1 by 10? Is each strip .1 (one tenth) of the whole square?

Now count the vertical lines. Into how many equal parts do both the horizontal and vertical lines divide the whole square? When you divide 1 by 100, you get one hundredth. The decimal numeral .01 stands for one hundredth. Study the diagram below, in which Ts represents tenths and Hs represents hundredths.



Which is the largest possible digit that can appear in tenths place? In hundredths place?

Oral Tell your classmates how you would use graph paper to show the meaning of each decimal numeral below. See below.

	a	*b*	*c*	*d*
1.	.1	.01	.5	2.5
2.	.9	1.0	.10	3.15
3.	.25	.75	1.00	4.10

Written Write decimal numerals to stand for each number expressed below.

1. Twelve hundredths .12
2. Twenty-five hundredths .25
3. Thirty hundredths .30
4. Ninety-eight hundredths .98
5. Seventy hundredths .70

Can you do this? Use graph paper to show the value of a number expressed in hundredths. Write the decimal numeral that stands for the number you illustrated.

1. 1 strip; 1 inner sq.; 5 strips; 2 whole sq. and 5 strips
2. 9 strips; 1 whole sq.; 10 inner sq.; 3 whole sq., 1 strip, 5 inner sq.
3. 2 strips and 5 inner sq.; 7 strips and 5 inner sq.; 1 whole sq.; 4 whole sq. and 1 strip

Using Smaller Decimal Units

On the graph paper on page 8, find one of the squares that is divided into hundredths. Think of that square as being divided by 10. How many of the small squares would you then have? You should have thought *one thousand.* When 1 is divided by 1000, the result is called thousandths.

You may remember that the first place-value position to the right of ones has a value of tenths and that the second place-value position has a value of hundredths.

The third place-value position to the right of the ones has a value of thousandths. Which position would you expect to have a value of ten-thousandths? Of hundred-thousandths?

Do you suppose it would be possible to divide 1 by ten thousand? By one hundred thousand? By one million? Could you use a picture to show the result of such divisions?

The names of the place-value positions for decimal numerals are shown below.

$1 \div 10$: tenths
$1 \div 100$: hundredths
$1 \div 1000$: thousandths
$1 \div 10{,}000$: ten-thousandths
$1 \div 100{,}000$: hundred-thousandths
$1 \div 1{,}000{,}000$: millionths

Oral Answer each question below.

1. What are the last three letters in each of these words: tenths, hundredths, thousandths? -ths

2. What is the value of 1 in .01? In .001? In .0001? 1 hundredth; 1 thousandth; 1 ten-thousandth

$\frac{1}{100}$ $\frac{1}{1000}$ $\frac{1}{10{,}000}$

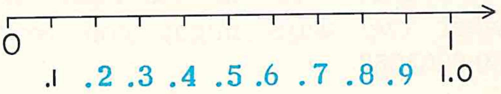
Written Express each of the following as a decimal.

1. Four tenths .4
2. Three hundredths .03
3. Two thousandths .002
4. One millionth .000001
5. Five tenths .5
6. Seven hundredths .07
7. Nine thousandths .009
8. Eight millionths .000008
9. Five ten-thousandths .0005
10. Eight hundred-thousandths .00008

Can you do this?

1. Write numerals to show a count by hundredths from 2.98 to 3.10. See below.

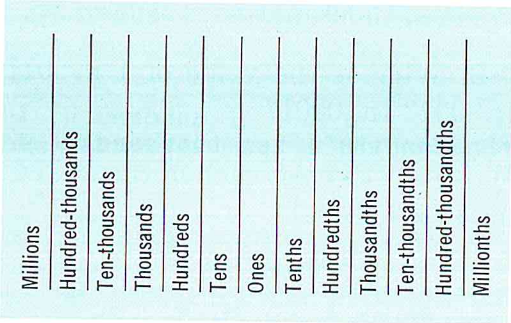
2. Draw a number line like the one below. Use decimal numerals to complete the scale below the line.



1. 2.98, 2.99, 3.00, 3.01, 3.02, 3.03, 3.04, 3.05, 3.06, 3.07, 3.08, 3.09, 3.10

Reading and Writing Decimals

Study the diagram below. How many places are at the left of ones place? At the right of ones place?



Use the diagram to help you read the numeral .0054. The decimal point in it shows that a decimal numeral is expressed. The zeros show that there are no tenths and no hundredths. They also help you know that 5 has a value of thousandths and that 4 has a value of ten-thousandths.

To read .0054, skip the two zeros and say *fifty-four*. Then name the place-value position of the digit at the right. Thus .0054 is read *fifty-four ten-thousandths.*

In reading 57.0158, you would probably say: *fifty-seven and one hundred fifty-eight ten-thousandths*. It would also be correct to say: *fifty-seven point zero one five eight.* In what two ways might you read 39.000125?

MORE PRACTICE
PAGE 295

Oral Read each decimal below.
Representative answers—column a onl

a *b* *c*

1. .75 .02 .069
Seventy-five hundredths

2. .005 .01376 .47325
Five thousandths

3. 5.5 3.07 8.75
Five and five tenths

4. 15.075 19.476 13.0321
Fifteen and seventy-five thousandths

5. 7.009 23.983 275.46105
Seven and nine thousandths

Written Write the words you would use in reading each decimal below.
Representative answers--column a onl

a *b* *c*

1. .1 .7 .03
One tenth

2. .001 .0009 .027
One thousandth

3. 3.9 17.15 35.07
Three and nine tenths

4. 12.0075 47.0705 78.3007
Twelve and seventy-five ten-thousandt

5. .21 .25301 99.099
Twenty-one hundredths

Write the following as decimals.

6. Eighty-five hundredths .85

7. Sixty-three thousandths .063

8. Two and twenty-five hundredths 2.25

9. Five million, one hundred and six hundredths 5,000,100.06

10. Nine million, six thousand fifty and fifteen thousandths
9,006,050.015

Practice in Reading Numerals

Number is everywhere. In the business world, the world of sports, science, education—everywhere you may look—you see number used in one or more of its many applications. The paragraphs below show some of these applications.

The Grand Canyon has been carved in the surface of the earth by the Colorado River during the hundreds of thousands of years that the river has been hurrying to the sea. The Canyon is 217 miles long, from 4 to 18 miles wide, and from 4000 to 5500 feet deep. In its floor, about 2300 feet above sea level, the river still carves away night and day. In a thousand years or so the Canyon may be a few inches deeper.

In 1910 the motor boat Dixie III won the Gold Cup. Her speed for her best 30-mile heat was thirty-three and six tenths (33.6) miles per hour. In 1960 the hydroplane, "Miss Thriftway," with a speed of one hundred ninety-two and one thousandth (192.001) miles per hour, set a new speed record for boats.

In 1960 the population of the United States was 179,323,175, and the population of Communist China was 582,603,000. In the same year, the population of California was 15,717,204.

In the 1961 World Series the Yankees beat Cincinnati 4 games to 1 game. In the summer of 1960 an Air Force captain jumped from a height of 102,800 feet in a free-fall parachute drop. In the same year the estimated elementary school enrollment was 34,155,000 pupils.

Oral Use the preceding paragraphs to help you answer each question below.

1. Which numeral tells how many miles long the Grand Canyon is? 217

2. Which numeral tells the estimated elementary school enrollment for the year 1960? 34,155,000

3. Which numeral tells the height from which the Air Force captain jumped? 102,800

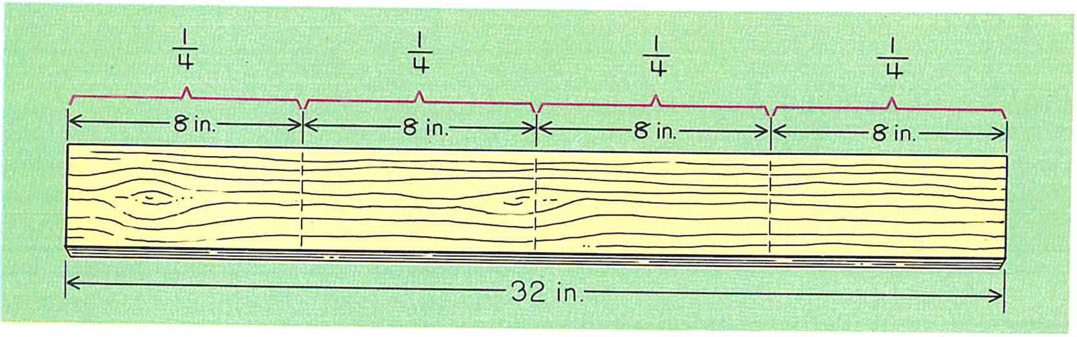
4. What is an easy way to compare the speed of the Gold Cup winners for the years 1910 and 1960?
Subtract 33.6 from 192.001

192.001
33.600
01

Read these numerals.
See below.

	a	*b*
5.	745,237,465.9	817,000,500
6.	19,010,050	240,000,750

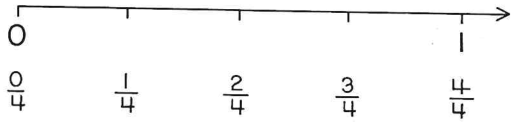
5. Seven hundred forty-five million, two hundred thirty-seven thousand, four hundred sixty-five and nine tenths; eight hundred seventeen million, five hundred
6. Nineteen million, ten thousand, fifty; two hundred forty million, seven hundred fifty


$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
8 in.
8 in.
8 in.
8 in.
32 in.

Fractional Numbers and Fractions

Into how many fourths will the board be divided after it is cut along the dotted lines? What number name is given to each of the fourths? Do you see $\frac{4}{4}$ as another name for 1?

By observing a number line, you can see what happens when you divide 1 by 4 and thus obtain a fraction.



The distance from 0 to 1 is used to represent the whole number 1. The distance from 0 to $\frac{1}{4}$ was determined by dividing the whole number 1 by 4. Which fraction represents the distance from 0 to $\frac{1}{4}$? From 0 to $\frac{2}{4}$? From 0 to $\frac{3}{4}$? From 0 to $\frac{4}{4}$? Do you see that each fraction corresponds to a single point on the number line shown above?

Take any other number. Divide 1 by the number you chose. Can you always use a fraction to name the result of the division?

Every fraction has a **numerator** and a **denominator.** The denominator tells you the number of parts of equal size into which a given quantity has been divided. The numerator tells you how many of these parts you are to think about.

In each of the fractions $\frac{0}{4}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$ the denominator is 4. What is the numerator in each?

Oral Answer each question below.

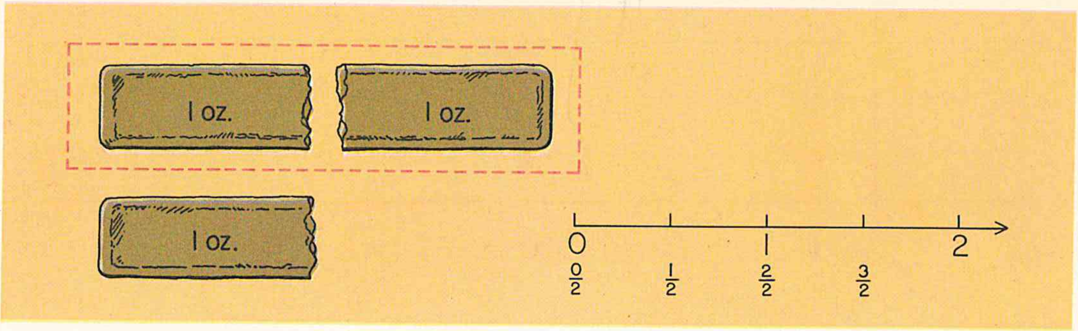
1. How many numerals that represent whole numbers are used in writing a fraction? How are the numerals separated? Two numerals; by a fraction line

2. Which fraction would you use to represent $1 \div 3$? $2 \div 3$? $5 \div 2$? $\frac{1}{3}$; $\frac{2}{3}$; $\frac{5}{2}$

3. What is the numerator and the denominator in $\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{2}{3}$? $\frac{5}{6}$? $\frac{5}{8}$? $\frac{3}{10}$? See below.

4. Which fraction would you use to express one half? Three fourths? Seven eighths? $\frac{1}{2}$; $\frac{3}{4}$; $\frac{7}{8}$

3. Numerators:
Denominators: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{2}{3}$; $\frac{5}{6}$; $\frac{5}{8}$; $\frac{3}{10}$



Kinds of Fractions

Suppose you had 1 candy bar that weighed 2 ounces and cut it exactly in half. Each piece would be $\frac{1}{2}$ of the whole bar. Does $\frac{1}{2}$ express a value less than 1? Fractions representing a value less than 1 are called **proper fractions.**

Suppose you took 2 of the halves. Would you have the same amount of candy as though you had 1 whole bar? Does $\frac{2}{2}$ have the same value as 1?

Suppose you took the $\frac{2}{2}$ of one candy bar and $\frac{1}{2}$ of another. You would have a value greater than 1. You could use $\frac{3}{2}$ to express the number of pieces you took. Fractions which name a number equal to or greater than one are called **improper fractions.**

Suppose you took 1 whole candy bar and $\frac{1}{2}$ of another. You would have $1+\frac{1}{2}$ candy bars. The numeral $1\frac{1}{2}$, which means $1+\frac{1}{2}$, would stand for the candy you took. Such numerals as $1\frac{1}{2}$ name a whole number and a fractional number. They are called **mixed fractions.** To read a mixed fraction, first name the whole number; then name the fractional number. To read $12\frac{1}{2}$, which means $12+\frac{1}{2}$, say: *twelve and one half.*

Oral Read each numeral below. Tell if it is a proper fraction, an improper fraction, or a mixed fraction and how you decided. See above.

	a	*b*	*c*
1.	$4\frac{1}{2}$ mixed	$\frac{2}{3}$ proper	$5\frac{2}{3}$ mixed
2.	$\frac{7}{4}$ improper	$\frac{5}{6}$ proper	$8\frac{3}{4}$ mixed
3.	$\frac{7}{6}$ improper	$3\frac{3}{4}$ mixed	$\frac{7}{5}$ improper
4.	$8\frac{1}{2}$ mixed	$\frac{3}{4}$ proper	$6\frac{2}{3}$ mixed
5.	$\frac{9}{2}$ improper	$\frac{2}{9}$ proper	$1\frac{7}{16}$ mixed

Written Follow these directions.
Representative answers below.

1. Write 5 proper fractions.
See below.

2. Write 5 fractions which have a value of 1. See below.

3. Write 5 improper fractions.
See below.

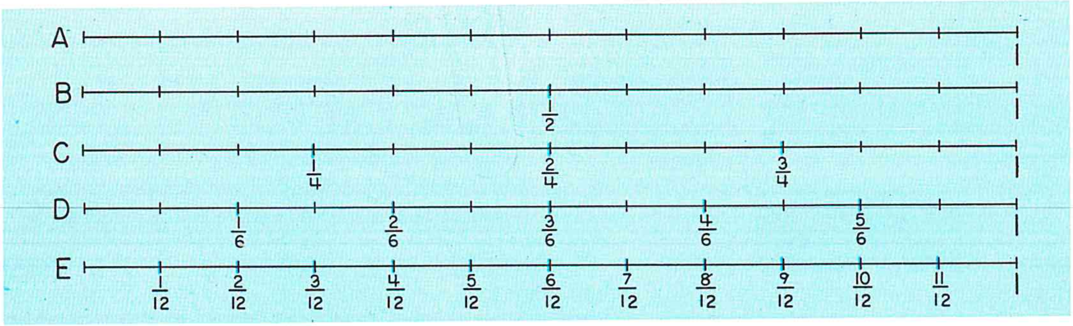
4. Write 5 mixed fractions.
See below.

1. $\frac{2}{3}$, $\frac{6}{7}$, $\frac{3}{10}$, $\frac{5}{11}$, $\frac{7}{12}$

2. $\frac{2}{2}$, $\frac{3}{3}$, $\frac{5}{5}$, $\frac{7}{7}$, $\frac{21}{21}$

3. $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{2}$, $\frac{6}{5}$, $\frac{43}{17}$

4. $1\frac{1}{2}$, $1\frac{1}{3}$, $2\frac{1}{2}$, $1\frac{1}{5}$, $2\frac{9}{17}$



Equivalent Fractions

Each line segment above represents a length of 1 foot. Line A has been divided into inches. 1 foot is equal in length to how many inches?

Now look at B above. Is $\frac{1}{2}$ of 12 inches equal in length to 6 inches? On C, do you see that $\frac{1}{4}$ of 12 inches is equal in length to 3 inches?

Find $\frac{2}{4}$ of 12 inches on C. Is $\frac{2}{4}$ of 12 equal in length to $\frac{1}{2}$ of 12? Do $\frac{1}{2}$ and $\frac{2}{4}$ represent the same length? In general, fractions which represent the same fractional number, like $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, and so on, are called **equivalent fractions.** Would you expect $\frac{1}{3}$ and $\frac{2}{6}$ to be equivalent fractions?

Now look at D. The 12 inches have been divided into sixths. Is $\frac{3}{6}$ of 12 inches equal in length to 6 inches? Is $\frac{1}{2}$ equivalent to $\frac{3}{6}$?

If you look at E, you will see that $\frac{6}{12}$ of 12 inches is equivalent in length to 6 inches. Which of the other fractions on A, B, C, and D above are equivalent to 6 inches?

Oral Tell which pairs of fractions below are equivalent fractions. Use the illustration for help.

	a	*b*	*c*
1.	$\frac{1}{2}$ and $\frac{2}{4}$	$\frac{1}{2}$ and $\frac{4}{8}$	$\frac{1}{2}$ and $\frac{2}{6}$
2.	$\frac{1}{6}$ and $\frac{2}{12}$	$\frac{3}{12}$ and $\frac{1}{6}$	$\frac{1}{2}$ and $\frac{3}{6}$
3.	$\frac{5}{6}$ and $\frac{10}{12}$	$\frac{3}{4}$ and $\frac{8}{12}$	$\frac{2}{3}$ and $\frac{1}{2}$

Written Copy. Then replace n by a numeral so that each statement below becomes true. Use the illustration for help.

	a	*b*	*c*
1.	$1=\frac{n}{2}$ 2	$1=\frac{n}{3}$ 3	$1=\frac{n}{4}$ 4
2.	$1=\frac{n}{6}$ 6	$1=\frac{n}{12}$ 12	$\frac{1}{2}=\frac{n}{4}$ 2
3.	$\frac{1}{2}=\frac{n}{6}$ 3	$\frac{4}{8}=\frac{n}{2}$ 1	$\frac{6}{12}=\frac{n}{2}$ 1

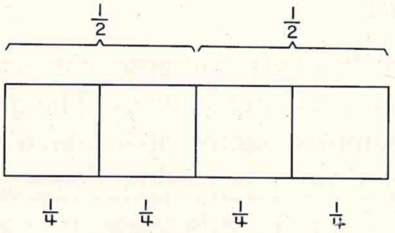
Something to do Draw 4 line segments that are equal in length. Label the endpoints 0 and 1.

Use your lines to show thirds, sixths, ninths, and twelfths. Then make a chart to show which fractions are equivalent to $\frac{1}{3}$. To $\frac{2}{3}$.

Changing Fractions to Higher Terms

A recipe that Nancy will use calls for $\frac{1}{2}$ pound of butter. The pound of butter she has is divided into fourths. How many fourths of a pound should she use?

The diagram may help you see the answer to the above problem.



1 pound is equivalent to $\frac{2}{2}$ of a pound. 1 pound is also equivalent to $\frac{4}{4}$ of a pound. Is $\frac{1}{2}$ of $\frac{2}{2}$ equivalent to $\frac{1}{2}$? Does $\frac{1}{2}$ of $\frac{4}{4}=\frac{2}{4}$? Does $\frac{1}{2}=\frac{2}{4}$?

In order to change $\frac{1}{2}$ into fourths without using a diagram as a help, you can use what you learned about the factor 1 when you studied multiplication. Do you recall that multiplying a number by 1 leaves the number unchanged? To illustrate, $1\times2=2$, $1\times3=3$, $1\times\frac{1}{2}=\frac{1}{2}$, $1\times\frac{2}{3}=\frac{2}{3}$. You also use the idea that 1 can be expressed as a fraction in many ways: $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and so on.

Now see what happens when you multiply $\frac{1}{2}$ by $\frac{2}{2}$. Does $\frac{2}{2}=1$?

$$\frac{1}{2}=1\times\frac{1}{2}=\frac{2}{2}\times\frac{1}{2}=\frac{2\times1}{2\times2}=\frac{2}{4}$$

Now look at the equation $\frac{1}{2}=\frac{n}{6}$. You are to replace n with a numeral to make the statement true. By which number would you multiply 2 to get 6? Did you say 3? If so, you were correct. Try thinking of 1 as $\frac{3}{3}$. Then see what happens.

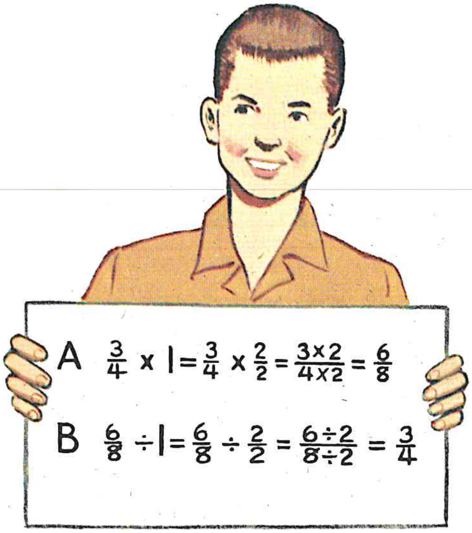
$$\frac{3}{3}\times\frac{1}{2}=\frac{3\times1}{3\times2}=\frac{3}{6}.\ \text{Does } \frac{1}{2}=\frac{3}{6}?$$

Oral What steps would you take in finding the value of n in each statement below? See column 1. Answers only.

	a	*b*	*c*
1.	$\frac{1}{6}=\frac{n}{12}$ 2	$\frac{3}{4}=\frac{n}{12}$ 9	$\frac{5}{8}=\frac{10}{n}$ 16
2.	$\frac{1}{3}=\frac{n}{6}$ 2	$\frac{3}{4}=\frac{n}{16}$ 12	$\frac{5}{6}=\frac{15}{n}$ 18

Written Copy. Replace n with the numeral that makes each equation true.

	a	*b*	*c*
1.	$\frac{2}{3}=\frac{n}{6}$ 4	$\frac{3}{4}=\frac{n}{8}$ 6	$\frac{1}{2}=\frac{2}{n}$ 4
2.	$\frac{1}{3}=\frac{n}{9}$ 3	$\frac{1}{4}=\frac{2}{n}$ 8	$\frac{1}{2}=\frac{n}{8}$ 4
3.	$\frac{1}{2}=\frac{3}{n}$ 6	$\frac{2}{3}=\frac{n}{9}$ 6	$\frac{1}{4}=\frac{3}{n}$ 12
4.	$\frac{4}{5}=\frac{n}{10}$ 8	$\frac{1}{2}=\frac{5}{n}$ 10	$\frac{3}{4}=\frac{n}{12}$ 9
5.	$\frac{1}{4}=\frac{4}{n}$ 16	$\frac{2}{5}=\frac{n}{10}$ 4	$\frac{1}{3}=\frac{5}{n}$ 15
6.	$\frac{1}{3}=\frac{4}{n}$ 12	$\frac{3}{8}=\frac{n}{16}$ 6	$\frac{5}{8}=\frac{n}{16}$ 10
7.	$\frac{2}{3}=\frac{8}{n}$ 12	$\frac{1}{6}=\frac{n}{18}$ 3	$\frac{7}{8}=\frac{21}{n}$ 24
8.	$\frac{3}{8}=\frac{9}{n}$ 24	$\frac{1}{5}=\frac{n}{10}$ 2	$\frac{2}{5}=\frac{n}{15}$ 6


A $\frac{3}{4} \times 1 = \frac{3}{4} \times \frac{2}{2} = \frac{3\times2}{4\times2} = \frac{6}{8}$
B $\frac{6}{8} \div 1 = \frac{6}{8} \div \frac{2}{2} = \frac{6\div2}{8\div2} = \frac{3}{4}$

Roger used multiplication to change $\frac{3}{4}$ to higher terms. Then he used division to change $\frac{6}{8}$ to lower terms. In both operations, what number name did he choose for 1?

Choose other names for the number 1. For example $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, and $\frac{6}{6}$. Take the fraction $\frac{3}{4}$. Repeat Roger's experiment enough times to satisfy yourself that changing a fraction to lower terms is the opposite of changing a fraction to higher terms. You should also convince yourself that dividing both of the numbers named by the numerator and the denominator by the same number, not 0 or 1, leaves the fractional number unchanged.

MORE PRACTICE PAGE 295

Changing a fraction to lower terms by dividing both numbers named in a fraction by the same number, not 0 or 1, is called **reducing a fraction.**

Reducing Fractions to Lowest Terms

To reduce a fraction to lowest terms, first determine the **greatest common factor.** The greatest common factor is the largest possible whole number by which both numbers named in the fraction are divisible. Divide both numbers named in the fraction by this factor. Write the answer.

To illustrate, suppose you are to reduce $\frac{3}{6}$ to lowest terms. The greatest common factor of 3 and 6 is 3. Proceed as shown below. Does $\frac{3}{6}=\frac{1}{2}$?

$$\frac{3}{6} \div 1 = \frac{3}{6} \div \frac{3}{3} = \frac{3\div3}{6\div3} = \frac{1}{2}$$

You may be able to perform the division mentally. If so, do not use pencil and paper.

Oral Tell the steps to take in changing the form of each fraction below to its lowest terms. See above. Answers only.

	a	*b*	*c*	*d*	*e*
1.	$\frac{4}{8}$ $\frac{1}{2}$	$\frac{8}{10}$ $\frac{4}{5}$	$\frac{3}{15}$ $\frac{1}{5}$	$\frac{4}{16}$ $\frac{1}{4}$	$\frac{5}{10}$ $\frac{1}{2}$
2.	$\frac{5}{20}$ $\frac{1}{4}$	$\frac{6}{9}$ $\frac{2}{3}$	$\frac{4}{12}$ $\frac{1}{3}$	$\frac{10}{12}$ $\frac{5}{6}$	$\frac{3}{9}$ $\frac{1}{3}$
3.	$\frac{6}{15}$ $\frac{2}{5}$	$\frac{2}{20}$ $\frac{1}{10}$	$\frac{12}{16}$ $\frac{3}{4}$	$\frac{8}{16}$ $\frac{1}{2}$	$\frac{9}{18}$ $\frac{1}{2}$
4.	$\frac{3}{12}$ $\frac{1}{4}$	$\frac{6}{8}$ $\frac{3}{4}$	$\frac{4}{10}$ $\frac{2}{5}$	$\frac{10}{16}$ $\frac{5}{8}$	$\frac{9}{12}$ $\frac{3}{4}$

Written Change the form of each fraction in rows 1 through 4 above to a fraction in lowest terms. Show each step.

Roman Numerals

The Romans invented symbols to represent numbers. The Roman symbols and their values are shown.

Our Numeral	Roman Numeral
1	I
5	V
10	X
50	L
100	C
500	D
1000	M

Observe how the Romans would have represented 132 and 1111.

CXXXII =
100 + (10 + 10 + 10) + (1 + 1) = 132
MCXI = 1000 + 100 + 10 + 1 = 1111

Look at CXXXII again. How many times is the symbol X used to represent 30? How many times is I repeated to represent 2? When symbols are repeated their values are meant to be added.

In the Roman numeration system, all fours and nines are written in a special way as shown.

IV = 4   XL = 40   CD = 400
IX = 9   XC = 90   CM = 900

Do you see that in each case the symbol at the left has the lesser value? Then the lesser value is subtracted from the greater value.

Except in writing fours and nines, another rule is used. It is this: Write the symbols from left to right in order of their decreasing value. Thus 49 is considered as 40 + 9 which equals (50 − 10) + (10 − 1) = XL + IX. XL + IX is written XLIX. 530 is considered as 500 + 30 and is written DXXX.

In writing Roman numerals, repetition may occur no more than 3 times. Subtraction may occur only once. 38 is written XXXVIII and not XXXIIX.

Oral Read each number expressed below. Tell how you decided. See above.

	a	*b*	*c*	*d*
1.	III	XX	CCC	MMM
	3	20	300	3000
2.	IV	XL	CD	CM
	4	40	400	900
3.	VIII	LIX	DXXXII	MCMLXIV
	8	59	532	1964

Written Write as Roman numerals.

	a	*b*	*c*	*d*
1.	9	72	69	90
	IX	LXXII	LXIX	XC
2.	150	1963	1492	3500
	CL	MCMLXIII	MCDXCII	MMMD

Can you do this? Use Roman numerals to add IX to XXIV. XXXIII

SIGN	MEANING
$+$	Add
$-$	Subtract
$\times$	Multiply
$\div$ $\overline{)}$ $\frac{1}{2}$	Divide
$=$	Equal to
$\neq$	Not equal to
$<$	Less than
$>$	Greater than

Which of the signs $+$, $-$, $\times$, $\div$, and $\overline{)}$ in the numerals below tell you that numbers are to be added? Multiplied? Divided? Subtracted?

$4+8$	3×4	$4\overline{)12}$
$12-4$	$12\div4$	$\frac{1}{4}\times12$

Read each number that is named above. First read it as expressed. Thus read $4+8$ as *four plus eight*. Then read it in simplest form, *twelve*. You are familiar with each sign used in the number names above. They are the signs that tell the operation to use in working with the numbers, and are called **operational signs.**

Read the following matching pairs of statements. Observe the signs that are used in place of words.

8 plus 5 is equal to 13	$8+5=13$
8 plus 5 is not equal to 12	$8+5\neq12$

Operations and Signs

3 is less than 5	$3<5$
5 is more than 3	$5>3$

Now read each statement below. Replace the signs $=$, $\neq$, $<$, and $>$ with the words shown on the chart on this page.

$8+4=12$	$12-4=8$	$3<5$
$4+8=12$	$4-12\neq8$	$5>3$

Notice that the open part of the sign $>$ and $<$ faces the numeral which names the greater number.

Oral Answer each question below.

1. What are some advantages of using signs rather than words? Answers will vary.

2. In $15\div3=5$ which sign tells the operation? $\div$

Written Copy. Write *true* or *false* after each statement below.

	a	*b*
1.	$9+5=5+9$ T	$17-9\neq9-17$
2.	$8\times7=7\times8$ T	$56\div7=56\div8$
3.	$9>7$ T	$8<5$ F
4.	$8+5>5+8$ F	$63\div9<72\div9$
5.	$48\div6\neq6\div48$ T	$8+7\neq15$ F
6.	$(9\times7)+2>2+(7\times9)$ F	
7.	$(17\times5)\times2\neq2\times(17\times5)$ F	

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Numerals are used to represent numbers. (5)

2. Each place-value position has a value that is ten times greater than the position to its right and one tenth that of the position to its left. (5, 9)

3. Zero is the symbol used to represent the empty set and means *not any* elements in that set. (5)

4. Such numerals as .9 and 587.9 are called decimal numerals. (7)

5. More than one name may be given to the same number. (18)

Words to Know

1. Digits, place value (5)

2. Numeration system (5)

3. Equivalent (14)

4. Operational signs (18)

Questions to Discuss

See T19 for answers.

1. What is the name of each place-value position beginning with ones and ending with billions? Beginning with tenths and ending with millionths? (6, 10)

2. Why is it a good idea to separate a numeral into periods? (6)

3. What makes 0 such an important numeral in our numeration system? (10)

4. What happens to the value of a number when you multiply it by 1? When you divide it by 1? (15, 16)

5. How do you change the form of a fraction to higher terms? To lower terms? (15, 16)

Written Practice

1. Name the smallest possible five-digit whole number using the numerals 5, 3, 2, 4, 1 only once. (5)
12,345

2. Write the names of the periods you would use in reading a whole number expressed in 12 digits. (6)
See below.

3. Write the words you would use in reading the numeral .0076; in reading the numeral 68.0275. (10)
See below.

Copy. Solve each equation below.

	a	*b*	*c*	
4.	$\frac{3}{6}=\frac{n}{2}$ 1	$\frac{6}{8}=\frac{n}{4}$ 3	$\frac{5}{10}=\frac{n}{2}$ 1	(15)
5.	$\frac{9}{12}=\frac{n}{4}$ 3	$\frac{6}{16}=\frac{n}{8}$ 3	$\frac{8}{12}=\frac{4}{n}$ 6	(15)
6.	$\frac{1}{2}=\frac{n}{8}$ 4	$\frac{3}{4}=\frac{n}{12}$ 9	$\frac{1}{3}=\frac{5}{n}$ 15	(16)

2. Billions, millions, thousands, ones
3. Seventy-six ten-thousandths; sixty-eight and two hundred seventy-five ten-thousandths

Self-Evaluation

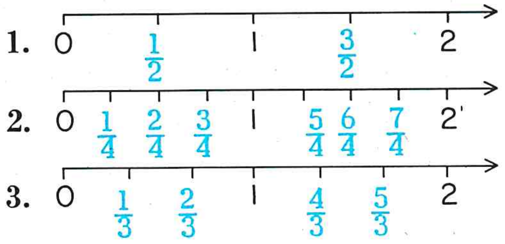
Part 1 Write each of the following as a numeral in our decimal numeration system.

1. Twenty-nine billion 29,000,000,000
2. One hundred twelve billion, three hundred million, one hundred five thousand, six hundred 112,300,105,600
3. Seven and nine tenths 7.9
4. Nine and five hundredths 9.05
5. Eight hundred-thousandths .00008

Part 2 Follow these directions.

1. Write the fraction you would use to represent 1 divided by 3. $\frac{1}{3}$
2. Write the fraction that names 3 of 4 equal parts of 1 pound. $\frac{3}{4}$
3. Write the numerator of $\frac{7}{16}$. 7
4. Write the denominator of $\frac{5}{12}$. 12
5. Write the Roman numeral used to express 45. XLV

Part 3 Copy each number line below. Supply the fractions for each point shown on the lines.



1. 0 $\frac{1}{2}$ 1 $\frac{3}{2}$ 2
2. 0 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ 1 $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ 2
3. 0 $\frac{1}{3}$ $\frac{2}{3}$ 1 $\frac{4}{3}$ $\frac{5}{3}$ 2

Part 4 Copy. Solve for n.

	a	*b*	*c*
1.	$\frac{3}{6}=\frac{n}{2}$ 1	$\frac{4}{8}=\frac{n}{2}$ 1	$\frac{3}{9}=\frac{n}{3}$ 1
2.	$\frac{2}{8}=\frac{n}{4}$ 1	$\frac{4}{6}=\frac{n}{3}$ 2	$\frac{4}{10}=\frac{n}{5}$ 2
3.	$\frac{6}{9}=\frac{n}{3}$ 2	$\frac{6}{10}=\frac{n}{5}$ 3	$\frac{6}{8}=\frac{n}{4}$ 3
4.	$\frac{2}{6}=\frac{n}{3}$ 1	$\frac{5}{10}=\frac{n}{2}$ 1	$\frac{6}{12}=\frac{n}{2}$ 1
5.	$\frac{2}{10}=\frac{n}{5}$ 1	$\frac{8}{10}=\frac{n}{5}$ 4	$\frac{9}{12}=\frac{n}{4}$ 3

Part 5 Copy. Solve for n.

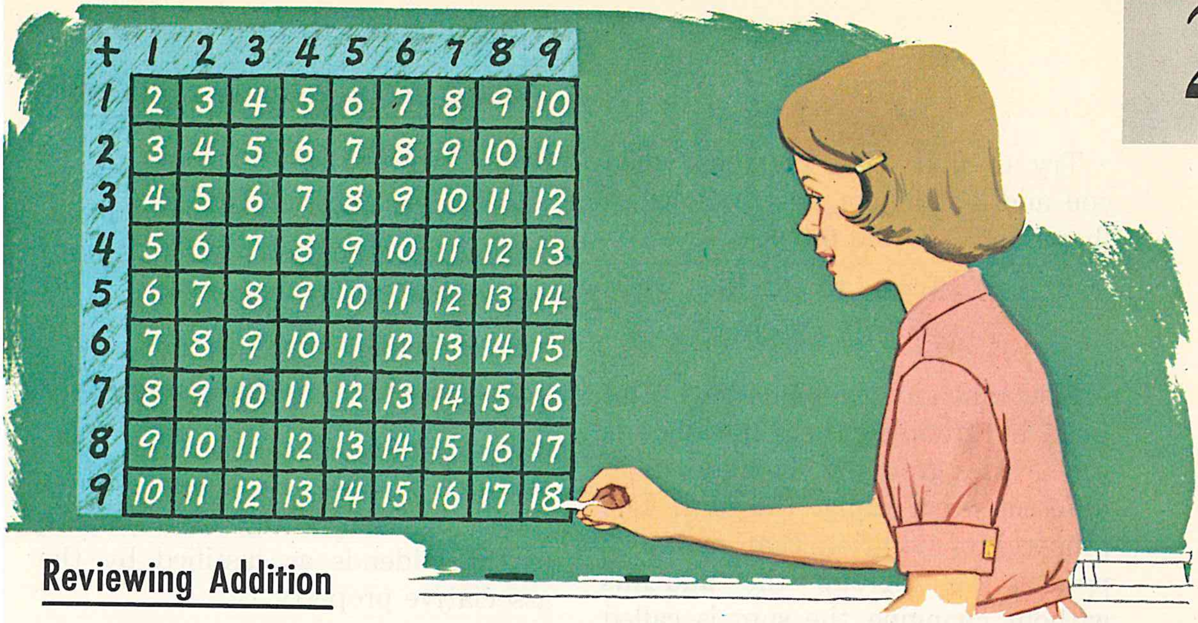
	a	*b*	*c*
1.	$\frac{1}{2}=\frac{n}{12}$ 6	$\frac{2}{5}=\frac{n}{20}$ 8	$\frac{1}{4}=\frac{n}{16}$ 4
2.	$\frac{3}{4}=\frac{9}{n}$ 12	$\frac{1}{3}=\frac{6}{n}$ 18	$\frac{3}{5}=\frac{12}{n}$ 20
3.	$\frac{2}{3}=\frac{4}{n}$ 6	$\frac{5}{6}=\frac{n}{12}$ 10	$\frac{7}{8}=\frac{14}{n}$ 16
4.	$\frac{3}{5}=\frac{n}{25}$ 15	$\frac{3}{8}=\frac{n}{16}$ 6	$\frac{9}{10}=\frac{n}{20}$ 18
5.	$\frac{2}{5}=\frac{n}{10}$ 4	$\frac{2}{3}=\frac{4}{n}$ 6	$\frac{1}{8}=\frac{5}{n}$ 40

Part 6 Write these Arabic numerals as Roman numerals.

	a	*b*	*c*	*d*
1.	9 IX	12 XII	20 XX	40 XL
2.	100 C	1000 M	550 DL	1965 MCMLXV

Write these Roman numerals as Arabic numerals.

	a	*b*	*c*
3.	XIX 19	XXVI 26	CCLVI 256
4.	MCML 1950	CM 900	DCCLXVIII 768



+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Reviewing Addition

Study the addition table. The first row was filled in by adding 1 in the left column to each number in the top row. How do you think each of the other rows was filled in?

If you know all of the basic addition combinations shown on the table and know how to regroup numbers, you can do any addition of whole numbers.

If an operation on any two numbers in a set produces a number already in the set, the set is **closed** under that operation. Is the sum of two whole numbers a whole number? The addition of whole numbers illustrates the **property of closure.**

Only two numbers were added to determine the sum for each square on the table. When you work with two numbers and get one number as the answer, you are using a **binary operation.** Thus addition is a binary operation. Add 2, 3, and 4. Did you add two numbers at a time?

On the table, find the diagonal or slanted line of 13's. Does 5+8=13? Does 8+5=13? Does 9+4=13? Does 4+9=13? Now try adding any pair of whole numbers. Then change the order of the addends and add again. The order in which you add two whole numbers makes no difference in the sum. This property is called the **commutative property** of addition.

(order)

Oral Answer these questions.

1. What are some other examples of binary operations? Subtraction, multiplication, division

2. How can you use the commutative property of addition to check addition? By adding the numbers in a different order

Adding More Than Two Addends

Try to find what happens when you add 3 numbers, 2 at a time, as shown for 7, 3, and 6 below.

$$(7+3)+6=10+6=16$$
$$7+(3+6)=7+9=16$$

Now take any three numbers. Add them by grouping them in different ways. Does grouping the addends in different ways change the sum? The property of addition that makes it possible to regroup the addends without changing the sum is called the **associative property.**

This property is used when you have more than two numbers to add. There are many ways to group or associate the addends in the equation $6+8+4+2=n$. One way is shown.

$$(6+8)+4+2=n$$

Why might it have been better to begin by associating 6 and 4 rather than 6 and 8?

Now choose any whole number. See what happens when you add it to 0 or add 0 to it. You will discover that the number obtained is identical to the number you chose whether you add zero to the number or add the number to zero.

This is why zero is called the **identity number** of addition.

MORE PRACTICE PAGE 296

Oral Answer these questions.

1. Which properties of addition do you use when you add 3 or more numbers? Associative, or associative and commutative

2. How might you use the associative property in checking the addition of more than two addends? Regroup and add again.

Written Copy. Find each sum. Check by using a different grouping of the addends as justified by the associative property.

	a	*b*
1.	$1+4+9=\overset{14}{n}$	$3+2+8=\overset{13}{n}$
2.	$2+1+9=\overset{12}{n}$	$5+4+6=\overset{15}{n}$
3.	$6+7+4=\overset{17}{n}$	$2+8+9=\overset{19}{n}$
4.	$3+1+7=\overset{11}{n}$	$4+6+1=\overset{11}{n}$
5.	$5+2+5=\overset{12}{n}$	$6+3+7=\overset{16}{n}$
6.	$5+5+4+2=\overset{16}{n}$	$4+5+2+3=\overset{14}{n}$
7.	$4+7+1+6=\overset{18}{n}$	$6+2+5+6=\overset{19}{n}$
8.	$7+9+6+1=\overset{23}{n}$	$8+6+7+5=\overset{26}{n}$
9.	$1+8+8+7=\overset{24}{n}$	$9+6+1+8=\overset{24}{n}$
10.	$7+3+3+4=\overset{17}{n}$	$8+1+9+9=\overset{27}{n}$
11.	$9+9+3+4+4=\overset{29}{n}$	
12.	$8+5+6+6+1=\overset{26}{n}$	
13.	$2+6+1+2+7=\overset{18}{n}$	

Practicing Addition

Suppose you were asked to add 6 to 52. You would probably immediately say *58* as the sum. If you weren't sure, you could add in a grid as shown in A. You can see at a glance that 2+6=8, and so 52+6=58. Which addition fact did you use? The sum is 5 tens and 8 ones or 58.

A

T	O
5	2
+	6
5	8

Now suppose you were to add 9 and 36. Which addition fact would you use? The addition is shown in grid B. 36+9=45. Will the sum of ones be greater than 9? In such cases there will be 1 more ten in the sum than in the larger addend. 36+9=45, because 6+9=15 and 1 ten+3 tens=4 tens. The sum is 4 tens and 5 ones or 45.

B

T	O
3	6
+	9
4	5

Sometimes the sum both of ones and of tens is greater than 9, as when you add 95 and 8. Study the addition in C. 95+8=103, because 5+8=13 and 1 ten+9 tens=10 tens=1 hundred and 0 tens. The sum is 1 hundred, 0 tens, and 3 ones or 103.

C

H	T	O
	9	5
	+	8
1	0	3

The addition facts may be used in adding any numbers. Tens, hundreds, thousands, and so on are added in the same way as ones.

Oral Read. Solve each equation.

	a	*b*
1.	38+4=n 42	27+6=n 33
2.	93+9=n 102	96+8=n 104

Written Copy. Write each sum. Check.

	a	*b*	*c*	*d*
1.	97 +6 103	8 +94 102	17 +62 79	46 +23 69
2.	69 +17 86	42 +29 71	97 +39 136	88 +96 184
3.	415 +31 446	60 +762 822	994 +83 1077	23 +519 542
4.	794 +17 811	56 +978 1034	985 +237 1222	484 +989 1473

Another way If you think of a single number such as 8 as representing a pair of numbers such as 3+5, you can do many additions mentally. To solve 97+8=n, think of 8 as 3+5. Then 97+8 becomes 97+(3+5). This becomes (97+3)+5 or 100+5 or 105. Use this way in solving equations in the *oral practice* lesson above.

Other Ways to Add

You can save time when adding 3 or more numbers by first looking for combinations of 10. Observe A and B below.

A $3+6+5+4+7=25$

B $7+3+6+4+5=25$

Are the same numbers added in both A and B? How have the numbers been regrouped in B?

The associative and commutative property of addition made the regrouping possible. This regrouping can be done mentally.

Suppose you are adding two-digit numbers. You can often save time by looking for combinations of 100. Thus $82+27=n$ is solved as shown below.

$$(80+2)+(20+7)=n$$
$$(80+20)+(2+7)=n$$
$$100+9=n$$
$$109=n$$

To solve the equation $87+76=n$, think of 76 as $3+73$. Then $87+76=n$ becomes $87+3+73=90+73$. Also, $90+73=90+70+3=160+3=163$, and $n=163$. How would you save time by looking for combinations of 1000 when adding 621 to 437?

MORE PRACTICE
PAGE 296

Oral Tell different ways to solve each equation below. Reminder: Make as much use as possible of the combinations of 10. See column 1.

	a		b	
1.	$8+5+7+2=n$	22	$4+6+8+7=n$	25
2.	$9+3+6+1=n$	19	$7+4+5+3=n$	19
3.	$6+2+4+5=n$	17	$1+5+5+9=n$	20
4.	$5+5+6+7=n$	23	$3+8+9+7=n$	27
5.	$2+6+9+8=n$	25	$9+3+1+9=n$	22
6.	$45+64=n$	109	$37+22=n$	59
7.	$98+36=n$	134	$76+89=n$	165

Written Copy. Write each sum. Check by using the associative property.

	a	b	c	d
1.	91 42 64 +31 228	13 11 14 +19 57	37 65 43 +76 221	26 83 56 +36 201
2.	22 12 90 +85 209	91 7 86 +42 226	984 231 323 +132 1670	468 432 987 +543 2430
3.	234 400 354 +800 1788	599 321 650 +402 1972	729 896 945 +208 2778	643 532 109 +358 1642

Adding Larger Numbers

When adding numbers with more than three digits, add the numbers in each place-value position as you add ones, tens, and hundreds.

Suppose you are to solve the equation 97,654+37,965+89,348=n. You would probably work in column form. Study the addition in the grid.

97654
37965
+89348

	HTh	TTh	Th	H	T	O
		9	7	6	5	4
		3	7	9	6	5
+		8	9	3	4	8
	2	2	4	9	6	7

After adding the ones, you change the form of 17 ones to 1 ten and 7 ones. After adding the tens, you change the form of 16 tens to 1 hundred and 6 tens. What change in form was made in thousands place? What change in form was made in ten-thousands place? The sum is 224,967.

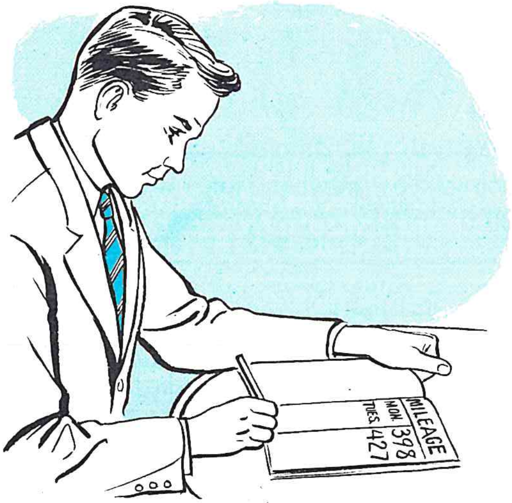
Oral Be ready to tell the steps to take in finding each sum. Wherever possible, save yourself time by looking for combinations of 10.
See above.

a	*b*	*c*
594 8614 +9276 18484	6783 4927 +9653 21363	74653 29874 +35698 140225

Written Write each sum. Check by using the associative property of addition.

	a	*b*	*c*
1.	6384 5577 7459 +3863 23283	3462 2643 6234 +9876 22215	2278 403 4095 +6045 12821
2.	4586 2583 469 326 +758 8722	2901 147 6296 108 +169 9621	3747 716 847 9413 +600 15323
3.	65327 41752 29649 13215 +4563 154506	60542 652 32700 9016 +75409 178319	5463 2000 4135 97254 +43048 151900
4.	46370 5710 9365 520 +32034 93999	3201 638 54350 475 +20843 79507	21724 63070 10935 82460 +9210 187399
5.	58000 35261 73582 90625 +9210 266678	12273 59146 46032 25309 +5432 148192	37527 84700 29348 42816 +6789 201180
6.	8476 3968 7405 1019 +2642 23510	39786 91591 27603 84932 +47685 291597	61897 88251 17973 43165 +29264 240550

Understanding Problems



The salesman wanted to know how far he had driven in two days. What information does he have that he can use? How might he solve his problem?

Any problem that can be solved by bringing two sets or collections together can be solved by adding the numbers of the sets. The salesman should add 398 and 427. The answer is 825 miles.

The following steps will help you solve a problem.

a. Read each problem carefully to determine how the statement and the question are related.

b. Decide what should be done with the sets to determine the answer. (If sets are to be joined, add the numbers of the sets.)

c. Write an equation, using a letter for the missing numeral.

d. Solve the equation to find a tentative answer.

e. Go over the problem again to see whether your tentative answer seems reasonable.

f. Write the answer to the question in the problem.

Oral Read each problem below. Translate it into an equation. Be ready to tell the class how you decided upon the equation to use.
See above.

1. A dramatics club had 109 guests at its first play, 129 guests at its second play, and 135 guests at its third play. How many guests came altogether? 109+129+135=n
373 guests

2. Miss Allen had 789 books in the library and added 350 books during the summer. How many books were then in the library? 789+350=n
1139 books

3. Mary and her father drove 419 miles one day, 398 miles the next day, and 438 miles the third day. How far did they drive in all?
419+398+438=n 1255 mi.

4. It is 713 miles by airplane from New York to Chicago; 1858 miles from Chicago to San Francisco; and 2407 miles from there to Honolulu. What is the distance between New York and Honolulu by this route?
713+1858+2407=n 4978 mi.

Written Solve problems 1 through 4.

Reviewing Subtraction

Think of any whole number. Add to it any other whole number. From the sum subtract the number you just added. You should always get the original whole number. Examples follow.

$8+5=13$ $\quad 13-5=8$
$8+6=14$ $\quad 14-6=8$
$9+0=9$ $\quad 9-0=9$
$0+9=9$ $\quad 9-9=0$

Does subtraction seem to undo what addition does? Operations that *undo* each other are called **inverse operations.** Observe that the sum obtained by adding 0 to a number is identical to the difference obtained by subtracting 0 from the sum.

Now take any whole number. Subtract it from itself. Try the experiment several times. Did you discover that subtracting a number from itself always leaves 0?

You use subtraction to answer such questions as these:

How much is left when part of a collection is removed?

How much larger or smaller is one amount than another amount?

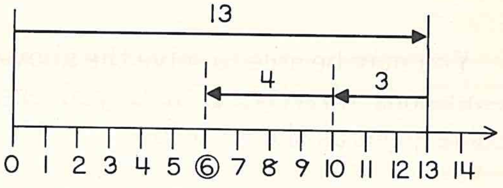
What is the difference between two numbers?

Oral Make up a problem to illustrate each question answered by subtraction. Translate your problem into an equation. Tell the class how you would solve for n.

Written Copy. Solve each equation below.

	a	*b*
1.	$17-9=\overset{8}{n}$	$\overset{24}{n}-8=16$
2.	$15-6=\overset{9}{n}$	$16-\overset{7}{n}=9$
3.	$16-7=\overset{9}{n}$	$\overset{14}{n}-8=6$
4.	$14-8=\overset{6}{n}$	$\overset{16}{n}-7=9$
5.	$15-\overset{6}{n}=9$	$17-\overset{9}{n}=8$
6.	$\overset{17}{n}-9=8$	$\overset{15}{n}-6=9$
7.	$13-\overset{6}{n}=7$	$\overset{13}{n}-7=6$

Another way Subtraction can be done in two steps as shown on the number line below.



$13-7=n \qquad (13-3)-4=6$

Something to do Make a number line similar to the one above. Show the solution of $17-9=n$. Of $15-7=n$.

Practicing Subtraction

Study A, B, and C. What steps were taken in solving for n?

A	B	C
$39-7=n$	$98-25=n$	$567-435=n$
$32=n$	$73=n$	$132=n$

In A, the ones were subtracted and the tens were left unchanged. In B, first the ones and then the tens were subtracted. What steps were taken in solving equation C?

Observe the equations below. How do the corresponding digits in each place-value position in the minuend and the subtrahend of each equation compare in number?

D	E	F
$37-9=n$	$95-28=n$	$835-567=n$

You may be able to solve the above equations directly. If not, you can solve them as shown below.

G	H	I
37 −9 28	95 −28 67	835 −567 268

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In G, the form of 3 tens and 7 ones was changed to 2 tens and 17 ones before subtracting. How was the form of 95 changed in H? How was the form of 835 changed in I?

Oral Tell how you would solve for n in each equation below. See above.

	a	*b*
1.	$49-8=n$ 41	$31-4=n$ 27
2.	$47-32=n$ 15	$75-35=n$ 40
3.	$82-20=n$ 62	$50-13=n$ 37
4.	$31-24=n$ 7	$546-82=n$ 464
5.	356 $n=435-79$	476 $n=841-365$

Written Copy. Find each difference. Use addition to check your work.

	a	*b*	*c*	*d*
1.	97 −9 88	62 −5 57	83 −8 75	45 −7 38
2.	65 −40 25	89 −67 22	42 −31 11	68 −47 21
3.	60 −21 39	83 −34 49	44 −16 28	92 −45 47
4.	875 −253 622	500 −48 452	980 −72 908	879 −24 855
5.	900 −354 546	612 −237 375	495 −367 128	723 −591 132

Subtracting Four- and Five-Digit Numbers

In subtracting larger numbers, use the same steps as those used in subtracting ones, tens, or hundreds. To illustrate, study the subtractions below. What changes in the form of each minuend were required?

A	B	C
8402	9213	65023
−326	−3875	−26415
8076	5338	38608

To check the accuracy of any subtraction, add the subtrahend and the difference. You should obtain the minuend. Can you tell why?

Oral Tell how to find the difference in each subtraction below. Tell the reason for each step. See above.

a	*b*	*c*
58729	93150	68005
−6814	−79368	−32758
51915	13782	35247

Written Copy. Write each difference. Check by using the inverse operation, addition.

	a	*b*	*c*
1.	4258 −256 4002	9613 −912 8701	5782 −591 5191
2.	7625 −870 6755	3982 −864 3118	8376 −649 7727
3.	5823 −379 5444	7605 −827 6778	9060 −314 8746
4.	2000 −548 1452	5003 −365 4638	7100 −321 6779
5.	3998 −1895 2103	6593 −2693 3900	2908 −1435 1473
6.	2834 −1867 967	9131 −4275 4856	6048 −2179 3869
7.	6054 −1396 4658	3067 −2578 489	7010 −3658 3352
8.	39679 −1265 38414	46968 −3857 43111	86890 −2960 83930
9.	49421 −9532 39889	62336 −7543 54793	72187 −4327 67860
10.	80708 −60809 19899	86307 −27002 59305	76894 −46587 30307
11.	56857 −23972 32885	65461 −20579 44882	58417 −31426 26991
12.	86093 −6185 79908	73021 −1205 71816	65837 −2416 63421
13.	38416 −3667 34749	60649 −2785 57864	27043 −1565 25478
14.	96375 −48567 47808	80000 −76945 3055	87906 −75904 12002

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Other Ways to Subtract

You can sometimes save time in subtracting if you subtract in two steps. To subtract 9 from 17, think of 9 as 7+2. Subtract 7 from 17 and 2 from the result, as shown.

$$(17-7)-2=n$$
$$10-2=n$$
$$8=n$$

Does $17-9=(17-7)-2$?

Try other subtractions in which you rename the subtrahend as a sum and subtract in two steps. Is the result the same whether you subtract in one step or in two steps?

Now study the solutions below. Decide why the subtrahend was renamed in the way chosen for it.

A	B
$45-18=n$	$100-47=n$
$45-(15+3)=n$	$100-(40+7)=n$
$(45-15)-3=n$	$(100-40)-7=n$
$30-3=n$	$60-7=n$
$27=n$	$53=n$

As the final step in each solution, a number was subtracted from a multiple of 10.

Whenever possible, subtract without using pencil and paper. In the equation $95685-45685=n$, you would simply subtract 45 thousand from 95 thousand. In the equation $97695-9999=n$ you would subtract 10,000 and add 1 to the difference. Can you think of a short way to subtract 97 from 101? 996 from 10,000?

Oral Tell the steps you would take in solving each equation below. See column 1.

1. $47-19=n$ 28
2. $1000-985=n$ 15
3. $65319-31319=n$ 34000
4. $73417-29000=n$ 44417
5. $25937-4537=n$ 21400
6. $10540-9240=n$ 1300
7. $93000-6375=n$ 86625
8. $25732-3732=n$ 22000
9. $58416-49000=n$ 9416

Written Solve each equation above.

Can you do this? Reconstruct the subtractions below by replacing each dot with a numeral. There is only one possible solution for each one.

a

$$\begin{array}{r} \bullet 3 \bullet \\ -\bullet\bullet 4 \\ \hline 765 \end{array} \quad \begin{array}{r} 939 \\ \underline{174} \end{array}$$

b

$$\begin{array}{r} \bullet\bullet 1 \\ -34\bullet \\ \hline 59 \end{array} \quad \begin{array}{r} 401 \\ \underline{-342} \end{array}$$

Understanding Problems

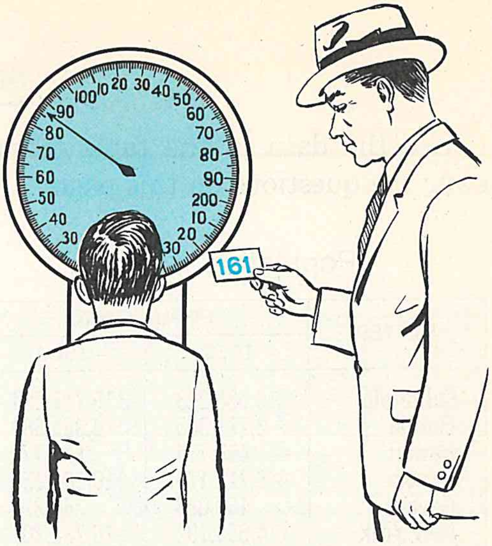
Each problem below tells you the number of two different sets. In some of the problems, the question can be answered by removing objects or units from a set. In others the question can be answered by finding by how many the number of one set is greater than that of the other. Both kinds of questions are answered by using subtraction.

To solve each problem, read it carefully and try to picture the situation. Decide upon the operation to use and express the problem as an equation. Solve the equation to determine a tentative answer. Read the problem again to see whether the answer seems reasonable. Check your computation. Then write the complete answer.

Oral Translate each problem below into an equation and tell why you decided as you did.

1. A parking lot at a shopping center had spaces for 1950 cars. A carnival on the lot took up 275 of the spaces. How many spaces were left for cars? 1950-275=n 1675 spaces

2. Tom collected 85 seashells and placed 59 of them in a showcase. How many of the shells were not placed in the showcase? 85-59=n 26 shells



3. Dick's uncle weighed 161 pounds; Dick weighed 87 pounds. How much less did Dick weigh than his uncle? 161-87=n 74 lb.

4. Jack's father has driven his car 11,058 miles and Bob's father has driven his car 9995 miles. How many more miles were driven by Jack's father than by Bob's father? 11,058-9995=n 1063 mi.

5. Ann had 171 stamps. She needed 500 stamps to complete her stamp collection. How many more stamps did she need to make her collection complete? 500-171=n 329 stamps

6. A filling station operator sold 1150 gallons of gas on a weekend. 895 gallons were regular and the rest was premium gas. How many gallons of premium gas did he sell? 1150-895=n 255 gal.

Written Solve the problems. Use the suggestions on this page.

Adding and Subtracting Larger Numbers

Use the data in the table to answer the questions on this page.

Population Table

STATES	POPULATION	
	1950	1960
California	2 10,586,223	2 15,717,204
Florida	7 2,771,305	6 4,951,560
Hawaii	8 499,794	8 632,772
Illinois	4 8,712,176	4 10,081,158
Nevada	10 160,083	10 285,278
New York	1 14,830,192	1 16,782,304
Ohio	5 7,946,627	5 9,706,397
Pennsylvania	3 10,498,012	3 11,319,366
Wisconsin	6 3,434,575	7 3,951,777
Wyoming	9 290,529	9 330,066

Oral Answer these questions.

1. In what order could you arrange the states so as to have the state with the largest population first and the one with the smallest population last according to the 1950 census? According to the 1960 census?
See population table above.

2. Which state made the greatest population gain in 10 years?
California

3. How many states had a gain of over 100,000? Of over 500,000?
9; 7

4. What is the population difference between Illinois and New York?
1950:6,118,016 1960:6,701,146

5. Which state made a population gain of more than 2 million but less than 3 million? Florida

Written Follow these directions.

1. Copy the population table, adding a fourth column called *Increase*. Complete the table by determining the population increase for each state. See below.

2. Find the total population of New York, Pennsylvania, Illinois, and California in 1950.
44,626,603

3. Find the total population of New York, Pennsylvania, Illinois, and California in 1960.
53,900,032

4. Find the total population of the six states in this table having the greatest population in 1960.
68,557,989

5. Find the total population of the six states in this table having the smallest population in 1960.
19,857,850

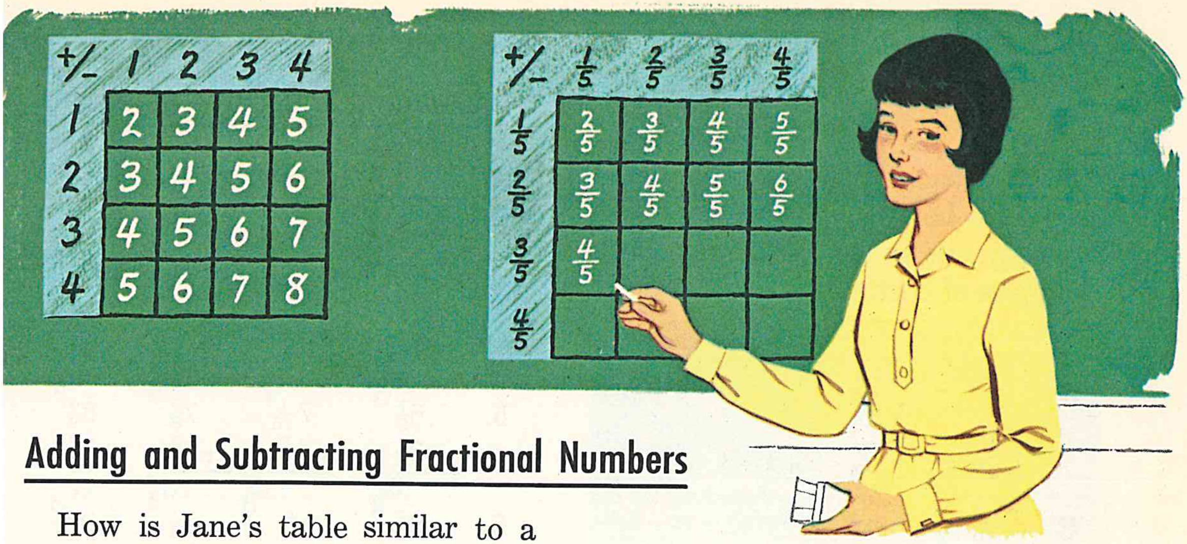
Can you do this? Use a reference book to help you answer these questions.

1. What is the difference between the populations of the United States and Canada?

2. What is the difference between the populations of the United States and Great Britain?

3. What is the difference between the populations of the United States and India?

1. Calif., 5,130,981; Fla., 2,180,255; Hawaii, 132,978; Ill., 1,368,982; Nev., 125,195; N.Y., 1,952,112; Ohio, 1,759,770; Penn., 821,354; Wisc., 517,202; Wyo., 39,537



+/−	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

+/−	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$	$\frac{6}{5}$
$\frac{3}{5}$	$\frac{4}{5}$			
$\frac{4}{5}$				

Adding and Subtracting Fractional Numbers

How is Jane's table similar to a table she might make for showing the addition and subtraction of whole numbers? Which fractions are needed to complete the table? Reminder: You add the numbers named by the numerators and use the common denominator.

The addition and subtraction operations are inverse operations for fractional numbers as well as for whole numbers. Changing the order of the addend has no effect on the sum of any pair of whole or of fractional numbers. Study examples A and B below.

A $\frac{2}{5}+\frac{1}{5}=\frac{2+1}{5}=\frac{3}{5}$ $\qquad$ $\frac{3}{5}-\frac{1}{5}=\frac{3-1}{5}=\frac{2}{5}$

B $\frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}=\frac{4}{5}$ $\qquad$ $\frac{1}{5}+\frac{3}{5}=\frac{1+3}{5}=\frac{4}{5}$

Does the commutative property of addition apply to the addition of fractional numbers?

Study examples C and D. Observe the order of the addends and note how they are grouped. The associative property of addition applies to the addition of fractional numbers.

C $\frac{2}{5}+\frac{3}{5}+\frac{4}{5}=\frac{(2+3)+4}{5}=\frac{5+4}{5}=\frac{9}{5}=1\frac{4}{5}$

D $\frac{2}{5}+\frac{3}{5}+\frac{4}{5}=\frac{2+(3+4)}{5}=\frac{2+7}{5}=\frac{9}{5}=1\frac{4}{5}$

Oral Tell how to solve each equation below.

	a	*b*	*c*
1.	$\frac{2}{5}+\frac{2}{5}=n$ $\frac{4}{5}$	$\frac{7}{10}+\frac{1}{10}=n$ $\frac{4}{5}$	$\frac{1}{7}+\frac{3}{7}=n$ $\frac{4}{7}$
2.	$\frac{1}{6}+\frac{1}{6}=n$ $\frac{1}{3}$	$\frac{2}{9}+n=\frac{6}{9}$ $\frac{4}{9}$	$n+\frac{2}{5}=\frac{4}{5}$ $\frac{2}{5}$
3.	$n+\frac{3}{7}=\frac{4}{7}$ $\frac{1}{7}$	$n+\frac{1}{10}=\frac{8}{10}$ $\frac{7}{10}$	$\frac{2}{5}+n=\frac{4}{5}$ $\frac{2}{5}$
4.	$\frac{6}{7}-\frac{4}{7}=n$ $\frac{2}{7}$	$\frac{7}{8}-\frac{3}{8}=n$ $\frac{1}{2}$	$\frac{7}{8}-\frac{5}{8}=n$ $\frac{1}{4}$
5.	$\frac{3}{5}-\frac{2}{5}=n$ $\frac{1}{5}$	$\frac{7}{8}-n=\frac{4}{8}$ $\frac{3}{8}$	$\frac{6}{7}-n=\frac{2}{7}$ $\frac{4}{7}$

Written Solve the above equations.

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Changing the Form of a Sum

Study each pair of completed additions below. Observe that the same *fractions* are used in both examples of each pair.

A

$$\begin{array}{r} \frac{1}{4} \\ +\frac{3}{4} \\ \hline \frac{4}{4}=1 \end{array} \qquad \begin{array}{r} 2\frac{1}{4} \\ +\frac{3}{4} \\ \hline 2\frac{4}{4}=2+1=3 \end{array}$$

B

$$\begin{array}{r} \frac{7}{10} \\ +\frac{4}{10} \\ \hline \frac{11}{10}=1\frac{1}{10} \end{array} \qquad \begin{array}{r} 3\frac{7}{10} \\ +2\frac{4}{10} \\ \hline 5\frac{11}{10}=5+1\frac{1}{10}=6\frac{1}{10} \end{array}$$

C

$$\begin{array}{r} \frac{3}{8} \\ +\frac{7}{8} \\ \hline \frac{10}{8}=1\frac{1}{4} \end{array} \qquad \begin{array}{l} \quad 4\frac{3}{8} \\ +7\frac{7}{8} \\ \hline 11\frac{10}{8}=11+1\frac{2}{8}=11+1\frac{1}{4} \\ \qquad =12\frac{1}{4} \end{array}$$

When the sum of the fractional numbers has a value of 1 or more the form of the sum can be changed. The whole number you obtain by this change is then added to the whole numbers already present.

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Oral Solve for n.

	a	*b*
1.	$\frac{1}{7}+\frac{6}{7}=n$ 1	$\frac{5}{7}+\frac{3}{7}=n$ $1\frac{1}{7}$
2.	$\frac{3}{4}+\frac{3}{4}=n$ $1\frac{1}{2}$	$1\frac{1}{3}+\frac{2}{3}=n$ 2
3.	$3\frac{5}{6}+2\frac{2}{6}=n$ $6\frac{1}{6}$	$5\frac{3}{4}+2\frac{3}{4}=n$ $8\frac{1}{2}$

Written Copy. Solve each equation in rows 1 through 3 above. Then copy and write the sum for each addition in rows 4 through 7.

	a	*b*	*c*	*d*
4.	$\frac{1}{5}$ $+3\frac{2}{5}$ $3\frac{3}{5}$	$8\frac{3}{7}$ $+9\frac{3}{7}$ $17\frac{6}{7}$	$3\frac{1}{5}$ $+7\frac{3}{5}$ $10\frac{4}{5}$	$4\frac{2}{9}$ $+5\frac{5}{9}$ $9\frac{7}{9}$
5.	$5\frac{1}{8}$ $+3\frac{1}{8}$ $8\frac{1}{4}$	$7\frac{1}{16}$ $+4\frac{9}{16}$ $11\frac{5}{8}$	$7\frac{3}{8}$ $+9\frac{3}{8}$ $16\frac{3}{4}$	$8\frac{4}{9}$ $+2\frac{2}{9}$ $10\frac{2}{3}$
6.	$3\frac{2}{9}$ $+4\frac{7}{9}$ 8	$\frac{4}{7}$ $+9\frac{3}{7}$ 10	$4\frac{1}{3}$ $+8\frac{2}{3}$ 13	$9\frac{7}{12}$ $+4\frac{5}{12}$ 14
7.	$2\frac{8}{15}$ $1\frac{1}{15}$ $+9\frac{11}{15}$ $13\frac{1}{3}$	$5\frac{11}{18}$ $6\frac{5}{18}$ $+7\frac{5}{18}$ $19\frac{1}{6}$	$7\frac{5}{9}$ $5\frac{8}{9}$ $+3\frac{8}{9}$ $17\frac{1}{3}$	$4\frac{9}{10}$ $2\frac{7}{10}$ $+5\frac{9}{10}$ $13\frac{1}{2}$

Another way Observe the solution of the equation $3\frac{3}{8}+5\frac{7}{8}=n$ in long form and in abbreviated form below.

$3\frac{3}{8}+5\frac{7}{8}=n$

$(3+\frac{3}{8})+(5+\frac{7}{8})=n$

$(3+5)+(\frac{3}{8}+\frac{7}{8})=n$

$8+\frac{3+7}{8}=n$

$8+\frac{10}{8}=n$

$(8+1)+\frac{2}{8}=n$

$9+\frac{2}{8}=n$

$9+\frac{1}{4}=n$

$9\frac{1}{4}=n$

$3\frac{3}{8}+5\frac{7}{8}=n$

$(3+5)+(\frac{3}{8}+\frac{7}{8})=n$

$8+\frac{10}{8}=n$

$8+1+\frac{2}{8}=n$

$9+\frac{1}{4}=n$

$9\frac{1}{4}=n$

Copy the additions in rows 4 through 7 in equation form. Solve each equation.

Changing to a Common Denominator

Look at the fractions $\frac{1}{8}$ and $\frac{1}{4}$. Are the denominators alike? Now rename $\frac{1}{4}$ as $\frac{2}{8}$. The fractions $\frac{1}{8}$ and $\frac{2}{8}$ have the same denominator, 8. It is called the **common denominator** because it is common to both fractions.

Any fractional number can be expressed by many fractions, as shown below.

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{7}{14}=\frac{8}{16}=\cdots$$

$$\frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{4}{16}=\frac{5}{20}=\frac{6}{24}=\frac{7}{28}=\frac{8}{32}=\cdots$$

$$\frac{1}{8}=\frac{2}{16}=\frac{3}{24}=\frac{4}{32}=\frac{5}{40}=\frac{6}{48}=\frac{7}{56}=\frac{8}{64}=\cdots$$

The dots after each row indicate that you could go on and on and on. Rows such as those above are helpful in finding a common denominator for two or more fractions. Is 4 the smallest denominator common to $\frac{1}{2}$ and $\frac{1}{4}$? What is the smallest denom-nator common to $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$? Did you say 8?

If the numbers named by the de-ominator of two fractions are such hat one number is divisible by the ther, the larger number is the small-st common denominator. For ex-mple, 8 is divisible by 4 so 8 is the nallest common denominator for $\frac{3}{4}$ nd $\frac{5}{8}$. What is the common denomi-ator of $\frac{2}{3}$ and $\frac{5}{9}$? Of $\frac{5}{6}$ and $\frac{11}{12}$? Of $\frac{3}{8}$ nd $\frac{5}{24}$?

Oral Tell the steps you would take in solving each equation below.

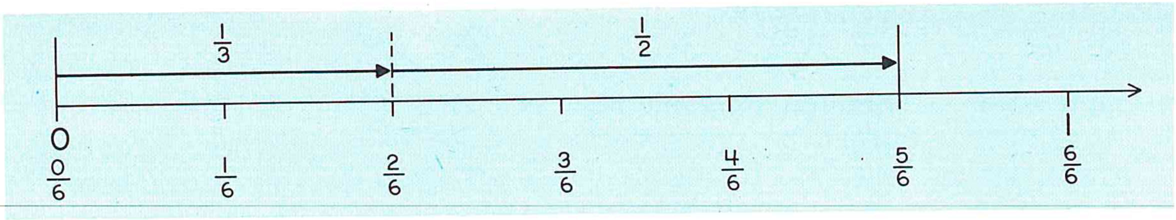
a	*b*	*c*
$\frac{1}{6}+\frac{1}{3}=n$ $\frac{1}{2}$	$\frac{3}{4}+\frac{1}{2}=n$ $1\frac{1}{4}$	$\frac{5}{12}+\frac{5}{6}+\frac{3}{4}=n$ 2

Written Copy. Write each sum. Check by using either the commutative or the associative property.

	a	*b*	*c*	*d*	*e*
1.	$\frac{3}{5}$ $+\frac{1}{10}$ $\frac{7}{10}$	$\frac{1}{12}$ $+\frac{1}{3}$ $\frac{5}{12}$	$\frac{2}{3}$ $+\frac{1}{6}$ $\frac{5}{6}$	$\frac{3}{5}$ $+\frac{3}{10}$ $\frac{9}{10}$	$\frac{1}{16}$ $+\frac{1}{2}$ $\frac{9}{16}$
2.	$\frac{1}{2}$ $+\frac{1}{14}$ $\frac{4}{7}$	$\frac{1}{10}$ $+\frac{1}{2}$ $\frac{3}{5}$	$\frac{1}{5}$ $+\frac{7}{15}$ $\frac{2}{3}$	$\frac{7}{15}$ $+\frac{1}{3}$ $\frac{4}{5}$	$\frac{5}{12}$ $+\frac{1}{3}$ $\frac{3}{4}$
3.	$\frac{7}{9}$ $+\frac{1}{3}$ $1\frac{1}{9}$	$\frac{1}{2}$ $+\frac{5}{8}$ $1\frac{1}{8}$	$\frac{7}{8}$ $+\frac{3}{4}$ $1\frac{5}{8}$	$\frac{9}{16}$ $+\frac{7}{8}$ $1\frac{7}{16}$	$\frac{8}{9}$ $+\frac{2}{3}$ $1\frac{5}{9}$
4.	$\frac{9}{10}$ $+\frac{1}{2}$ $1\frac{2}{5}$	$\frac{2}{3}$ $+\frac{11}{15}$ $1\frac{2}{5}$	$\frac{9}{20}$ $+\frac{4}{5}$ $1\frac{1}{4}$	$\frac{1}{4}$ $+\frac{11}{12}$ $1\frac{1}{6}$	$\frac{13}{15}$ $+\frac{1}{3}$ $1\frac{1}{5}$
5.	$\frac{1}{6}$ $\frac{1}{2}$ $+\frac{1}{3}$ 1	$\frac{5}{12}$ $\frac{1}{3}$ $+\frac{1}{4}$ 1	$\frac{1}{3}$ $\frac{4}{15}$ $+\frac{1}{5}$ $\frac{4}{5}$	$\frac{1}{4}$ $\frac{1}{12}$ $+\frac{1}{6}$ $\frac{1}{2}$	$\frac{2}{15}$ $\frac{1}{5}$ $+\frac{1}{3}$ $\frac{2}{3}$

Copy in column form. Solve each equation.

	a	*b*
6.	$4\frac{1}{8}+\frac{5}{16}=n$ $4\frac{7}{16}$	$6\frac{3}{10}+\frac{3}{5}=n$ $6\frac{9}{10}$
7.	$1\frac{1}{2}+8\frac{7}{10}=n$ $10\frac{1}{5}$	$6\frac{7}{8}+5\frac{1}{2}=n$ $12\frac{3}{8}$
8.	$8\frac{3}{4}+2\frac{1}{2}+\frac{1}{12}=n$ $11\frac{1}{3}$	$9\frac{3}{5}+3\frac{3}{10}+2\frac{1}{2}=n$ $15\frac{2}{5}$
9.	$7\frac{1}{6}+2\frac{1}{3}+\frac{1}{12}=n$ $9\frac{7}{12}$	$4\frac{1}{8}+\frac{3}{4}+2\frac{1}{2}=n$ $7\frac{3}{8}$



Changing to the Least Common Denominator

Look at the illustration above. Is $\frac{1}{3}$ equal in length to $\frac{1}{6}+\frac{1}{6}$? Is $\frac{1}{2}$ equal to $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$?

You can add $\frac{1}{3}$ to $\frac{1}{2}$ by renaming $\frac{1}{3}$ as $\frac{2}{6}$ and $\frac{1}{2}$ as $\frac{3}{6}$. $\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$.

When you renamed $\frac{1}{2}$ and $\frac{1}{3}$ as sixths, you were using the **least common denominator.** The least common denominator of two or more fractions is the smallest number divisible by all of the denominators. What is the smallest number divisible by 3 and by 4? By 5 and by 2?

A good way to find the least common denominator of several fractions is to rename them as shown below. What are some other names for $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$?

$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{7}{14}=\frac{8}{16}= \ldots$

$\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}=\frac{7}{21}=\frac{8}{24}= \ldots$

$\frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{4}{16}=\frac{5}{20}=\frac{6}{24}=\frac{7}{28}=\frac{8}{32}= \ldots$

What is the smallest number that appears as a denominator in all 3 rows? Is it 12? 12 is the least common denominator of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. What is the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$? Of $\frac{1}{3}$ and $\frac{1}{4}$?

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Oral Tell how you would solve each equation below. See column 1.

a	*b*	*c*
$\frac{3}{4}+\frac{1}{3}=n$ $1\frac{1}{12}$	$\frac{1}{2}+\frac{3}{4}+\frac{5}{6}=n$ $2\frac{1}{12}$	$1\frac{3}{4}+1\frac{5}{6}=n$ $3\frac{7}{12}$

Written Copy. Write each sum.

	a	*b*	*c*	*d*
1.	$\begin{array}{r}\frac{5}{6}\\+\frac{1}{4}\\\hline 1\frac{1}{12}\end{array}$	$\begin{array}{r}\frac{1}{2}\\+\frac{5}{7}\\\hline 1\frac{3}{14}\end{array}$	$\begin{array}{r}\frac{3}{5}\\+\frac{1}{2}\\\hline 1\frac{1}{10}\end{array}$	$\begin{array}{r}\frac{2}{3}\\+\frac{3}{4}\\\hline 1\frac{5}{12}\end{array}$
2.	$\begin{array}{r}\frac{2}{3}\\+\frac{1}{4}\\\hline \frac{11}{12}\end{array}$	$\begin{array}{r}\frac{3}{4}\\+\frac{1}{6}\\\hline \frac{11}{12}\end{array}$	$\begin{array}{r}\frac{1}{3}\\+\frac{1}{4}\\\hline \frac{7}{12}\end{array}$	$\begin{array}{r}\frac{1}{2}\\+\frac{1}{5}\\\hline \frac{7}{10}\end{array}$
3.	$\begin{array}{r}1\frac{1}{4}\\\frac{5}{6}\\+\frac{3}{4}\\\hline 2\frac{5}{6}\end{array}$	$\begin{array}{r}\frac{1}{8}\\\frac{1}{3}\\+5\frac{7}{8}\\\hline 6\frac{1}{3}\end{array}$	$\begin{array}{r}7\frac{3}{4}\\\frac{5}{6}\\+\frac{1}{6}\\\hline 8\frac{3}{4}\end{array}$	$\begin{array}{r}\frac{1}{8}\\4\frac{2}{3}\\+\frac{1}{2}\\\hline 5\frac{7}{24}\end{array}$
4.	$\begin{array}{r}\frac{5}{6}\\1\frac{2}{3}\\+3\frac{3}{4}\\\hline 6\frac{1}{4}\end{array}$	$\begin{array}{r}5\frac{3}{8}\\6\frac{1}{6}\\+\frac{3}{4}\\\hline 12\frac{7}{24}\end{array}$	$\begin{array}{r}2\frac{1}{2}\\1\frac{3}{4}\\+2\frac{2}{3}\\\hline 6\frac{11}{12}\end{array}$	$\begin{array}{r}8\\6\frac{2}{3}\\+\frac{1}{2}\\\hline 15\frac{1}{6}\end{array}$
5.	$\begin{array}{r}4\frac{1}{6}\\+8\frac{9}{10}\\\hline 13\frac{1}{15}\end{array}$	$\begin{array}{r}3\frac{2}{3}\\+4\frac{2}{5}\\\hline 8\frac{1}{15}\end{array}$	$\begin{array}{r}7\frac{1}{3}\\+5\frac{1}{4}\\\hline 12\frac{7}{12}\end{array}$	$\begin{array}{r}4\frac{1}{2}\\+3\frac{2}{7}\\\hline 7\frac{11}{14}\end{array}$
6.	$\begin{array}{r}9\frac{3}{4}\\+5\frac{1}{6}\\\hline 14\frac{11}{12}\end{array}$	$\begin{array}{r}3\frac{7}{12}\\+7\frac{3}{8}\\\hline 10\frac{23}{24}\end{array}$	$\begin{array}{r}7\frac{2}{3}\\+5\frac{1}{8}\\\hline 12\frac{19}{24}\end{array}$	$\begin{array}{r}1\frac{1}{4}\\+3\frac{3}{5}\\\hline 4\frac{17}{20}\end{array}$
7.	$\begin{array}{r}9\frac{3}{10}\\7\frac{1}{3}\\+6\frac{3}{5}\\\hline 23\frac{7}{30}\end{array}$	$\begin{array}{r}4\frac{1}{6}\\3\frac{3}{4}\\+7\frac{2}{3}\\\hline 15\frac{7}{12}\end{array}$	$\begin{array}{r}6\frac{3}{8}\\9\frac{2}{3}\\+3\frac{1}{12}\\\hline 19\frac{1}{8}\end{array}$	$\begin{array}{r}6\frac{1}{4}\\4\frac{3}{10}\\+9\frac{3}{5}\\\hline 20\frac{3}{20}\end{array}$

Kathy
Carlton

Subtracting Fractional Numbers

Suppose you are to solve the equation $3\frac{2}{3}-\frac{1}{3}=n$. Can you subtract $\frac{1}{3}$ from $\frac{2}{3}$? There are no ones to subtract from 3 ones; so leave it unchanged. $3\frac{2}{3}-\frac{1}{3}=3\frac{1}{3}$.

To subtract $\frac{1}{4}$ from 1, as in $1-\frac{1}{4}=n$, express 1 as $\frac{4}{4}$. Does $\frac{4}{4}-\frac{1}{4}=\frac{3}{4}$?

In column form the subtraction of $\frac{1}{4}$ from 1 may be done as shown at the right. Are any ones left to subtract?

$$\begin{array}{rcl} 1 & = & \frac{4}{4} \\ -\frac{1}{4} & = & \frac{1}{4} \\ \hline & & \frac{3}{4} \end{array}$$

Study the subtractions in column form below.

$$\textbf{A}\quad \begin{array}{rcl} 9 & = & 8\frac{6}{6} \\ -2\frac{5}{6} & = & 2\frac{5}{6} \\ \hline & & 6\frac{1}{6} \end{array} \qquad \textbf{B}\quad \begin{array}{rcl} 5\frac{2}{9} & = & 4\frac{11}{9} \\ -1\frac{8}{9} & = & 1\frac{8}{9} \\ \hline & & 3\frac{3}{9}=3\frac{1}{3} \end{array}$$

In A, 9 ones is expressed as $8\frac{6}{6}$. How was $5\frac{2}{9}$ expressed in B?

Study the subtraction below.

$$\begin{array}{rcccl} 8\frac{1}{2} & = & 8\frac{3}{6} & = & 7\frac{9}{6} \\ -4\frac{5}{6} & = & 4\frac{5}{6} & = & 4\frac{5}{6} \\ \hline & & & & 3\frac{4}{6}=3\frac{2}{3} \end{array}$$

he minuend and the subtrahend ere renamed so that the fractions ad a common denominator. Why as $8\frac{3}{6}$ then expressed as $7\frac{9}{6}$?

Oral Tell the steps you would take in solving each equation below.

	a	*b*
1.	$1\frac{9}{10}-\frac{7}{10}=n$ $1\frac{1}{5}$	$2\frac{1}{7}-1\frac{3}{7}=n$ $\frac{5}{7}$
2.	$\frac{5}{6}-\frac{1}{3}=n$ $\frac{1}{2}$	$4\frac{1}{4}-2\frac{1}{2}=n$ $1\frac{3}{4}$

Written Copy. Solve each equation.

	a	*b*
1.	$16\frac{7}{18}-7\frac{5}{18}=n$ $9\frac{1}{9}$	$11\frac{13}{16}-2\frac{3}{16}=n$ $9\frac{5}{8}$
2.	$8\frac{1}{16}-7\frac{9}{16}=n$ $\frac{1}{2}$	$6\frac{1}{12}-5\frac{5}{12}=n$ $\frac{2}{3}$
3.	$5\frac{2}{12}-2\frac{13}{15}=n$ $2\frac{3}{10}$	$10\frac{5}{9}-3\frac{7}{9}=n$ $6\frac{7}{9}$
4.	$18\frac{1}{12}-7\frac{7}{12}=n$ $10\frac{1}{2}$	$24\frac{11}{16}-20\frac{15}{16}=n$ $3\frac{3}{4}$
5.	$8\frac{4}{5}-2\frac{7}{10}=n$ $6\frac{1}{10}$	$9\frac{5}{6}-6\frac{1}{2}=n$ $3\frac{1}{3}$
6.	$16\frac{5}{6}-14\frac{7}{12}=n$ $2\frac{1}{4}$	$8\frac{7}{12}-1\frac{1}{3}=n$ $7\frac{1}{4}$
7.	$5\frac{5}{6}-2\frac{1}{12}=n$ $3\frac{3}{4}$	$7\frac{11}{15}-3\frac{2}{5}=n$ $4\frac{1}{3}$
8.	$67\frac{3}{8}-66\frac{3}{4}=n$ $\frac{5}{8}$	$36\frac{1}{2}-12\frac{5}{8}=n$ $23\frac{7}{8}$

Copy. Write each difference.

	a	*b*	*c*	*d*
9.	$\begin{array}{r} 6\frac{3}{8} \\ -5\frac{1}{4} \\ \hline 1\frac{1}{8} \end{array}$	$\begin{array}{r} 8\frac{5}{8} \\ -7\frac{7}{16} \\ \hline 1\frac{3}{16} \end{array}$	$\begin{array}{r} 4\frac{1}{3} \\ -2\frac{1}{15} \\ \hline 2\frac{4}{15} \end{array}$	$\begin{array}{r} 3\frac{1}{2} \\ -1\frac{1}{8} \\ \hline 2\frac{3}{8} \end{array}$
10.	$\begin{array}{rl} 47\frac{2}{3} & \frac{4}{6} \\ -39\frac{1}{6} & \frac{1}{6} \\ \hline 8\frac{1}{2} & \end{array}$	$\begin{array}{rl} 18\frac{4}{5} & \frac{8}{10} \\ -4\frac{3}{10} & \frac{3}{10} \\ \hline 14\frac{1}{2} & \end{array}$	$\begin{array}{rl} 8\frac{11}{18} & \frac{11}{18} \\ -4\frac{1}{9} & \frac{2}{18} \\ \hline 4\frac{1}{2} & \end{array}$	$\begin{array}{rl} 6\frac{19}{20} & \frac{19}{20} \\ -5\frac{3}{4} & \frac{15}{20} \\ \hline 1\frac{1}{5} & \end{array}$
11.	$\begin{array}{rl} 61\frac{3}{10} & \frac{3}{10} \\ -38\frac{2}{5} & \frac{4}{10} \\ \hline 22\frac{9}{10} & \end{array}$	$\begin{array}{rl} 17\frac{1}{9} & \frac{3}{27} \\ -16\frac{2}{3} & \frac{18}{27} \\ \hline \frac{4}{9} & \end{array}$	$\begin{array}{rl} 37\frac{2}{5} & \frac{6}{15} \\ -29\frac{7}{15} & \frac{7}{15} \\ \hline 7\frac{14}{15} & \end{array}$	$\begin{array}{rl} 75\frac{1}{4} & \frac{2}{8} \\ -57\frac{3}{8} & \frac{3}{8} \\ \hline 17\frac{7}{8} & \end{array}$

Using the Least Common Denominator in Subtracting

Which numeral would you use to express the least common denominator of $\frac{3}{4}$ and $\frac{2}{3}$? Of $\frac{3}{10}$ and $\frac{1}{6}$? Of $\frac{1}{6}$ and $\frac{4}{15}$?

Study the solution of $\frac{3}{4}-\frac{2}{3}=n$ and of $\frac{3}{10}-\frac{1}{6}=n$ shown in A and B.

A
$$\begin{array}{r} \frac{3}{4}=\frac{9}{12} \\ -\frac{2}{3}=\frac{8}{12} \\ \hline \frac{1}{12} \end{array}$$

B
$$\begin{array}{r} \frac{3}{10}=\frac{9}{30} \\ -\frac{1}{6}=\frac{5}{30} \\ \hline \frac{4}{30}=\frac{2}{15} \end{array}$$

The fractions in A and B were renamed before the subtraction was done. Each difference is expressed by a fraction in lowest terms.

Oral Tell the steps taken in solving the equations $5\frac{1}{3}-\frac{1}{2}=n$ and $6\frac{1}{4}-3\frac{1}{3}=n$ as shown below. See above.

1. $$\begin{array}{r} 5\frac{1}{3}=5\frac{2}{6}=4\frac{8}{6} \\ -\frac{1}{2}=\ \frac{3}{6}=\ \frac{3}{6} \\ \hline 4\frac{5}{6} \end{array}$$

2. $$\begin{array}{r} 6\frac{1}{4}=6\frac{3}{12}=5\frac{15}{12} \\ -3\frac{1}{3}=3\frac{4}{12}=3\frac{4}{12} \\ \hline 2\frac{11}{12} \end{array}$$

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Written Copy. Solve each equation.

	a	*b*	*c*
1.	$\frac{1}{4}-\frac{1}{5}=n$ $\frac{1}{20}$	$\frac{1}{2}-\frac{1}{5}=n$ $\frac{3}{10}$	$\frac{4}{5}-\frac{1}{2}=n$ $\frac{3}{10}$
2.	$\frac{7}{8}-\frac{2}{3}=n$ $\frac{5}{24}$	$\frac{2}{3}-\frac{1}{2}=n$ $\frac{1}{6}$	$\frac{1}{2}-\frac{1}{3}=n$ $\frac{1}{6}$
3.	$\frac{1}{2}-\frac{2}{5}=n$ $\frac{1}{10}$	$\frac{3}{4}-\frac{3}{10}=n$ $\frac{9}{20}$	$\frac{4}{5}-\frac{2}{3}=n$ $\frac{2}{15}$
4.	$\frac{4}{5}-\frac{3}{4}=n$ $\frac{1}{20}$	$\frac{3}{4}-\frac{1}{6}=n$ $\frac{7}{12}$	$\frac{5}{8}-\frac{1}{3}=n$ $\frac{7}{24}$
5.	$\frac{1}{3}-\frac{1}{4}=n$ $\frac{1}{12}$	$\frac{2}{3}-\frac{3}{5}=n$ $\frac{1}{15}$	$\frac{3}{8}-\frac{1}{3}=n$ $\frac{1}{24}$
6.	$\frac{4}{5}-\frac{1}{3}=n$ $\frac{7}{15}$	$\frac{3}{8}-\frac{1}{6}=n$ $\frac{5}{24}$	$\frac{1}{2}-\frac{3}{7}=n$ $\frac{1}{14}$
7.	$\frac{7}{15}-\frac{1}{6}=n$ $\frac{3}{10}$	$\frac{5}{6}-\frac{8}{15}=n$ $\frac{3}{10}$	$\frac{9}{10}-\frac{1}{6}=n$ $\frac{11}{15}$

Copy. Find each difference.

	a	*b*	*c*	*d*
8.	$1\frac{1}{4}$ $-\frac{3}{5}$ $\frac{13}{20}$	$1\frac{1}{10}$ $-\frac{5}{6}$ $\frac{4}{15}$	$1\frac{2}{15}$ $-\frac{5}{6}$ $\frac{3}{10}$	$1\frac{1}{6}$ $-\frac{7}{10}$ $\frac{7}{15}$
9.	$73\frac{1}{6}$ $-59\frac{4}{15}$ $13\frac{9}{10}$	$7\frac{1}{15}$ $-2\frac{1}{6}$ $4\frac{9}{10}$	$11\frac{1}{15}$ $-3\frac{1}{6}$ $7\frac{9}{10}$	$7\frac{5}{6}$ $-4\frac{9}{10}$ $2\frac{14}{15}$
10.	$9\frac{1}{8}$ $-8\frac{2}{3}$ $\frac{11}{24}$	$41\frac{2}{3}$ $-16\frac{3}{5}$ $25\frac{1}{15}$	$5\frac{2}{5}$ $-3\frac{5}{6}$ $1\frac{17}{30}$	$13\frac{1}{2}$ $-10\frac{6}{7}$ $2\frac{9}{14}$
11.	$10\frac{1}{6}$ $-3\frac{5}{9}$ $6\frac{11}{18}$	$17\frac{3}{8}$ $-4\frac{5}{6}$ $12\frac{13}{24}$	$8\frac{1}{2}$ $-1\frac{4}{5}$ $6\frac{7}{10}$	$5\frac{3}{7}$ $-3\frac{1}{2}$ $1\frac{13}{14}$

Solve these problems.

12. There were $3\frac{1}{2}$ dozen cookies in a box. After $1\frac{2}{3}$ dozen were eaten, how many dozen cookies were left?
$3\frac{1}{2}-1\frac{2}{3}=n$ $1\frac{5}{6}$ doz.

13. Mrs. Kohl bought $7\frac{1}{4}$ yards of cloth. She used $3\frac{1}{3}$ yards. How many yards were left? $7\frac{1}{4}-3\frac{1}{3}=n$ $3\frac{11}{12}$ yd.

14. Jack cut $\frac{5}{6}$ of a foot of tape from a piece that was $1\frac{3}{4}$ feet long. How much tape was left?
$1\frac{3}{4}-\frac{5}{6}=n$ $\frac{11}{12}$ ft.

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. The same properties apply to the addition of both whole and fractional numbers. (21, 22, 33)

2. Subtraction is the inverse operation of addition. It undoes what addition does. (27, 33)

3. Fractional numbers that are to be added or subtracted must have a common denominator. (35-38)

Words to Know

1. Closure, binary operation, commutative property (21)

2. Associative property, identity number (22)

3. Common denominator (35)

4. Least common denominator (36)

Questions to Discuss

See T39 for answers.

1. How can you use the commutative property of addition to check the addition of a pair of numbers? (21)

2. How can you use the associative property of addition to check addition? (22)

3. What steps should you take in solving a story problem? (26)

4. How do you use addition to check subtraction? (29)

5. What are some of the properties that are the same regardless of whether you are adding whole or fractional numbers? (33)

6. How do you find a common denominator for two fractions? (35)

Written Practice

Copy. Write each sum or difference. Check your work.

	a	*b*	*c*	
1.	$\begin{array}{r}7506\\6341\\8900\\+6750\\\hline 29497\end{array}$	$\begin{array}{r}1001\\3203\\2101\\+1302\\\hline 7607\end{array}$	$\begin{array}{r}2291\\8407\\6233\\+8110\\\hline 25041\end{array}$	(25)
2.	$\begin{array}{r}7359\\-3582\\\hline 3777\end{array}$	$\begin{array}{r}5007\\-4631\\\hline 376\end{array}$	$\begin{array}{r}8790\\-2346\\\hline 6444\end{array}$	(29)
3.	$\begin{array}{r}7\frac{7}{12}\\+\frac{1}{12}\\\hline 7\frac{2}{3}\end{array}$	$\begin{array}{r}\frac{1}{3}\\+\frac{4}{9}\\\hline \frac{7}{9}\end{array}$	$\begin{array}{r}\frac{1}{12}\\+\frac{3}{4}\\\hline \frac{5}{6}\end{array}$	(34, 35)
4.	$\begin{array}{r}\frac{1}{2}\\+\frac{2}{3}\\\hline 1\frac{1}{6}\end{array}$	$\begin{array}{r}3\frac{2}{3}\\+7\frac{1}{5}\\\hline 10\frac{13}{15}\end{array}$	$\begin{array}{r}4\frac{9}{10}\\+1\frac{1}{6}\\\hline 6\frac{1}{15}\end{array}$	(36)
5.	$\begin{array}{r}5\frac{7}{10}\\-4\frac{9}{10}\\\hline \frac{4}{5}\end{array}$	$\begin{array}{r}\frac{8}{9}\\-\frac{1}{3}\\\hline \frac{5}{9}\end{array}$	$\begin{array}{r}\frac{2}{3}\\-\frac{5}{12}\\\hline \frac{1}{4}\end{array}$	(37)
6.	$\begin{array}{r}7\frac{4}{5}\\-2\frac{1}{3}\\\hline 5\frac{7}{15}\end{array}$	$\begin{array}{r}8\frac{1}{2}\\-3\frac{4}{5}\\\hline 4\frac{7}{10}\end{array}$	$\begin{array}{r}14\frac{3}{4}\\-7\frac{5}{6}\\\hline 6\frac{11}{12}\end{array}$	(38)

Self-Evaluation

Part 1 Copy. Write each sum.

	a	*b*	*c*	*d*
1.	8031 7321 5068 +6245 26665	7284 8543 3829 +2849 22505	4532 5200 4616 +4325 18673	6945 7211 2553 +1356 18065
2.	$27\frac{1}{2}$ $+65\frac{1}{2}$ 93	$6\frac{3}{4}$ $+8\frac{2}{5}$ $15\frac{3}{20}$	$27\frac{2}{3}$ $+45\frac{7}{12}$ $73\frac{1}{4}$	$16\frac{2}{5}$ $+42\frac{1}{2}$ $58\frac{9}{10}$
3.	$1\frac{1}{2}$ $1\frac{3}{4}$ $+5\frac{1}{2}$ $8\frac{3}{4}$	$4\frac{1}{8}$ $5\frac{1}{2}$ $+4\frac{3}{4}$ $14\frac{3}{8}$	$\frac{2}{3}$ $\frac{1}{3}$ $+\frac{1}{2}$ $1\frac{1}{2}$	$7\frac{5}{6}$ $1\frac{1}{2}$ $+2\frac{1}{3}$ $11\frac{2}{3}$
4.	$3\frac{1}{3}$ $2\frac{1}{2}$ $+4\frac{3}{4}$ $10\frac{7}{12}$	$8\frac{3}{16}$ $4\frac{5}{8}$ $+3\frac{1}{4}$ $16\frac{1}{16}$	$10\frac{1}{3}$ $14\frac{5}{9}$ $+11\frac{2}{3}$ $36\frac{5}{9}$	$7\frac{1}{8}$ $4\frac{5}{6}$ $+2\frac{2}{3}$ $14\frac{5}{8}$

Part 2 Copy. Write each difference.

	a	*b*	*c*	*d*
1.	6832 −2678 4154	2539 −1099 1440	8204 −4576 3628	6523 −3286 3237
2.	$\frac{2}{3}$ $-\frac{1}{3}$ $\frac{1}{3}$	$3\frac{3}{4}$ $-2\frac{1}{4}$ $1\frac{1}{2}$	1 $-\frac{1}{8}$ $\frac{7}{8}$	$1\frac{2}{5}$ $-\frac{4}{5}$ $\frac{3}{5}$
3.	$4\frac{1}{3}$ $-2\frac{2}{3}$ $1\frac{2}{3}$	$3\frac{1}{8}$ $-2\frac{5}{8}$ $\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{4}$ $\frac{1}{4}$	$2\frac{3}{4}$ $-\frac{1}{8}$ $2\frac{5}{8}$
4.	$\frac{3}{8}$ $-\frac{1}{8}$ $\frac{1}{4}$	$6\frac{3}{10}$ $-\frac{1}{10}$ $6\frac{1}{5}$	$1\frac{2}{5}$ $-\frac{3}{5}$ $\frac{4}{5}$	$4\frac{2}{9}$ $-\frac{5}{9}$ $3\frac{2}{3}$
5.	$3\frac{5}{8}$ $-1\frac{5}{16}$ $2\frac{5}{16}$	$2\frac{1}{2}$ $-1\frac{7}{8}$ $\frac{5}{8}$	$7\frac{1}{2}$ $-2\frac{7}{10}$ $4\frac{4}{5}$	$24\frac{1}{2}$ $-13\frac{2}{3}$ $10\frac{5}{6}$

Part 3 Copy. Write each answer.

	a	*b*	*c*
1.	15671 34750 71933 43465 59748 +46187 271754	674208 832671 29544 3786 597413 +987654 3125276	15798432 150905 4321507 79321459 89786 +8143215 107825304
2.	97564 −32874 64690	847000 −369475 477525	32961425 −18345216 14616209
3.	94521 −59879 34642	895613 −694972 200641	68953214 −34967159 33986055

Part 4 Solve these problems.

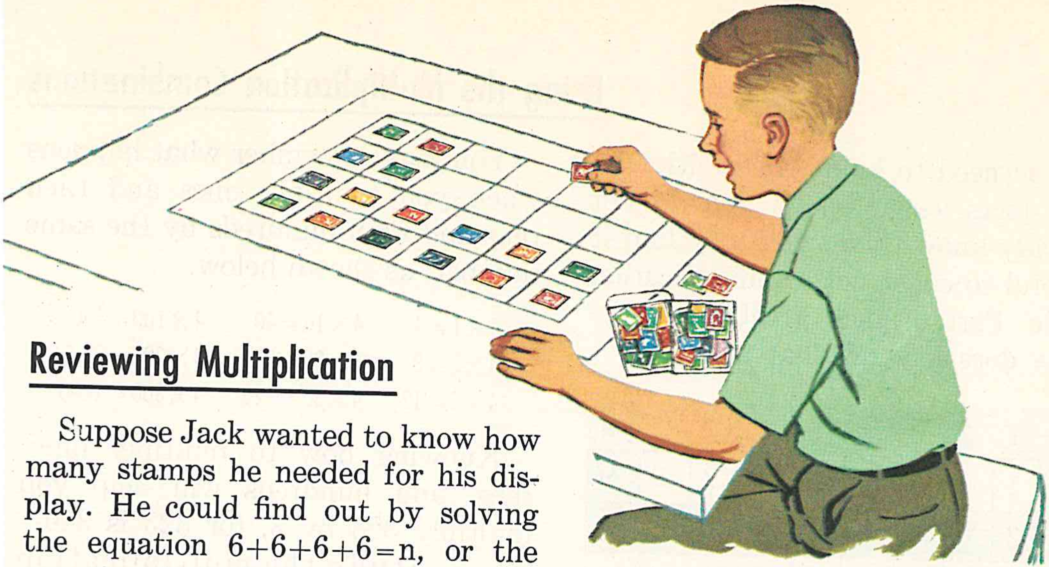
1. Bob and his father went to a baseball game. The attendance was 36,011. There were 28,111 permanent seats in the stadium. How many temporary seats were used?
36,011-28,111=n 7900 seats

2. In one city, there were 1737 pupils enrolled in elementary school and 713 pupils enrolled in high school. How many pupils were enrolled in the schools?
1737+713=n 2450 pupils

3. Walter hiked $2\frac{1}{2}$ miles on Saturday, $2\frac{2}{3}$ miles on Sunday morning, and $1\frac{3}{4}$ miles on Sunday afternoon. How far did he hike in all?
$2\frac{1}{2}+2\frac{2}{3}+1\frac{3}{4}=n$ $6\frac{11}{12}$ mi.

4. Bob weighs $90\frac{1}{2}$ pounds. His father weighs $186\frac{1}{4}$ pounds. How much less does Bob weigh than his father? $186\frac{1}{4}-90\frac{1}{2}=n$ $95\frac{3}{4}$ lb.

3



Reviewing Multiplication

Suppose Jack wanted to know how many stamps he needed for his display. He could find out by solving the equation 6+6+6+6=n, or the equation 4+4+4+4+4+4=n.

The rapid way to find the sum of four 6's is to multiply 6 by 4. To find the sum of six 4's, you would multiply 4 by 6. Does 6×4=24? Does 4×6=24? Does 6×4=4×6?

Now choose any two whole numbers and multiply them. The number you obtain is called the **product.** Change the order of the numbers you chose and multiply them again. You should get the same product both times, for the order in which you multiply two whole numbers makes no difference in the product. Thus multiplication, as well as addition, has the *commutative property.*

When two or more numbers are to be multiplied, each of them is called a **factor.** In the expression 3×6=18, 3 is a factor of 18 and 6 is a factor of 18, because when you multiply 6 and 3 you obtain 18. 6 and 3 are a pair of factors of 18. Another pair of factors of 18 is 2 and 9, since 2×9=18. What are other pairs of factors of 18?

Sometimes one of the factors is 0, as in 9×0=n. To find the value of n, add 0+0+0+0+0+0+0+0+0. Does 9×0=0? Try multiplying 0 by other numbers. You will find that any number times 0 is equal to 0. Since the order of factors does not change the product, 9×0=0×9. But 9×0=0; so 0×9=0. In fact, 0 times any number equals 0.

Oral Answer each question.

1. What pairs of factors can you name for 16? For 24? For 60?
See below.

2. Which numeral or numerals represent the factors and which the product in 7×6=42? In 9×7=63?
F F P F F P

16: 2x8, 8x2, 4x4, 1x16, 16x1
24: 2x12, 12x2, 3x8, 8x3, 6x4, 4x6, 1x24, 24x1
60: 2x30, 30x2, 3x20, 20x3, 4x15, 15x4, 5x12, 12x5, 6x10, 10x6, 1x60, 60x1

Using the Multiplication Combinations

You need to know the multiplication facts very well. If you do not already know them, you may find it helpful to construct a multiplication table. Part of such a table follows. How does it show that 9×2=18?

×	1	2	3	4	5	6	7	8	⑨
1	1	2	3	4	5	6	7	8	9
②	2	4	6	8	10	12	14	16	(18)
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54

Observe that the numerals used in the row labeled 1 are found by counting by 1's. The entries in the next four rows are found by counting by 2's, 3's, 4's, and 5's. How would you expect to find the entries for rows 6, 7, 8, and 9?

Observe the use of 1 in the following equations.

$1\times3=3\times1=3$ $\quad$ $1\times4=4\times1=4$

$1\times7=7\times1=7$ $\quad$ $1\times8=8\times1=8$

Now choose any other whole number. Multiply it by 1. Then reverse the factors and multiply again. You will always get the whole number you chose. This is why 1 is called the **identity number of multiplication.**

You may remember what happens when you multiply ones and then tens and then hundreds by the same number, as shown below.

$4\times1=4$ $\quad$ $4\times10=40$ $\quad$ $4\times100=400$

$4\times2=8$ $\quad$ $4\times20=80$ $\quad$ $4\times200=800$

$4\times3=12$ $\quad$ $4\times30=120$ $\quad$ $4\times300=1200$

Knowing how to multiply ones, tens, and hundreds will help you multiply 323 by 3, for 323 is 300+20+3. Study the multiplication below.

A

$$\begin{array}{lrl} & 323 & \\ & \times3 & \\ \hline 3\times3=9 & 9 & \text{first partial product} \\ 3\times20=60 & 60 & \text{second partial product} \\ 3\times300=900 & 900 & \text{third partial product} \\ \hline \text{Add} & 969 & \text{product} \end{array}$$

Can you multiply 323 by 3 without writing each partial product? What steps would you take?

Suppose you are to multiply 635 by 4. The partial products are greater than 9 when you multiply 5 by 4, greater than 99 when you multiply 30 by 4, and greater than 999 when you multiply 600 by 4. Observe the multiplication of 635 by 4 as shown in B and C on the next page.

In B, how was the form of 20 ones, of 12 tens, and of 24 hundreds changed?

B

Th	H	T	O
	6	3	5
		×	4
		2	0
	1	2	0
2	4	0	0
2	5	4	0

C

Th	H	T	O
	6	3	5
		×	4
2	5	4	0

Study C. When 5 ones were multiplied by 4, 20 ones were changed to 2 tens and 0 ones. Where was 0 written? The 2 tens were remembered and then added to the product of 4×3 tens. What other steps were taken?

Oral Tell the steps to take in solving the equation 6×975=n.
See examples A, B, and C.

Written Copy. Write each product.

	a	*b*	*c*	*d*
1.	81 ×5 405	63 ×3 189	90 ×6 540	72 ×4 288
2.	45 ×2 90	19 ×5 95	26 ×3 78	48 ×2 96
3.	64 ×3 192	87 ×4 348	62 ×5 310	83 ×6 498
4.	300 ×4 1200	163 ×3 489	490 ×5 2450	109 ×6 654

Another way Suppose you want to solve the equation 4×237=n. Rename the factor 237 as a sum of hundreds, tens, and ones; that is, 2 hundreds and 3 tens and 7 ones, or 200+30+7. Then the partial products are easy to find.

4×237=n becomes 4×(200+30+7)=n. This means that the 200, the 30, and the 7 are all to be multiplied by 4. Study the solution below.

$$\begin{aligned} 4\times237 &= 4\times(200+30+7)\\ &= (4\times200)+(4\times30)+(4\times7)\\ &= 800+120+28\\ &= 948 \end{aligned}$$

When you multiplied 200 by 4, 30 by 4, and 7 by 4, you found the partial products 800, 120, and 28. Are these partial products the same as those you find when you multiply 237 by 4 in column form? Why?

Can you do this? Solve for n, using the equation form. Check by using the column form.

	a	*b*
1.	3×210=n 630	4×485=n 1940
2.	7×539=n 3773	2×617=n 1234
3.	5×721=n 3605	9×706=n 6354
4.	8×692=n 5536	6×320=n 1920
5.	7×564=n 3948	2×398=n 796

Multiplying Larger Numbers

Why might you expect to multiply thousands in the same way as hundreds, tens, and ones? Would you also expect to multiply ten thousands, hundred thousands, or even millions as you multiply hundreds, tens, and ones?

Suppose you are to multiply 9875 by 6. First think of 9875 as 9000+800+70+5, or 9 Th+8 H+7 T+5 O. Then you can multiply as in grid A. Study the multiplication.

A

	TTh	Th	H	T	O
		9	8	7	5
				×	6
6×5 O=30 O=				3	0
6×7 T=42 T=			4	2	0
6×8 H=48 H=		4	8	0	0
6×9 Th=54 Th=	5	4	0	0	0
Add	5	9	2	5	0

See how much time you save if you do not write each partial product, as in B. The digits in each place-value position are multiplied separately. What changes in form were necessary in each place-value position?

B

9875
×6
59250

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Oral Tell the steps to take in finding each partial product and product.

See examples A and B.

	a	*b*	*c*
	2758 ×3 8274	21030 ×6 126180	419500 ×7 2936500

Written Copy. Write each product. Use the way that is fastest for you.

	a	*b*	*c*
1.	6024 ×2 12048	1402 ×7 9814	8720 ×4 34880
2.	3291 ×2 6582	7283 ×3 21849	1640 ×5 8200
3.	2119 ×4 8476	5327 ×3 15981	1516 ×6 9096
4.	4605 ×7 32235	1148 ×5 5740	6019 ×8 48152
5.	1329 ×7 9303	2487 ×4 9948	3456 ×3 10368
6.	9675 ×6 58050	6543 ×8 52344	3589 ×9 32301
7.	31605 ×5 158025	62435 ×3 187305	86429 ×4 345716
8.	76149 ×9 685341	48632 ×7 340424	54975 ×8 439800
9.	419560 ×2 839120	327175 ×4 1308700	869543 ×9 7825887

Regrouping Factors

You may remember that you can associate, or regroup, addends in any way without affecting the sum. Do you suppose that in the same way you can associate factors in various ways without affecting the product?

Suppose you wish to multiply 2 by 3 and this product by 4. You can indicate this work by writing $(2\times3)\times 4$. Would the product be the same if you treated the work as $2\times(3\times4)$? In both cases the product is 24.

The property of multiplication that makes it possible to regroup the factors without changing the product is called the **associative property for multiplication.** You may recall that addition, too, has this same property.

Suppose you are to multiply 29 by 5 and the product by 2. Two solutions are shown below. Which is easier? Why?

A	B
$(29\times5)\times2=n$	$29\times(5\times2)=n$
$145\times2=n$	$29\times10=n$
$290=n$	$290=n$

The associative property of multiplication makes it possible to use your knowledge of 10 in solving many equations. Suppose, for example, you wanted to multiply 14 by 5. You might think of 14 as 2×7. Then the work becomes $5\times(2\times7)$. If you associate 2 and 5, the work becomes $(5\times2)\times7$ or 10×7 or 70.

Study C and D. How is the associative property used in each one?

C	D
$5\times24=n$	$2\times55=n$
$5\times(2\times12)=n$	$2\times(5\times11)=n$
$(5\times2)\times12=n$	$(2\times5)\times11=n$
$10\times12=n$	$10\times11=n$
$120=n$	$110=n$

Oral How would you regroup the factors in 1 and 2 below? Which factor would you rename in 3, 4, and 5?

	a	*b*
1.	$2\times(19\times5)=n$ 190	$(27\times2)\times5=n$ 270
2.	$5\times(38\times2)=n$ 380	$2\times(5\times65)=n$ 650
3.	$5\times\underline{36}=n$ 180	$2\times\underline{35}=n$ 70
4.	$5\times\underline{14}=n$ 70	$\underline{18}\times5=n$ 90
5.	$2\times\underline{45}=n$ 90	$5\times\underline{32}=n$ 160

Written Solve each equation in rows 1 through 5 above using the associative property of multiplication. Then solve for n in each equation below.

	a	*b*
6.	$n=(86\times5)+9$ 439	$n=(2\times65)-7$ 123
7.	$n=(5\times28)+8$ 148	$n=8+(37\times8)$ 304

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To find this number:	Use this operation
How many in all of the sets with different numbers?	Addition
How many in all of the sets with the same number?	Multiplication
How many are left?	Subtraction
By how many the number of one set differs from that of another?	Subtraction

Choosing the Correct Operation

Write the equation you decide to use in solving problems 1 through 6. Reminder: The signs $ and . are not part of an equation.

1. The driver of a school bus drove the bus 38 miles each day. How many miles would he drive the bus in 9 days? 9x38=n 342 mi.

2. Betty bought 6 records at $.89 each. How much did she spend for records? 6x89=n $5.34

3. A parking lot had room for 8 rows of cars. There were spaces for 35 cars in each row. How many cars could be parked in the lot?
8x35=n 280 cars

4. The skis that Bob wants cost $14.95. He has saved $9.98. How much more money does he need before he can buy the skis?
1495-998=n $4.97

5. Roger had 81 papers to deliver. As he neared the end of his route he counted 9 papers in the carrier on his bicycle. How many papers had he delivered? 81-9=n 72 papers

6. Sally studied geography for 35 minutes, history for 45 minutes, and arithmetic for 35 minutes. How much time did she spend on her studies?
35+45+35=n 1 hr.55 min.

Oral Tell the equation you decided to use in solving each problem and why you decided as you did.

Written Solve problems 1 through 6. Check each answer for accuracy.

A quick review

	a	*b*
1.	$(6\times9)+12=\overset{66}{n}$	$\overset{14}{n}=(4\times5)-6$
2.	$7\times(9+1)=\overset{70}{n}$	$\overset{17}{n}=80-(9\times7)$
3.	$8+(9-0)=\overset{17}{n}$	$\overset{20}{n}=(7+0)+13$
4.	$(0\times9)+\overset{9}{n}=9$	$\overset{78}{n}=30+(6\times8)$
5.	$(\overset{4}{n}\times9)+12=48$	$(7\times\overset{8}{n})+10=66$
6.	$(5\times12)-18=\overset{42}{n}$	$(\overset{9}{n}-6)\times7=21$
7.	$75-(6\times\overset{12}{n})=3$	$(\overset{10}{n}-4)\times8=48$
8.	$\overset{100}{n}-(9\times8)=28$	$\overset{12}{n}+(6\times8)=60$

Multiplying by Two-Digit Numbers

See what happens when you multiply ones by tens and then tens by tens.

$10\times1=10$ $10\times10=100$

$20\times4=80$ $20\times40=800$

$40\times6=240$ $40\times60=2400$

Choose other numbers between 1 and 9. Multiply them by tens. Did you convince yourself that ones multiplied by tens results in tens? Choose any other numbers from 10 through 99. Multiply them by tens. The result is always in hundreds.

Suppose you are to multiply 18 by 53. You can proceed as shown in A.

The partial products 24 and 30 are found by multiplying 18 by 3. You can probably do this multiplication mentally.

The partial product 400 is found by multiplying 8 ones by 5 tens. 5 tens×8 ones=40 tens =4 hundreds. The 4 hundreds are written as the partial product 400.

A

H	T	O
	1	8
×	5	3
	2	4
	3	0
4	0	0
5	0	0
9	5	4

The partial product 500 is obtained by multiplying 1 ten by 5 tens. 1 ten×5 tens=5 hundreds. The 5 hundreds are written as the partial product 500. Perhaps you can multiply 18 by 5 tens mentally. Then you need not write each partial product.

The shorter form of doing the multiplication is shown below.

18	factors	
×53		
54	partial product	add these
90	partial product	
954	product	

Oral Tell the steps you would use in solving the equation $13\times23=n$. In solving the equation $39\times97=n$.

`See examples in tint blocks.`

Written Copy. Find each product.

	a	*b*	*c*	*d*
1.	15 ×13 195	25 ×23 575	54 ×12 648	23 ×39 897
2.	19 ×46 874	72 ×21 1512	34 ×32 1088	25 ×63 1575
3.	71 ×71 5041	14 ×87 1218	33 ×52 1716	91 ×81 7371
4.	47 ×24 1128	19 ×53 1007	95 ×83 7885	52 ×46 2392
5.	93 ×43 3999	82 ×34 2788	57 ×72 4104	73 ×29 2117

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Multiplying Three-Digit Numbers

You would probably expect to multiply hundreds as you multiply tens and ones. The illustration below shows that this is true.

2×200=400 20×200=4000

5×300=1500 50×300=15000

Do you see that ones times hundreds results in hundreds and that tens times hundreds results in thousands?

Suppose you are to multiply 917 by 24. You can do the work in column form as shown.

TTh	Th	H	T	O
		9	1	7
		×	2	4
	3	6	6	8
1	8	3	4	
2	2	0	0	8

Notice especially that 4 ones×9 hundreds=36 hundreds or 3 thousands and 6 hundreds. Observe that 2 tens×9 hundreds=18 thousands.

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In such multiplications as 50×789, multiply by 5 tens. Write 0 in ones place in the product as shown. Why? What is the product?

789
×50
39450

Whenever possible do as much of each multiplication as you can without using pencil and paper. However, take sufficient time to work accurately and to check your computation.

Oral Be ready to tell the steps taken in finding each partial product and product below. See column 1.

a	*b*	*c*
764	907	960
×35	×86	×70
3820	5442	67200
2292	7256	
26740	78002	

Written Copy in column form. Solve each equation for n.

	a	*b*
	6440	5886
1.	23×280=n	54×109=n
	27950	17100
2.	43×650=n	36×475=n
	21225	40066
3.	25×849=n	67×598=n
	13248	63270
4.	48×276=n	90×703=n
	14560	30000
5.	70×208=n	50×600=n
	94525	26214
6.	95×995=n	51×514=n
	57780	69084
7.	60×963=n	76×909=n
	67200	13600
8.	75×896=n	34×400=n
	18699	19395
9.	23×813=n	45×431=n

Multiplying Larger Numbers

Jack counted 25 grain bins. Each bin had a capacity of 1275 bushels. How many bushels of grain could be stored in all of the bins?

You are to think of 25 sets of 1275 bushels each as being joined; so you multiply 1275 by 25. The equation is 25×1275=n. The multiplication can be done in column form as shown.

TTh	Th	H	T	O
	1	2	7	5
		×	2	5
	6	3	7	5
2	5	5	0	
3	1	8	7	5

Do you see that thousands are multiplied in the same way as hundreds, tens, and ones? 5 ones×1 thousand=5 thousand and 5 thousand plus 1 thousand remembered equals 6 thousand. 2 tens×1 thousand=2 ten thousands.

You might expect to multiply ten thousands in the same way as thousands. Observe the results of such multiplications by ones and then by tens.

1×10000=10000 10×10000=100000
2×20000=40000 20×20000=400000
5×50000=250000 50×50000=2500000

Oral Be ready to tell the steps taken in finding each partial product and product below. See column 1.

a	*b*	*c*
8257	59327	87750
×96	×87	×70
49542	415289	6142500
74313	474616	
792672	5161449	

Written Copy. Find each product.

	a	*b*	*c*
1.	2312 ×23 53176	6100 ×45 274500	3000 ×90 270000
2.	8610 ×53 456330	4031 ×40 161240	5700 ×30 171000
3.	7310 ×60 438600	9786 ×52 508872	8743 ×75 655725
4.	21200 ×32 678400	41010 ×72 2952720	72130 ×43 3101590
5.	96875 ×50 4843750	87432 ×95 8306040	49327 ×67 3304909

A quick review

	a	*b*
1.	(3×20)×50=n 3000	(n+7)−5=9 7
2.	n−(9×40)=20 380	0×(5+6)=n 0
3.	(4×12)×35=n 1680	(n+9)−9=0 0
4.	n+(3×18)=60 6	(n×6)+3=9 1

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Using Three-Digit Multipliers

A warehouse stored 175 cases of eggs. Each case held 12 dozen, or 144 eggs. How many eggs were stored in the warehouse?

You are to think of the 175 sets of 144 eggs each as being joined; so you multiply. The equation is 175×144=n. The multiplication is shown below.

TTh	Th	H	T	O
		1	4	4
	×	1	7	5
		7	2	0
1	0	0	8	
1	4	4		
2	5	2	0	0

Do you see that hundreds×ones = hundreds, hundreds×tens = thousands, and hundreds×hundreds = ten thousands?

How many zeros have been omitted at the end of the second partial product? At the end of the third partial product? Why is it unnecessary to write these zeros?

Oral Tell the steps to take in finding each partial product and product.
See above.

a	b	c	d
213 ×301 64113	241 ×122 29402	3421 ×512 1751552	1201 ×683 820283

Written Copy. Find each product. Check by using the commutative property.

	a	b	c	d
1.	214 ×122 26108	421 ×512 215552	683 ×211 144113	521 ×351 182871
2.	934 ×622 580948	743 ×462 343266	872 ×562 490064	587 ×628 368636
3.	724 ×493 356932	465 ×834 387810	978 ×253 247434	658 ×586 385588
4.	735 ×395 290325	824 ×392 323008	593 ×217 128681	234 ×102 23868
5.	320 ×301 96320	106 ×407 43142	300 ×204 61200	765 ×302 231030
6.	607 ×809 491063	970 ×708 686760	800 ×907 725600	312 ×330 102960
7.	430 ×260 111800	206 ×480 98880	700 ×500 350000	608 ×700 425600

Can you do this? Tell how you would use the associative property of multiplication in solving for n.

	a	b
1.	(19×(25)×4) = n 1900	(35×(5)×20) = n 3500
2.	(25×(4)×29) = n 2900	(2×(50)×17) = n 1700
3.	(5×(87)×2) = n 870	(5×(19)×20) = n 1900
4.	(4×(28)×25) = n 2800	(17×(50)×2) = n 1700

Problems to Solve

Read each problem carefully and decide upon the operation to use in solving it. Translate each problem into an equation.

1. Each of the 72 children in the sixth-grade class had to bring 85¢ to help pay for the two city buses the class had to rent for a field trip. How much did it cost the class to rent the buses? 72x85=n $61.20

2. After Janet had earned $3.75, she still needed $1.98 to pay for a skirt that she wanted. What was the price of the skirt? 375+198=n
$5.73

3. Each of the 27 children in Miss Brown's room planned to bring 250 pounds of newspaper to sell so as to arn money for a party. How many pounds of old newspaper had they et as the room goal? 27x250=n
6750 lb.

4. A school superintendent transferred 275 pupils out of a school aving an enrollment of 1080 pupils. How many pupils were left?
1080-275=n 805 pupils

5. Jerry counted 444 names listed on a page in the telephone directory, and there were 343 pages in the book. How many telephone subscribers were listed in his directory?
343x444=n 152292 subscribers

6. Mary spent $9.98 for shoes and $39.75 for a coat. The tax was $1.74. How much money did she need altogether? 998+3975+174=n
$51.47

7. The Girl Scouts sold 462 boxes of buttered popcorn at 25¢ per box at the charity bazaar. How much money did they collect to contribute to the charity fund? 462x25=n
$115.50

Oral Tell why you decided upon using each of the equations you wrote for problems 1 through 7.
Answers will vary.

Written Solve each equation for problems 1 through 7. As you work, write a tentative answer for each problem. When you finish, read the problem again to see whether your tentative answer is sensible. Write your answer.

Multiplying Large Numbers by Three-Digit Multipliers

An underground parking garage needs at least 5100 customers a day to pay expenses. How many customers does it need in a year to pay the cost of operating the garage?

The equation is 365×5100 = n. Why? In column form the multiplication may be done as shown.

M	HTh	TTh	Th	H	T	O
			5	1	0	0
			×	3	6	5
		2	5	5	0	0
	3	0	6	0	0	
1	5	3	0	0		
1	8	6	1	5	0	0

Notice that ones×thousands = thousands, tens×thousands = ten thousands, and hundreds×thousands = hundred thousands.

The garage needs 1,861,500 customers a year merely to pay expenses.

If you can multiply thousands by ones, tens, and hundreds, you should be able to multiply ten thousands, hundred thousands, or even millions by the same numbers. Why?

MORE PRACTICE PAGE 299

Oral What result would you expect if you were to multiply hundred thousands by ones? By tens? By hundreds? See above.

Written Copy. Write each product.

	a	*b*	*c*
1.	1432 ×231 330792	3002 ×443 1329886	9300 ×323 3003900
2.	8965 ×487 4365955	4321 ×595 2570995	5427 ×687 3728349
3.	3461 ×620 2145820	2571 ×139 357369	7460 ×782 5833720
4.	1798 ×243 436914	6897 ×567 3910599	5352 ×876 4688352
5.	41378 ×262 10841036	35760 ×303 10835280	68922 ×687 47349414
6.	92689 ×150 13903350	57811 ×500 28905500	89243 ×796 71037428

Another way You can sometimes save time in multiplying large numbers if you change the order of the factors. Which property of multiplication makes this possible? Suppose you are to multiply 7000 by 645. Think: 645×7000 = 7000×645. Then multiply 645 by 7 and affix 3 zeros.

Use the shortest possible way to solve each equation below.

	a	*b*
1.	7600000 950×8000 = n	3125 625×5000 = n
2.	3380000 845×4000 = n	65700 730×9000 = n

Problem Solving

Study each problem carefully by reading it several times. Decide upon the operation to use in solving each problem. Then write the equation you would use in solving each problem. Also write a sentence telling why you decided upon the operation you plan to use.

1. A trailer was loaded with 4 automobiles weighing 3185 pounds each. What was the total weight of the load? 4x3185=n 12740 lb.

2. Mr. King can get an $89.89 overcoat for $71.98 on sale. How much less is the sale price than the regular price? 8989-7198=n $17.91

3. Bob's older brother earns $11.75 on each Saturday. How much money can he earn at this rate if he works for 52 Saturdays? 52x1175=n $611.00

4. Jack bought a suit for $22.50, shoes for $10.98, and a shirt for $4.98. What was the total cost? 2250+1098+498=n $38.46

5. A truck and its load of coal weighed 14,875 pounds. The empty truck weighed 5996 pounds. Find the weight of the coal. 14875-5996=n 8879 lb.

6. Mr. Johnson sold 28 bushels of apples for $1.75 a bushel. How much money did he receive for the apples? 28x175=n $49.00

7. Jack saved his money as follows: September, $5.25; October, $3.50; November, $3.85; and December, $5.19. What were his total savings for those four months? 525+350+385+519=n $17.79

8. At the tree nursery, Tom counted 28 rows of pine trees. The forester said that there were at least 575 trees in each row. At least how many trees were in the 28 rows of trees? 28x575=n 16100 trees

9. Ann had $119.25 in the club treasury before paying a bill for $38.98. How much money was still in the club treasury? 11925-3898=n $80.27

10. Bob needs $49.50 for a new bicycle. He has saved $21.85. How much more money must he have before he can buy the bicycle? 4950-2185=n $27.65

11. After being given a special allowance of $170.87, Mr. Zeman owed $1876.85 on a car. How much would he have owed if no special allowance had been made? 17087+187685=n $2047.72

Oral Tell the equation you would use in solving each problem on this page and why you would use it.

Written Solve each equation you wrote for the problems. Read each problem again to see whether your answer is sensible. Write the answer to each problem.

Review and Practice

You should be able to do the computations on this page without any help.

Part 1 Copy. Find each sum.

	a	b	c	d
1.	687 49 768 +385 1889	834 913 226 +101 2074	632 786 398 +854 2670	$9.13 4.55 5.77 +3.69 $23.14
2.	4218 595 2633 +474 7920	3929 2462 3978 +4184 14553	8286 5007 3828 +6719 23840	4352 6251 4879 +9278 24760

Part 2 Copy. Find each sum.

	a	b	c
1.	6273 841 3962 +4832 15908	4631 2843 3866 +4763 16103	$91.82 26.65 62.28 +71.43 $252.18
2.	74961 36622 4913 28411 +9726 154633	38912 84329 39654 83275 +7864 254034	$931.47 406.95 327.23 810.28 +219.85 $2695.78

Part 3 Copy. Find each difference.

	a	b	c	d
1.	7651 −973 6678	3233 −984 2249	5004 −2195 2809	$6.85 −2.97 $3.88
2.	2683 −958 1725	6954 −3076 3878	6001 −4346 1655	9649 −8775 874

Part 4 Copy. Find each difference.

	a	b	c
1.	39473 −9865 29608	49346 −8500 40846	89000 −9575 79425
2.	76854 −21096 55758	917809 −430918 486891	705069 −321478 383591
3.	92165 −74520 17645	831601 −300909 530692	654321 −439482 214839

Part 5 Copy. Find each product.

	a	b	c
1.	15 ×67 1005	30 ×98 2940	78 ×54 4212
2.	615 ×47 28905	376 ×79 29704	3049 ×82 250018
3.	6895 ×73 503335	4568 ×59 269512	98976 ×67 6631392
4.	437 ×256 111872	292 ×462 134904	376 ×459 172584
5.	195 ×459 89505	483 ×937 452571	7600 ×257 1953200

Part 6 Solve for n.

	a	b
1.	198 n = 98 + 47 + 53	3634 46 × 79 = n
2.	155 (17 × 85) + n = 1600	4336 54 × 803 = n
3.	1309 1525 − (12 × 18) = n	1702 37 × 46 = n

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. The order of the factors does not change the product. (41)

2. Whenever 0 is one of the factors, the product is 0. (41)

3. Whenever 1 is one of two factors, the other factor and the product are identical. (42)

4. Three or more factors may be associated, or grouped, in any order for multiplication. (45)

Words to Know

1. Factor, product (41)

2. Commutative and associative properties for multiplication (41, 45)

Questions to Discuss

See T55 for answers.

1. How would you construct and use a multiplication table? (42)

2. Why might you want to regroup the factors in an equation? (45)

3. How do you decide upon the correct operation to use in solving a problem? (46)

4. How many zeros do you expect in the product when you multiply 5 tens by 7 tens? 5 hundreds by 7 hundreds? (47, 48)

Written Practice

Copy. Write each product.

	a	*b*	
1.	954 $\times 7$ 6678	685 $\times 9$ 6165	(43)
2.	9863 $\times 8$ 78904	947000 $\times 5$ 4735000	(44)
3.	93 $\times 89$ 8277	64 $\times 17$ 1088	(47)
4.	809 $\times 75$ 60675	760 $\times 80$ 60800	(48)
5.	4896 $\times 35$ 171360	72500 $\times 90$ 6525000	(49)
6.	312 $\times 214$ 66768	903 $\times 785$ 708855	(50)
7.	7200 $\times 628$ 4521600	28950 $\times 850$ 24607500	(52)

Solve for n.

890
8. $(89\times5)\times2=n$ (45)

10730
9. $29\times370=n$ (48)

3245625
10. $75\times43275=n$ (49)

7500
11. $25\times(4\times75)=n$ (50)

34250000
12. $685\times50000=n$ (52)

Self-Evaluation

Part 1 Copy. Find each answer as indicated by the operation sign.

	a	*b*	*c*
1.	605 ×29 17545	856 ×28 23968	934 ×67 62578
2.	785 ×508 398780	927 ×439 406953	216 ×789 170424
3.	1495 ×356 532220	2405 ×397 954785	36327 ×350 12714450
4.	9867 7654 3589 6540 +7694 35344	23415 9327 18546 49042 +2390 102720	915728 614683 42017 879589 +4013 2456030
5.	4937 −3989 948	3461 −2679 782	8561 −7495 1066
6.	93257 −42869 50388	391528 −109150 282378	876412 −157243 719169

Part 2 Solve for n.

	a	*b*
1.	9752 9080+672=n	9205 n=8562+643
2.	6909 7463−554=n	2778 n=9347−6569
3.	120597 183×659=n	30030 n=65×462
4.	140220 95×1476=n	2154555 n=87×24765
5.	94872 354×268=n	114597 n=321×357
6.	3395000 350×9700=n	74298000 n=854×87000

Part 3 Copy. Write A, S, or M to indicate whether each word makes you think of addition, subtraction, or multiplication.

	a		*b*	
1.	sum	A	factor	M
2.	product	M	difference	S
3.	subtrahend	S	addend	A
4.	minuend	S	plus	A
5.	times	M	minus	S

Part 4 Express each problem below in equation form. Solve the equation. Write the answer.

1. For a special program, 1085 chairs were set up in the school gym. A total of 1195 tickets had been sold for the program. How many more chairs should be set up? 1195-1085=n
110 chairs

2. A railroad bought 75 boxcars each with a capacity of 134,500 pounds. What was the total capacity, in tons, of all of the boxcars? (Each ton is equivalent to 2000 pounds.)
75x134500=n in lb. $5043\frac{3}{4}$ T.

3. An electric motor ran for 24 hours a day for 2 years. How many hours did the motor run? (Use 365 days as being equivalent to 1 year.)
730x24=n 17,520 hr.

4. A bushel of corn weighs 56 lb. How much will 1725 bushels weigh?
1725x56=n 96,600 lb.

4

Reviewing Division

Jane has 18 birthday candles to use on birthday cakes for her little sister's birthday party. She will put 3 candles on each cake. How many cakes can she decorate with candles?

If you let n represent the number of sets of 3, the problem can be translated into the equation $n \times 3 = 18$. Of course you know that $6 \times 3 = 18$; so $n = 6$. Jane can decorate 6 cakes.

Suppose you didn't know that $6 \times 3 = 18$. Then you would use the operation of division to find the value of n in $n \times 3 = 18$. The equation is solved by dividing 18 by 3.

Do you remember the name for each part of a division equation? The names are given in the following equation.

$$\underset{\text{dividend}}{\underset{\downarrow}{18}} \div \underset{\text{divisor}}{\underset{\downarrow}{3}} = \underset{\text{quotient}}{\underset{\downarrow}{6}} \qquad \begin{array}{r} 6 \\ 3\overline{)18} \end{array}$$

The equation $18 \div 3 = 6$ is a division equation. It is read like this: Eighteen divided by three equals six. The number being divided is 18. What name is it given? What name is given to the number by which you divide? To the number obtained as the result of the division?

Oral Name the factors and the product in each equation in column *a* and the dividend, divisor, and quotient in each equation in column *b*. What do you notice about the order of the numerals in each pair of equations? Order is reversed. See below.

	a	*b*
1.	$7 \times 13 = 91$	$91 \div 13 = 7$
2.	$35 \times 50 = 1750$	$1750 \div 50 = 35$
3.	$125 \times 30 = 3750$	$3750 \div 30 = 125$
4.	$25 \times 45 = 1125$	$1125 \div 45 = 25$
5.	$35 \times 75 = 2625$	$2625 \div 75 = 35$

a. factor x factor = product
b. dividend ÷ divisor = quotient

The Properties of Division

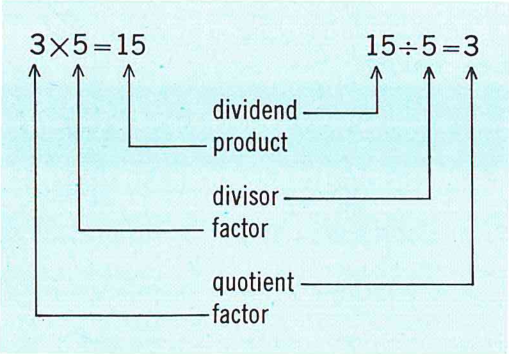
Suppose you bought 3 boxes of soap, with 6 bars in each box. You would have 18 bars of soap. Now suppose you had 18 bars of soap and separated the set of 18 into 3 sets of equal size. You would have 6 bars in each set.

The first operation was one of putting sets together; the second was one of taking a set apart. Did the second operation undo the first?

You may remember that operations that undo each other are *inverse operations*. Is division the inverse operation of multiplication? Observe a few more examples to convince yourself that your answer is correct.

$5\times6=30$ $30\div6=5$
$7\times8=56$ $56\div8=7$

Can you find the pairs of corresponding numbers in the diagram below?



Study the table below.

×/÷	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

To multiply 7 by 6, find the square at which column 7 and row 6 intersect. The product, 42, is in that square.

To divide 42 by 7, select the numeral 42 in the column headed 7. By looking to the left, you see that 42 is in row 6. $42\div7=6$.

Observe that any whole number multiplied by 1 produces that number. Thus $1\times7=7$. Any whole number, other than zero, when divided by itself yields a quotient of 1. Thus $7\div7=1$.

Do you change a whole number when you multiply it by 1? When you divide it by 1?

Observe each equation and its solution below. In which equations is n a whole number?

$3\times5=n$	$15\div5=n$	$3\div5=n$
$15=n$	$3=n$	$\frac{3}{5}=n$

Do you always get a whole number when you multiply two whole numbers? When you divide two whole numbers?

The set of whole numbers is *closed* under multiplication. This means that the product of any two whole numbers is again a whole number. The set of whole numbers is *not* closed under division. This means that certain equations, like $5\div7=n$, cannot be solved in whole numbers.

Does the commutative property apply to division? That is, can you change the order of the divisor and the dividend without changing the quotient? Try a few examples as follows. Observe that $\neq$ is read *is not equal to.*

$24\div6\neq6\div24$ $18\div3\neq3\div18$

Try other examples to convince yourself that division does not have the commutative property.

You may remember that the product is 0 whenever any of the factors is 0. But a quotient is 0 only if the dividend is 0 and the divisor is not 0.

Division by zero is meaningless. Therefore you will not be asked to divide by 0.

Oral Which sign, $=$, $\neq$, $+$, $\times$, $-$, or $\div$, belongs in place of each blank?

	a	*b*
1.	$4\times3\,\underline{\;=\;}\,3\times4$	$18\div3\,\underline{\;\neq\;}\,3\div18$
2.	$9\times7\,\underline{\;=\;}\,63$	$63\div7\,\underline{\;=\;}\,9$
3.	$35\div7\,\underline{\;=\;}\,5$	$7\div35\,\underline{\;=\;}\,\frac{7}{35}$
4.	$35\,\underline{\;+\;}\,6=41$	$41\,\underline{\;-\;}\,6=35$
5.	$9\,\underline{\;\times\;}\,7=63$	$63\,\underline{\;\div\;}\,7=9$
6.	$112\,\underline{\;-\;}\,48=64$	$64\,\underline{\;+\;}\,48=112$
7.	$85\,\underline{\;+\;}\,27=112$	$112\,\underline{\;-\;}\,27=85$
8.	$0\,\underline{\overset{\text{or }\div}{\;\times\;}}\,9=0$	$9\,\underline{\;\times\;}\,0=0$
9.	$150\,\underline{\;\div\;}\,5=30$	$30\,\underline{\;\times\;}\,5=150$
10.	$100\,\underline{\;\div\;}\,10=10$	$10\,\underline{\;\times\;}\,10=100$

Read. Solve for n.

	a	*b*
11.	$\overset{6}{n}\times7=42$	$42\div7=\overset{6}{n}$
12.	$\overset{5}{n}\times12=60$	$60\div12=\overset{5}{n}$
13.	$0\div5=\overset{0}{n}$	$90\div15=\overset{6}{n}$
14.	$\overset{4}{n}\times25=100$	$100\div25=\overset{4}{n}$

Written Copy. Complete each statement in rows 1 through 10. Solve for n in rows 11 through 14.
See above.

Dividing by a One-Digit Divisor

One summer, 135 boys signed up for baseball. 9 boys were put on each team. How many teams were formed?

You are to think of a set of 135 boys as being separated into subsets of 9 boys each, and you are to find the number of subsets. Thus you are to divide. The equation is $135 \div 9 = n$. It can be solved and checked as shown.

$$\begin{array}{r} 15 \\ 9\overline{)135} \\ \underline{9} \\ 45 \\ \underline{45} \end{array} \qquad \begin{array}{r} \text{Check} \\ 15 \\ \times 9 \\ \hline 135 \end{array}$$

In the division above, you cannot divide 1, the hundreds digit, by 9, the divisor; so think of 1 hundred as 10 tens and add them to the tens already in tens place. 9 is contained in 13 tens at least 10 times. That is why 1 is written in tens place of the quotient. Then proceed as follows.

Multiply: 1 ten × 9 ones = 9 tens.

Subtract: 13 tens − 9 tens = 4 tens.

Divide: 45 ones ÷ 9 ones = 5 ones. Write 5 in ones place in the quotient.

Multiply: $5 \times 9 = 45$.

The quotient is 15.

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Dividing by a one-digit divisor involves the steps *divide, multiply,* and *subtract.* These steps are repeated over and over until the final remainder is either 0 or another number less than the divisor.

Whenever the remainder is 0, the dividend is said to be a **multiple** of the divisor. Thus 135 is a multiple of 9. Other multiples of 9 include 9, 18, 27, 36, and so on.

Observe the equation $27 \div 9 = 3$. Since 27 is a multiple of 9 it is said to be **divisible** by 9. Any multiple of a number is divisible by that number. Is 36 a multiple of 4? Is it divisible by 4? 1; 1,2; 1,2,3,4; 1-6; 1-8

Oral What are the possible remainders when you divide by 2? By 3? By 5? By 7? By 9?

Written Copy. Write each quotient.

	a	*b*	*c*
1.	72r2 $5\overline{)362}$	31r1 $5\overline{)156}$	69r3 $4\overline{)279}$
2.	302 $2\overline{)604}$	107r4 $5\overline{)539}$	140r3 $5\overline{)703}$
3.	400 $5\overline{)2000}$	941r3 $6\overline{)5649}$	4234 $2\overline{)8468}$
4.	1220r5 $8\overline{)9765}$	1136 $7\overline{)7952}$	2975 $3\overline{)8925}$
5.	7043 $4\overline{)28172}$	11963r2 $8\overline{)95706}$	9255r3 $9\overline{)83298}$

Finding Trial Quotients

John has 60 old textbooks to pack in boxes for storage. Each box will hold 20 books. How many boxes does he need?

You are to separate a set of 60 into subsets of equal size; so you divide. The equation is 60÷20=n. 60 is a multiple of 20 because 3×20=60. Thus 60 is divisible by 20. The quotient is 3. 20 is contained in 60 exactly 3 times.

In solving an equation like 98÷24=n, the value of n is not so obvious. You might save time by dividing as shown.

$$\begin{array}{r} 4\text{ r}2 \\ 24\overline{)98} \\ \underline{96} \\ 2 \end{array}$$

Check

4×24=96
96+2=98

The quotient is 4 and the remainder is 2.

Suppose you didn't know the multiples of 24 through 96. You would proceed by finding a **trial quotient.** A trial quotient is the number which you think most nearly tells how many times the divisor is contained in the dividend.

You can find the trial quotient for 98 divided by 24 by dividing the tens of the dividend by the tens of the divisor, like this: 9 T÷2 T=n. Think: How many times is 2 contained in 9? The answer is *not more than* 4 times. The trial quotient is 4. Is 4 the correct quotient?

Suppose you were asked to solve the equation 53÷17=n. To find the trial quotient, use the equation 5 T÷1 T=n. The trial quotient is 5. Check it by multiplying mentally as shown below.

5×17=85	4×17=68	3×17=51
85>53	68>53	51<53

Note: The sign > is read *is more than,* the sign < is read *is less than.* Since 51 is the multiple of 17 that is closest to but less than 53, the quotient is 3. 53÷17=3 r2.

See column 1 and above.

Oral Tell the trial quotient and what you did to find it.

	a	*b*
1.	$72 \div 12 = \overset{6}{n}$	$45 \div 15 = \overset{3}{n}$
2.	$56 \div 14 = \overset{4}{n}$	$78 \div 13 = \overset{6}{n}$
3.	$87 \div 29 = \overset{3}{n}$	$64 \div 16 = \overset{4}{n}$
4.	$54 \div 17 = \overset{3r3}{n}$	$99 \div 14 = \overset{7r1}{n}$
5.	$59 \div 18 = \overset{3r5}{n}$	$57 \div 19 = \overset{3}{n}$

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Written Solve the equations in rows 1 through 5 above. See above.

Dividing Three-Digit Numbers

Suppose that you are to divide 210 by 42. About how many times do you think that 42 is contained in 210?

To divide 210 by 42, you may proceed as shown.

$$\begin{array}{r} 5\\ 42\overline{)210}\\ \underline{210} \end{array}$$

Trial Quotient

21 T÷4 T = n

To determine the trial quotient, think: 21 T÷4 T = 5 plus a remainder. However, the remainder has nothing to do with determining the trial quotient. The trial quotient is 5. It tells you that 4 tens is contained in 21 tens not more than 5 times.

Sometimes the tens digit of the divisor will divide the hundreds digit of the three-digit dividend as in 643÷32 = n. About how many times do you think 32 is contained in 643? Your answer is greater than 10, because 10×32 = 320 and 320 < 643. But your answer is less than 100, because 100×32 = 3200 and 3200 > 643. So your answer must be a two-digit number; that is, a number between 10 and 100.

Observe the solution of 643÷32 = n in the next column. Do you see that 3 tens is contained in 3 ones less than 1 time? That is why 0 appears in ones place of the quotient.

$$\begin{array}{r} 20\ \text{r}3\\ 32\overline{)643}\\ \underline{64}\\ 3 \end{array}$$

Trial Quotient

6 H÷3 T = n

3 0÷3 T = n

Oral What steps would you take in finding each trial quotient? Why?
See examples in tint blocks.

	a	*b*
1.	140÷20 = $\overset{7}{n}$	200÷40 = $\overset{5}{n}$
2.	288÷72 = $\overset{4}{n}$	159÷53 = $\overset{3}{n}$
3.	800÷20 = $\overset{40}{n}$	960÷32 = $\overset{30}{n}$
4.	150÷25 = $\overset{6}{n}$	360÷42 = $\overset{8r24}{n}$

Written Solve each equation above. Then find each quotient in rows 5 through 10 below.

	a	*b*	*c*
5.	$\begin{array}{r}3\\92\overline{)276}\end{array}$	$\begin{array}{r}9\\30\overline{)270}\end{array}$	$\begin{array}{r}8\\93\overline{)744}\end{array}$
6.	$\begin{array}{r}9\\42\overline{)378}\end{array}$	$\begin{array}{r}4\\65\overline{)260}\end{array}$	$\begin{array}{r}9\\63\overline{)567}\end{array}$
7.	$\begin{array}{r}3\\37\overline{)111}\end{array}$	$\begin{array}{r}9\\89\overline{)801}\end{array}$	$\begin{array}{r}9\\12\overline{)108}\end{array}$
8.	$\begin{array}{r}8\text{r}9\\12\overline{)105}\end{array}$	$\begin{array}{r}8\text{r}5\\18\overline{)149}\end{array}$	$\begin{array}{r}30\\18\overline{)540}\end{array}$
9.	$\begin{array}{r}20\text{r}5\\40\overline{)805}\end{array}$	$\begin{array}{r}30\text{r}7\\23\overline{)697}\end{array}$	$\begin{array}{r}20\text{r}9\\38\overline{)769}\end{array}$
10.	$\begin{array}{r}60\text{r}9\\13\overline{)789}\end{array}$	$\begin{array}{r}30\text{r}3\\17\overline{)513}\end{array}$	$\begin{array}{r}40\text{r}3\\15\overline{)603}\end{array}$

A B C

$29\overline{)928}$

A: $\begin{array}{r} \text{3 tens} \\ 29\overline{)\,92 \text{ tens}} \\ \underline{87} \\ 5 \end{array}$ and B: $\begin{array}{r} \text{2 ones} \\ 29\overline{)\,58 \text{ ones}} \\ \underline{58} \end{array}$ therefore C: $\begin{array}{r} 32 \\ 29\overline{)928} \\ \underline{87} \\ 58 \\ \underline{58} \end{array}$

Dividing Tens and Ones

Observe the solution of the equa-ion 928÷29=n shown on the board. How can you tell that the quotient vill be a two-digit number? In A, here is a remainder of 5 tens. 29 s contained in 50 a whole number of imes. That is why the ones must be livided by 29. Observe the division f the 58 ones in B. Is there any emainder of ones? The completed olution is shown in C.

The steps in the division are outned below.

a. Think of 9 hundreds as 90 tens nd add them to the tens already in ens place. Round 29 to 30. Why?

b. Divide 92 tens by 30 or H÷3 T=n.

c. Multiply and subtract.

d. Think of 5 tens as 50 ones and ld them to the ones already in es place.

e. Divide 58 ones by 30 or T÷3 T=n. Multiply 29 by 2.

Oral Tell the steps to take in dividing 840 by 20. In dividing 730 by 20. In dividing 975 by 34.
See column 1.

Written Copy. Write each quotient.

	a	*b*	*c*
1.	$35\overline{)770}$ 22	$46\overline{)966}$ 21	$23\overline{)529}$ 23
2.	$40\overline{)840}$ 21	$90\overline{)990}$ 11	$20\overline{)680}$ 34
3.	$18\overline{)810}$ 45	$14\overline{)756}$ 54	$13\overline{)481}$ 37
4.	$10\overline{)730}$ 73	$13\overline{)299}$ 23	$15\overline{)480}$ 32
5.	$39\overline{)875}$ 22r17	$46\overline{)970}$ 21r4	$23\overline{)490}$ 21r7

Can you do this? Complete the divisions by replacing each dot with a numeral. See T63 for solutions.

a

$\begin{array}{r} 1● \\ 2●\overline{)●19} \\ \underline{2●} \\ 189 \\ \underline{●●●} \\ 5 \end{array}$

b

$\begin{array}{r} 1● \\ ●7\overline{)●●●} \\ \underline{3●} \\ 11● \\ \underline{111} \\ 1 \end{array}$

c

$\begin{array}{r} 6● \\ 1●\overline{)8●●} \\ \underline{●8} \\ 10● \\ \underline{104} \\ 5 \end{array}$

MORE PRACTICE PAGE 300

A Special Property of Multiplication and Division

You may remember that you can solve the equation $6\times25=n$ by first renaming 25 as the sum $20+5$ and then multiplying 20 by 6 and 5 by 6 as in A.

A
$$\begin{aligned}6\times25&=(6\times20)+(6\times5)\\&=120+30\\&=150\end{aligned}$$

Do you suppose that it is possible to rename a dividend as a sum and to divide each of its addends separately? Observe how you divide 150 by 6 when you first rename 150 as $120+30$.

B
$$\begin{aligned}150\div6&=(120+30)\div6\\&=(120\div6)+(30\div6)\\&=20+5\\&=25\end{aligned}$$

Since 120 and 30 are both multiples of 6, $120+30$ was a good way of renaming 150 as a sum. Why would $75+75$ have been a poor way for our purpose?

Try a few other divisions in which you first rename the dividend as a sum. For example, divide 161 by 7 by first renaming 161 as $(140+21)$. Then divide 140 by 7 and 21 by 7. Add the quotients. Did you obtain 23 as a quotient?

MORE PRACTICE PAGE 300

Oral Tell how you would complete each multiplication and division below. See column 1.

1. $7\times21=7\times(20+1)=n$ 147
2. $9\times65=9\times(60+5)=n$ 585
3. $68\div4=(60+8)\div4=n$ 17
4. $520\div8=(480+40)\div8=n$ 65
5. $72\div12=(60+12)\div12=n$ 6
6. $156\div12=(144+12)\div12=n$ 13

Written Solve for n. Do the computation without using pencil and paper.

	a	*b*
1.	$81\div3=n$ 27	$96\div4=n$ 24
2.	$84\div7=n$ 12	$46\div2=n$ 23
3.	$99\div7=n$ 14r1	$93\div3=n$ 31
4.	$128\div4=n$ 32	$176\div8=n$ 22
5.	$132\div6=n$ 22	$198\div9=n$ 22
6.	$72\div12=n$ 6	$45\div15=n$ 3
7.	$48\div16=n$ 3	$160\div20=n$ 8
8.	$372\div12=n$ 31	$750\div25=n$ 30
9.	$750\div50=n$ 15	$960\div30=n$ 32

Finding Averages

Al said, "I sold an average of 77 papers each day." Do you know what he meant by the word **average?**

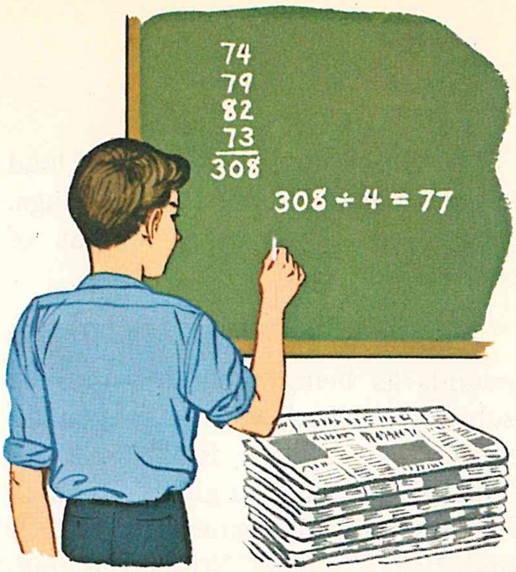
He sold 308 papers, but may not have sold the same number on any of the days. His division, 308 ÷ 4 = 77, tells us that he would have sold the same total amount *if* he had sold 77 papers each day. Thus 77 is called the average for the 4 days.

Al found the average by adding the numbers that expressed his daily sales. He divided the sum by 4, the number of addends used. The quotient, 77, is called the average.

Suppose you are told the sum and the number of addends. For example, "3 pupils sold 117 tickets. Each sold a different number of tickets." To find the average, divide 117 by 3. Why?

Study the example of how to find the average of five numbers.

$$\begin{array}{r} 139\\ 150\\ 125\\ 130\\ +143\\ \hline 687 \end{array} \qquad \begin{array}{r} 137\frac{2}{5}\\ 5\overline{)687}\\ \underline{5}\\ 18\\ \underline{15}\\ 37\\ \underline{35}\\ 2 \end{array}$$



The average is $137+\frac{2}{5}$ or $137\frac{2}{5}$. How many of the numbers are less than the average? More than the average?

Oral Tell how you would find the average of each set of numbers below.

1. $(45,^{+}67,^{+}50,^{+}39,^{+}46) \div 5 = 49\frac{2}{5}$
2. $(27,^{+}43,^{+}35,^{+}49,^{+}30,^{+}25) \div 6 = 34\frac{5}{6}$
3. $(19,^{+}17,^{+}21,^{+}30,^{+}29,^{+}16) \div 6 = 22$

Written Find the average of the numbers in each set in rows 1 through 3 above. Then solve these problems.

4. Tom and Roy tied newspapers in bundles. The bundles weighed 23 lb., 27 lb., 35 lb., and 31 lb. What was the average weight per bundle?
(23+27+35+31)÷4=n   29 lb.

5. Dick's father drove 96 miles in 3 hours. What was his average speed per hour? 96÷3=n   32 mph.

MORE PRACTICE PAGE 301

Dividing Larger Numbers

Mr. Nelson had a 2240-pound load of corn which he used to fill 32 bags. What was the average weight of each bag of corn?

You are to think of a set of 2240 pounds as being separated into 32 subsets of equal weight; so you divide. The equation is $2240 \div 32 = n$. The quotient will be greater than 10 but less than 100 because $10 \times 32 = 320$ and $100 \times 32 = 3200$. Study the complete solution shown below.

		Trial Quotients
	70	
	32)2240	22 H÷3 T=n
A	224	
	0	0 0÷3 T=n

The equations for the trial quotients are solved below.

22 H÷3 T=7 T r1 H

0 0÷3 T=0

You may remember that you disregard the remainders in finding a trial quotient. The first trial quotient is 7. Why is the second trial quotient 0?

You use the same steps in dividing larger numbers that you use in dividing smaller ones. You simply repeat the steps as often as needed.

The division process depends on these four important steps.

a. Determining a trial quotient.

b. Multiplying mentally by 10, 100, 1000, and so on. This is an easy way to decide in which place-value position to write the trial quotient. Use > and < to help you see tens.

c. Multiplying the divisor by the trial quotient.

d. Subtracting this product from the dividend.

Observe how the equation $17488 \div 34 = n$ is solved.

		Trial Quotients
	514 r12	
	34)17488	17 Th÷3 T=n
	170	
	48	4 H÷3 T=n
B	34	
	148	14 T÷3 T=n
	136	
	12	

Now observe how to multiply mentally to find the number of digits in the quotient in B.

$100 \times 34 = 3400$ and $3400 < 17488$

$1000 \times 34 = 34000$ and $34000 > 17488$

Do you see why n must be a three-digit number?

Now study the divisions in C and D. Observe that the quotient is a four-digit number in C and a five-digit number in D.

```
           3420                  17254
C   60)205200        D   41)707414
       180                   41
        252                  297
        240                  287
         120                  104
         120                   82
           0                   221
                               205
                                164
                                164
```

To determine how many digits the quotient in C will have, multiply mentally as shown.

$1000 \times 60 = 60000$ and $60000 < 205200$

$10000 \times 60 = 600000$ and $600000 > 205200$

Therefore n must be a four-digit number, and the first digit of the quotient must be written in thousands place.

To determine how many digits the quotient in D will have, multiply mentally again.

$10000 \times 41 = 410000$ and $410000 < 707414$

$100000 \times 41 = 4100000$ and $4100000 > 707414$

Therefore n must be a five-digit number, and the first digit of the quotient must be written in ten thousands place.

Oral Answer these questions.

1. To what number was the divisor rounded in finding the trial quotient in A? In B? In D? 30; 30; 40

2. To what number might you round a divisor of 39 when you are finding a trial quotient? 40

3. Which equations would you use in finding the other trial quotients in C? In D? See below.

4. Which equation would you use in finding the first trial quotient when solving the equation 310500÷23?
3HTh÷2T=n

Written Copy. Solve for n.

	a	*b*
1.	3420÷57=n 60	6320÷71=n 89r1
2.	3538÷58=n 61	1247÷29=n 43
3.	3230÷35=n 92r10	5458÷67=n 81r31
4.	39605÷24=n 1650r5	43610÷12=n 3634r2
5.	21571÷14=n 1540r11	37775÷19=n 1988r3

Copy. Write each quotient.

	a	*b*
6.	19)30782 1620r2	32)64992 2031
7.	20)16080 804	63)31563 501
8.	12)119258 9938r2	14)135957 9711r3
9.	21)659409 31400r9	13)103747 7980r7

MORE PRACTICE PAGE 301

. C: 25Th÷6T=n; 12H÷6T=n
D: 29TTh÷4T=n; 10Th÷4T=n; 22H÷4T=n; 16T÷4T=n

Using a Three-Digit Divisor

Suppose that a travel agent were asked to make travel arrangements for 672 students planning to visit Washington. He chartered jets with a capacity of 112 passengers. How many planes did he charter?

You are to think of a set of 672 as being separated into subsets of 112; so you divide. The equation is $672 \div 112 = n$. Since $1 \times 112 = 112$ and $112 < 672$, and since $10 \times 112 = 1120$ and $1120 > 672$, n must be greater than 1 but less than 10. The quotient will have only one digit.

To find the trial quotient, divide 6 hundreds by 1 hundred. 6 H÷1 H= 6. Does $6 \times 112 = 672$? Is 112 contained in 672 exactly 6 times?

Suppose you were to divide 1020 by 204. To find the trial quotient think: How many times are 2 hundreds contained in 10 hundreds? The answer is 5. Does $5 \times 204 = 1020$?

You use the same steps in dividing by a three-digit number as you use when dividing by a two-digit number. First estimate the quotient for the equation $7509 \div 212 = n$ as shown.

$$10 \times 212 = 2120 \text{ and } 2120 < 7509$$
$$100 \times 212 = 21200 \text{ and } 21200 > 7509$$

Therefore n must have 2 digits and be a number between 10 and 100.

Study the example below. Why are two different equations required in finding the trial quotients?

$$\begin{array}{r} 35\ r89 \\ 212\overline{)7509} \\ \underline{636} \\ 1149 \\ \underline{1060} \\ 89 \end{array}$$

Trial Quotients

7 Th÷2 H = n

11 H÷2 H = n

Oral Tell the steps to take in solving each equation below. Tell the reason for taking each step.
See column 1.

	a	*b*
1.	$4900 \div 245 = \overset{20}{n}$	$12800 \div 320 = \overset{40}{n}$
2.	$1842 \div 614 = \overset{3}{n}$	$11750 \div 235 = \overset{50}{n}$

Written Copy. Find each quotient. Check your computation.

	a	*b*
1.	$\begin{array}{r} 6 \\ 325\overline{)1950} \end{array}$	$\begin{array}{r} 9 \\ 230\overline{)2070} \end{array}$
2.	$\begin{array}{r} 8r30 \\ 150\overline{)1230} \end{array}$	$\begin{array}{r} 6r100 \\ 591\overline{)3646} \end{array}$
3.	$\begin{array}{r} 30 \\ 313\overline{)9390} \end{array}$	$\begin{array}{r} 50 \\ 149\overline{)7450} \end{array}$
4.	$\begin{array}{r} 80r6 \\ 112\overline{)8966} \end{array}$	$\begin{array}{r} 30r169 \\ 247\overline{)7579} \end{array}$
5.	$\begin{array}{r} 70r100 \\ 539\overline{)37830} \end{array}$	$\begin{array}{r} 20 \\ 985\overline{)19700} \end{array}$
6.	$\begin{array}{r} 70r100 \\ 700\overline{)49100} \end{array}$	$\begin{array}{r} 20r353 \\ 606\overline{)12473} \end{array}$

A

$140839 \div 144 = n$

$n \times 144 = 140839$

$100 \times 144 = 14400$ and $14400 < 140839$

$1000 \times 144 = 144000$ and $144000 > 140839$

Then $100 < n < 1000$ and n will have 3 digits

Dividing Larger Numbers by a Three-Digit Divisor

If you can multiply by 10, 100, 1000, and 10000, you should have little difficulty in determining the number of digits in any quotient. Study A. $100<n<1000$ is read 100 is less than n and n is less than 1000. Do you see that n must have 3 digits?

Study the examples below.

B

```
       978 r7
144)140839
    1296
     1123
     1008
      1159
      1152
         7
```

C

```
       2175
293)637275
    586
     512
     293
     2197
     2051
      1465
      1465
```

To find the first trial quotient in B, divide 1408 by 144. Since 10×144 is greater than 1408, the first trial quotient is 9. Which number do you divide by 144 to find the other trial quotients? How would you check the computation in B?

In C, the following equations are used in finding the trial quotients.

6 HTh ÷ 3 H = n

21 Th ÷ 3 H = n

5 TTh ÷ 3 H = n

14 H ÷ 3 H = n

Did each of the equations above give the correct trial quotient? Do you see that it was a good idea to round off 293 to 300 before attempting to find any of the trial quotients?

Oral Tell the multiplications you would do mentally to estimate a quotient for each division below. See explanation for A, B, and C on this page.

	a	*b*
1.	614)14736 — 24	348)11832 — 34
2.	161)12195 — 75r120	320)96000 — 300
3.	208)84340 — 405r100	230)115000 — 500
4.	340)136120 — 400r120	518)311847 — 602r11
5.	805)195620 — 243r5	150)450075 — 3000r75

Written Copy. Find the answer for each division in rows 1 through 5 above. Use multiplication to check your work.

MORE PRACTICE PAGE 301

Reviewing Addition, Subtraction, Multiplication, and Division

You should be able to do all of the computation on this page without referring to other parts of the book.

Part 1 Solve each equation for n.

	a	*b*
1.	52260 $65\times804=n$	69216 $96\times721=n$
2.	32886 $54\times609=n$	11456 $32\times358=n$
3.	446452 $956\times467=n$	241794 $342\times707=n$
4.	704811 $983\times717=n$	97230 $463\times210=n$
5.	20r30 $670\div32=n$	71r11 $3987\div56=n$
6.	690r78 $64248\div93=n$	1992r4 $75700\div38=n$
7.	16r150 $9446\div581=n$	39r44 $7532\div192=n$
8.	9746 $1984+7762=n$	5357 $5746-389=n$
9.	18489 $8624+9865=n$	5206 $4231+975=n$
10.	10548 $2563+7985=n$	50850 $75\times678=n$
11.	5r516 $4676\div832=n$	58r46 $4976\div85=n$
12.	1005 $865+93+47=n$	54r41 $3821\div70=n$
13.	844 $93+28+723=n$	103500 $225\times460=n$
14.	19102 $19969-867=n$	1807r20 $72300\div40=n$
15.	49733 $50000-267=n$	6177 $6940-763=n$
16.	15 $(72\div9)+n=23$	345750 $461\times750=n$
17.	9 $n-(96\div12)=1$	218r25 $8963\div41=n$

Part 2 Copy. Solve for n.

1. 144 $(1760\div20)+56=n$
2. 513 $(1431\div53)\times19=n$
3. 241548 $(905\times267)-87=n$
4. 669776 $(717\times934)+98=n$
5. 752376 $(763\times986)+58=n$
6. 61 $(3654\div87)+19=n$
7. 54 $4266\div79=n$
8. 89012 $(256\times348)-76=n$
9. 829797 $(903\times919)-60=n$
10. 19r13 $(846+934)\div93=n$

Part 3 Copy the pairs of equations. In column 2, replace n with one of the numerals in column 1.

	If	*Then*
1.	$34\times27=918$	27 $918\div n=34$
2.	$47\times56=2632$	47 $2632\div56=n$
3.	$4680\div65=72$	65 $72\times n=4680$
4.	$9724\div68=143$	9724 $68\times143=n$
5.	$5625\div25=225$	5625 $25\times225=n$
6.	$3400\div100=34$	100 $n\times34=3400$
7.	$7260\div30=242$	30 $n\times242=7260$

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Multiplication and division are inverses of each other. (58)

2. Zero divided by a number other than 0 will always give a quotient of 0. (59)

3. Dividing by a one-digit divisor involves the steps divide, multiply, and subtract. (60)

4. To find the trial quotient when the first digit of the divisor is less than or equal to the first digit of the dividend, divide the first digit of the dividend by the first digit of the divisor. In other cases divide the first two digits of the dividend by the first digit of the divisor. (62, 63)

5. Multiplying the divisor by 10, 100, 1000, and so on is an easy way to determine the number of digits in the quotient. (66-69)

Words to Know

1. Dividend, divisor, quotient (57)

2. Multiple of, divisible by (60)

3. Trial quotient (61)

4. Greater than, less than (61)

5. Average (65)

Questions to Discuss

See T71 for answers.

1. Does division have the commutative property? What example can you give to show that your answer is correct? (59)

2. How do you check your work in division? (60, 61)

3. Why is it a good idea to multiply mentally when you are deciding whether or not a trial quotient is correct? (61)

4. How is dividing by a three-digit divisor similar to dividing by a two-digit divisor? (68)

Written Practice

Copy. Find each quotient and remainder.

	a	*b*	
1.	1155r2 $6\overline{)6932}$	43662r3 $7\overline{)305637}$	(60)
2.	7r1 $21\overline{)148}$	9r44 $74\overline{)710}$	(62)
3.	32 $30\overline{)960}$	32r5 $15\overline{)485}$	(63)
4.	80 $60\overline{)4800}$	17359r5 $18\overline{)312467}$	(67)
5.	3r200 $233\overline{)899}$	20r353 $606\overline{)12473}$	(68)
6.	2358r83 $415\overline{)978653}$	236r651 $792\overline{)187563}$	(69)

Self-Evaluation

Part 1 Copy. Solve for n.

	a	b
1.	40000 1000×40 = n	1000 n = 40000÷40
2.	54000 1000×54 = n	1000 n = 54000÷54
3.	500 30000÷60 = n	3000 n = 60000÷20
4.	3r150 750÷200 = n	107r400 n = 86000÷800

Part 2 Each division in 1 through 4 is followed by four statements. Two of them are true. Write the letters indicating the true statements.

1. 14736 divided by 24

T **a.** The quotient has 3 digits.

b. The first trial quotient is 7.

c. 24 is the dividend.

T **d.** The remainder is 0.

2. 884896 divided by 32

a. The dividend has 5 digits.

b. The first trial quotient is obtained by dividing 8 TTh by 3 T.

T **c.** The remainder is 0.

T **d.** The quotient has 5 digits.

3. 7579 divided by 247

T **a.** The divisor is contained in the dividend at least 10 times.

b. The quotient is greater than 100 but less than 1000.

c. The remainder is 0.

T **d.** The first trial quotient is 3.

4. 617529 divided by 379

T **a.** 379 may be rounded to 400 to help find a trial quotient.

b. 379 multiplied by 1000 results in a number greater than the dividend.

T **c.** The quotient has 4 digits.

d. The remainder is 0.

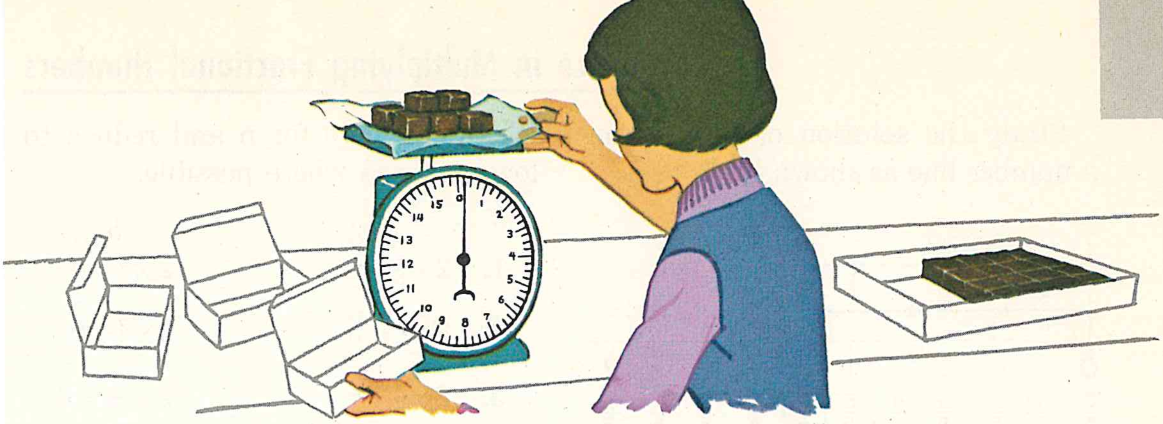
Part 3 Copy. Complete the following division.

	a	b
1.	62r9 340)21089	86 406)34916
2.	5r65 224)1185	5008 125)626000
3.	310r79 159)49369	300r15 900)270015
4.	902 194)174988	423 314)132822
5.	324r25 356)115369	1200 214)256800

Part 4 Copy. Solve each problem.

1. As an advertising stunt a new car was driven 2880 miles in 24 hours. What was its average speed per hour?
2880÷24=n 120 mph

2. The estimated September school enrollment of a city was 12,400 pupils and there was to be an average of 28 pupils per classroom. How many classrooms were needed? 12,400÷28=n 442r24 or 443 classrooms



Multiplying a Fractional Number by a Whole Number

Betsy plans to fill 3 boxes of candy, using $\frac{1}{4}$ pound of candy in each box. How much candy will she use?

You are to think of three $\frac{1}{4}$'s of a pound as being joined; so you multiply the numbers. The equation is $3\times\frac{1}{4}=n$.

The rapid way to find the sum of three $\frac{1}{4}$'s is to multiply 1 fourth by 3. Just as 3 times 1 *one* equals 3 ones, 3 times 1 *fourth* equals 3 fourths. In each case you obtained the result by multiplying 1 by 3.

You add and subtract fractional numbers as you add and subtract whole numbers. You might expect to multiply a fractional number by a whole number as you multiply a whole number by a whole number. See if this is true.

$$3\times\frac{1}{4} \text{ means } \frac{1}{4}+\frac{1}{4}+\frac{1}{4}$$

$$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{1+1+1}{4}=\frac{3}{4} \qquad 3\times\frac{1}{4}=\frac{3\times1}{4}=\frac{3}{4}$$

Which numbers would you multiply if you were asked to multiply 1 fifth by 5? 1 sixth by 5? How would you use repeated addition to check your answer to each of the above questions?

Oral Complete each statement below. Express the value of n in reduced form. See below for 4, 5, and 6.

1. $3\times\frac{1}{6}$ means $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$ $\quad 3\times\frac{1}{6}=n$ $\frac{1}{2}$
2. $3\times\frac{1}{3}$ means $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ $\quad 3\times\frac{1}{3}=n$ 1
3. $2\times\frac{1}{4}$ means $\frac{1}{4}+\frac{1}{4}$ $\quad 2\times\frac{1}{4}=n$ $\frac{1}{2}$
4. $6\times\frac{1}{4}$ means ____ $\quad 6\times\frac{1}{4}=n$ $1\frac{1}{2}$
5. $9\times\frac{1}{4}$ means ____ $\quad 9\times\frac{1}{4}=n$ $2\frac{1}{4}$
6. $8\times\frac{2}{3}$ means ____ $\quad 8\times\frac{2}{3}=n$ $5\frac{1}{3}$

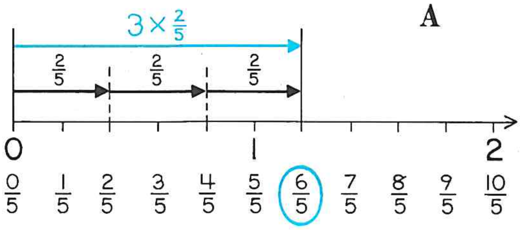
Written Solve each equation for n in rows 1 through 6 above. Check your work in the form of repeated addition.

4. $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$
5. $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$
6. $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$

Practice in Multiplying Fractional Numbers

Study the solution of $3\times\frac{2}{5}=n$ on a number line as shown in A.

A


$3\times\frac{2}{5}$
$\frac{2}{5}$
$\frac{2}{5}$
$\frac{2}{5}$
0
1
2
$\frac{0}{5}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$ $\frac{6}{5}$ $\frac{7}{5}$ $\frac{8}{5}$ $\frac{9}{5}$ $\frac{10}{5}$

$$3\times\frac{2}{5} \text{ means } \frac{2}{5}+\frac{2}{5}+\frac{2}{5}$$

$$\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=\frac{6}{5}=\frac{5+1}{5}=1+\frac{1}{5}=1\frac{1}{5}$$

Therefore $3\times\frac{2}{5}=\frac{3\times2}{5}=\frac{6}{5}=\frac{5+1}{5}=1+\frac{1}{5}=1\frac{1}{5}$.

The fast way to multiply 2 fifths by 3 is to multiply 2 by 3 and to then express the result as fifths like this: $3\times\frac{2}{5}=\frac{6}{5}=1\frac{1}{5}$.

Observe B, C, and D. What steps are taken in each multiplication? How was the result of each multiplication reduced to lowest terms?

B $4\times\frac{3}{4}=\frac{4\times3}{4}=\frac{12}{4}=12\div4=3$

C $5\times\frac{3}{14}=\frac{5\times3}{14}=\frac{15}{14}=15\div14=1\frac{1}{14}$

D $10\times\frac{5}{6}=\frac{10\times5}{6}=\frac{50}{6}=50\div6=8\frac{2}{6}=8\frac{1}{3}$

Oral How would you use a number line to show that the following statements are true? See A above.

	a	*b*
1.	$2\times\frac{4}{5}=1\frac{3}{5}$	$4\times\frac{2}{5}=1\frac{3}{5}$
2.	$3\times\frac{5}{6}=2\frac{1}{2}$	$5\times\frac{2}{3}=3\frac{1}{3}$

Written Solve for n and reduce to lowest terms where possible.

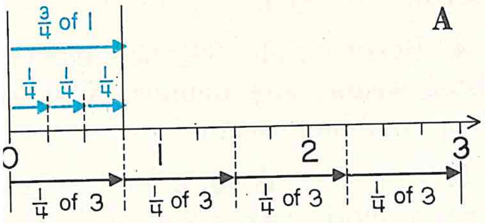
	a	*b*
1.	$2\times\frac{2}{5}=n$ $\frac{4}{5}$	$2\times\frac{3}{7}=n$ $\frac{6}{7}$
2.	$2\times\frac{4}{9}=n$ $\frac{8}{9}$	$3\times\frac{2}{7}=n$ $\frac{6}{7}$
3.	$2\times\frac{3}{8}=n$ $\frac{3}{4}$	$2\times\frac{5}{12}=n$ $\frac{5}{6}$
4.	$2\times\frac{3}{16}=n$ $\frac{3}{8}$	$3\times\frac{2}{15}=n$ $\frac{2}{5}$
5.	$5\times\frac{2}{5}=n$ 2	$6\times\frac{2}{3}=n$ 4
6.	$8\times\frac{2}{3}=n$ $5\frac{1}{3}$	$9\times\frac{5}{9}=n$ 5
7.	$4\times\frac{2}{3}=n$ $2\frac{2}{3}$	$6\times\frac{3}{7}=n$ $2\frac{4}{7}$
8.	$3\times\frac{7}{10}=n$ $2\frac{1}{10}$	$8\times\frac{5}{7}=n$ $5\frac{5}{7}$
9.	$3\times\frac{5}{9}=n$ $1\frac{2}{3}$	$2\times\frac{3}{4}=n$ $1\frac{1}{2}$
10.	$6\times\frac{4}{15}=n$ $1\frac{3}{5}$	$6\times\frac{7}{8}=n$ $5\frac{1}{4}$
11.	$2\times\frac{2}{9}=n$ $\frac{4}{9}$	$4\times\frac{2}{9}=n$ $\frac{8}{9}$
12.	$2\times\frac{3}{10}=n$ $\frac{3}{5}$	$5\times\frac{2}{15}=n$ $\frac{2}{3}$
13.	$5\times\frac{4}{5}=n$ 4	$9\times\frac{2}{3}=n$ 6
14.	$5\times\frac{5}{12}=n$ $2\frac{1}{12}$	$4\times\frac{3}{5}=n$ $2\frac{2}{5}$

A quick review Express each of the following fractions as mixed fractions in simplest form.

	a	*b*	*c*	*d*
1.	$\frac{8}{3}$ $2\frac{2}{3}$	$\frac{45}{6}$ $7\frac{1}{2}$	$\frac{14}{9}$ $1\frac{5}{9}$	$\frac{55}{8}$ $6\frac{7}{8}$
2.	$\frac{4}{2}$ 2	$\frac{45}{3}$ 15	$\frac{5}{4}$ $1\frac{1}{4}$	$\frac{56}{4}$ 14

ultiplying a Whole Number by a Fractional Number

Study the solution of $\frac{1}{4}\times3=n$ on number line as shown in A.



A

The diagram in A shows that $\frac{3}{4}$ of represents the same length as $\frac{1}{4}$ of . Therefore $\frac{3}{4}\times1=\frac{1}{4}\times3$.

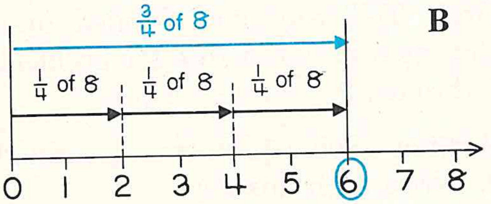
$$\frac{3}{4}\times1=\frac{3\times1}{4}=\frac{3}{4}$$

$$\frac{1}{4}\times3=\frac{1\times3}{4}=\frac{3}{4}$$

Is the multiplication of fractional umbers commutative?

You might have expected that hanging the order of the factors vould not change the result, since vou have already learned that operaions with fractional numbers are imilar to operations with whole umbers.

Suppose you wanted to find $\frac{3}{4}$ of 8 miles. You could find the answer by using diagram B.



B

A faster way to proceed would be to write the equation $\frac{3}{4}\times8=n$ and to solve for n. Since $\frac{3}{4}\times8=8\times\frac{3}{4}$, solve the equation $\frac{3}{4}\times8=n$ by multiplying 3 and 8. Divide the result by 4.

Oral What steps would you take in solving the equation $\frac{3}{4}\times8=n$? In solving the equation $\frac{2}{3}\times15=n$?

See column 1.

Written Copy. Solve for n.

	a	*b*
1.	$\frac{1}{3}\times6=n$ 2	$\frac{1}{4}\times84=n$ 21
2.	$\frac{1}{9}\times36=n$ 4	$\frac{1}{7}\times49=n$ 7
3.	$\frac{1}{3}\times10=n$ $3\frac{1}{3}$	$\frac{1}{2}\times5=n$ $2\frac{1}{2}$
4.	$\frac{1}{10}\times35=n$ $3\frac{1}{2}$	$\frac{1}{4}\times18=n$ $4\frac{1}{2}$
5.	$\frac{3}{4}\times12=n$ 9	$\frac{3}{5}\times15=n$ 9
6.	$\frac{3}{10}\times10=n$ 3	$\frac{2}{3}\times12=n$ 8
7.	$\frac{2}{3}\times14=n$ $9\frac{1}{3}$	$\frac{2}{9}\times19=n$ $4\frac{2}{9}$
8.	$\frac{3}{4}\times22=n$ $16\frac{1}{2}$	$\frac{7}{10}\times8=n$ $5\frac{3}{5}$
9.	$\frac{1}{6}\times12=n$ 2	$\frac{1}{2}\times28=n$ 14
10.	$\frac{1}{8}\times17=n$ $2\frac{1}{8}$	$\frac{1}{8}\times12=n$ $1\frac{1}{2}$
11.	$\frac{3}{8}\times16=n$ 6	$\frac{2}{5}\times25=n$ 10
12.	$\frac{3}{8}\times17=n$ $6\frac{3}{8}$	$\frac{5}{8}\times20=n$ $12\frac{1}{2}$
13.	$\frac{2}{3}\times19=n$ $12\frac{2}{3}$	$\frac{3}{4}\times27=n$ $20\frac{1}{4}$

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Solving Problem

You determine the operations to use in solving problems containing fractional numbers in a way similar to that you have used with whole numbers. Begin by reading the problem several times to help you decide whether to add, subtract, multiply, or divide the numbers in the problem. These suggestions may help you decide.

a. Add to find the sum of two or more different numbers.

b. Multiply to find the product of two numbers or to find a fractional part of a number.

c. Subtract to find the difference between two numbers.

d. Divide to find the quotient of two numbers.

After you determine the operation to use, write the equation.

1. Jane practices her music lesson $\frac{3}{4}$ of an hour each day. How many hours does she practice in 5 days?

$5\times\frac{3}{4}=n$ $3\frac{3}{4}$ hr.

2. A restaurant owner plans to serve about $\frac{2}{3}$ of a pound of turkey to each guest. He estimates that 24 guests will order turkey. How many pounds of turkey does he need?

$24\times\frac{2}{3}=n$ 16 lb.

3. Larry needs a board 10 inches wide for a window box. The widest board that he has is $7\frac{1}{4}$ inches wid By how much is his board to narrow?

$10-7\frac{1}{4}=n$ $2\frac{3}{4}$ in.

4. Betty weighs $86\frac{1}{4}$ pounds. He sister weighs $93\frac{3}{4}$ pounds. What i their combined weight?

$86\frac{1}{4}+93\frac{3}{4}=n$ 180 lb.

5. Ann is making a quilt. Eac strip of cloth makes $\frac{7}{8}$ of an inch o quilt. How many inches wide will th quilt be after she uses 72 strips?

$72\times\frac{7}{8}=n$ 63 in.

6. Bob cut a strip $1\frac{3}{4}$ inches wid from a piece of cardboard $9\frac{1}{2}$ inches wide. What is the width of the remaining piece of cardboard?

$9\frac{1}{2}-1\frac{3}{4}=n$ $7\frac{3}{4}$ in.

7. Nancy's baby sister is gaining an average of 4 ounces, or $\frac{1}{4}$ of a pound, a week. How many pounds will she gain in 13 weeks?

$13\times\frac{1}{4}=n$ $3\frac{1}{4}$ lb

8. Mary used $1\frac{1}{3}$ cups of milk in baking bread and $\frac{3}{4}$ of a cup of milk in a batch of cookies. How much milk did she use in the bread and the cookies together?

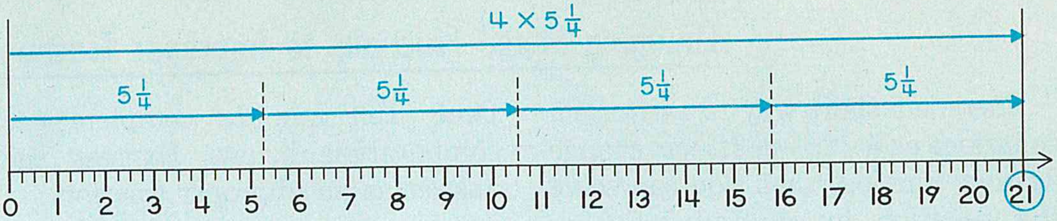
$1\frac{1}{3}+\frac{3}{4}=n$ $2\frac{1}{12}$ cups

9. Billy needs one $4\frac{2}{3}$-foot board and one $6\frac{1}{4}$-foot board. How much waste will he have if he buys a 12-foot board?

$12-(4\frac{2}{3}+6\frac{1}{4})=n$ $1\frac{1}{12}$ ft.

Oral Tell why you decided upon the equation you wrote for problems 1 through 9. See column 1.

Written Solve problems 1 through 9. Write each answer.



Multiplication Using Mixed Fractions

John bought 4 baseballs that weighed $5\frac{1}{4}$ ounces each. How much in all did the baseballs weigh?

You are to think of 4 objects of $5\frac{1}{4}$ ounces each as being joined; so you multiply. The equation is $4\times5\frac{1}{4}=n$.

Recall that $5\frac{1}{4}$ means $5+\frac{1}{4}$. Then the equation becomes $4\times(5+\frac{1}{4})=n$ which may be solved as in A.

A

$4\times5\frac{1}{4}=n$

Rename $5\frac{1}{4}$ — $4\times(5+\frac{1}{4})=n$

Multiply — $(4\times5)+(4\times\frac{1}{4})=n$

Add — $20+1=n$

$21=n$

The baseballs weighed 21 ounces.

In solving $4\times(5+\frac{1}{4})=n$ as shown in A, you see again that fractional numbers behave like whole numbers. You can multiply 5 by 4 and $\frac{1}{4}$ by 4 separately just as you can multiply 10 by 4 and 5 by 4 separately when solving the equation $4\times15=n$.

You may wish to work in column form when one or both factors have more than one digit. To illustrate, $18\times15\frac{1}{3}=n$ may be solved as in B.

Observe that some of the partial products were omitted.

B

$$\begin{array}{rr} & 15\frac{1}{3} \\ & \times 18 \\ \hline 18\times\frac{1}{3}= & 6 \\ 8\times15= & 120 \\ 10\times15= & 150 \\ \hline \text{Add}= & 276 \end{array}$$

Oral Tell the steps you would take in solving for n in column *a*.
See above.

	a	*b*
1.	27 $6\times4\frac{1}{2}=n$	184 $12\times15\frac{1}{3}=n$
2.	24 $9\times2\frac{2}{3}=n$	729 $18\times40\frac{1}{2}=n$
3.	$6\frac{1}{2}$ $2\times3\frac{1}{4}=n$	1314 $24\times54\frac{3}{4}=n$
4.	14 $8\times1\frac{3}{4}=n$	3054 $36\times84\frac{5}{6}=n$

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Written Solve for n in rows 1 through 4 above.

Changing Mixed Fractions to Improper Fractions

There is a short way to solve such equations as $4\times2\frac{2}{3}=n$. If you change $2\frac{2}{3}$ to a fraction, then you can solve the equation by multiplying fractional numbers. You know that any whole number multiplied by 1 remains unchanged. You also know that 1 can be expressed in many different forms as a fraction. One form is $\frac{3}{3}$. Then:

$$2\times1=2\times\frac{3}{3}=\frac{2\times3}{3}=\frac{6}{3}$$

You choose $\frac{3}{3}$ as the name for 1 because the fractional part of the mixed fraction $2\frac{2}{3}$ is $\frac{2}{3}$. Always choose the fractional form for 1 that has the same denominator as the fractional part of the mixed fraction.

Study the steps used in changing $2\frac{2}{3}$ to a fraction.

$$\begin{aligned}2\tfrac{2}{3}=2+\tfrac{2}{3}&=(2\times1)+\tfrac{2}{3}\\&=(2\times\tfrac{3}{3})+\tfrac{2}{3}\\&=\tfrac{2\times3}{3}+\tfrac{2}{3}\\&=\tfrac{6}{3}+\tfrac{2}{3}=\tfrac{8}{3}\end{aligned}$$

You may be able to do several of the above steps mentally.

Suppose you are to change $5\frac{3}{4}$ to an improper fraction. You would use $\frac{4}{4}$ as the name for 1. By which number would you multiply $\frac{4}{4}$? Does $\frac{20}{4}+\frac{3}{4}=\frac{23}{4}$? Why?

Oral Tell how to complete each computation below. Express each answer as an improper fraction.

1. $2\frac{2}{5}=2+\frac{2}{5}=(2\times\frac{5}{5})+\frac{2}{5}=\frac{12}{5}$
2. $3\frac{2}{3}=3+\frac{2}{3}=(3\times\frac{3}{3})+\frac{2}{3}=\frac{11}{3}$
3. $4\frac{5}{6}=4+\frac{5}{6}=(4\times\frac{6}{6})+\frac{5}{6}=\frac{29}{6}$
4. $6\frac{3}{8}=6+\frac{3}{8}=(6\times\frac{8}{8})+\frac{3}{8}=\frac{51}{8}$
5. $5\frac{1}{4}=5+\frac{1}{4}=(5\times\frac{4}{4})+\frac{1}{4}=\frac{21}{4}$

Written Write each of the following mixed fractions as an improper fraction in lowest terms.

	a	*b*	*c*	*d*
1.	$3\frac{5}{6}$ $\frac{23}{6}$	$2\frac{7}{10}$ $\frac{27}{10}$	$9\frac{3}{8}$ $\frac{75}{8}$	$4\frac{3}{4}$ $\frac{19}{4}$
2.	$4\frac{1}{4}$ $\frac{17}{4}$	$7\frac{1}{6}$ $\frac{43}{6}$	$8\frac{2}{3}$ $\frac{26}{3}$	$10\frac{6}{7}$ $\frac{76}{7}$
3.	$5\frac{1}{2}$ $\frac{11}{2}$	$3\frac{7}{9}$ $\frac{34}{9}$	$12\frac{2}{3}$ $\frac{38}{3}$	$21\frac{3}{5}$ $\frac{10}{5}$
4.	$9\frac{1}{6}$ $\frac{55}{6}$	$18\frac{2}{3}$ $\frac{56}{3}$	$7\frac{3}{4}$ $\frac{31}{4}$	$5\frac{6}{7}$ $\frac{41}{7}$
5.	$32\frac{3}{4}$ $\frac{131}{4}$	$17\frac{2}{5}$ $\frac{87}{5}$	$27\frac{2}{3}$ $\frac{83}{3}$	$19\frac{5}{6}$ $\frac{119}{6}$
6.	$9\frac{2}{3}$ $\frac{29}{3}$	$17\frac{1}{5}$ $\frac{86}{5}$	$23\frac{4}{5}$ $\frac{119}{5}$	$11\frac{7}{8}$ $\frac{95}{8}$

Can you do this? Find a replacement for n such that each statement becomes true.

	a	*b*
1.	$\frac{2}{3}$ $5\times n=\frac{10}{3}$	3 $n\times\frac{5}{6}=\frac{15}{6}$
2.	$\frac{3}{4}$ $7\times n=\frac{21}{4}$	9 $n\times\frac{3}{5}=\frac{27}{5}$
3.	$\frac{4}{5}$ $9\times n=\frac{36}{5}$	9 $n\times\frac{7}{8}=\frac{63}{8}$

Multiplication Using Improper Fractions

Suppose you were asked to multiply $3\frac{2}{5}$ by 7. You might proceed by multiplying 3 by 7 and $\frac{2}{5}$ by 7 and then adding the results. Another way to perform the multiplication is to rename $3\frac{2}{5}$ as $\frac{17}{5}$. Then the problem becomes one of multiplying a fractional number by a whole number as shown below.

$$7\times\frac{17}{5}=\frac{7\times17}{5}=\frac{119}{5}=119\div5=23\frac{4}{5}$$

In solving equations in which one factor is named by a mixed fraction and the other factor is a whole number, decide on the best way to proceed. Then choose the way that permits you to do the greatest possible amount of work mentally. In $6\times1\frac{1}{3}=n$, you will probably multiply 1 by 6 and $\frac{1}{3}$ by 6. In $6\times1\frac{5}{7}=n$, you will probably decide first to change $1\frac{5}{7}$ to an improper fraction and then to multiply $\frac{12}{7}$ by 6. Why?

Oral Tell the steps you would use in solving for n by using two different methods. See above.

	a	*b*
1.	$6\times1\frac{1}{3}=n$ 8	$6\times1\frac{1}{2}=n$ 9
2.	$2\times3\frac{2}{3}=n$ $7\frac{1}{3}$	$2\times1\frac{4}{5}=n$ $3\frac{3}{5}$
3.	$1\frac{1}{4}\times2=n$ $2\frac{1}{2}$	$2\frac{2}{3}\times6=n$ 16

Written Solve each equation below. Check for accuracy.

	a	*b*
1.	$9\times1\frac{2}{3}=n$ 15	$6\times1\frac{5}{6}=n$ 11
2.	$3\times1\frac{3}{7}=n$ $4\frac{2}{7}$	$4\times2\frac{1}{6}=n$ $8\frac{2}{3}$
3.	$5\times2\frac{3}{10}=n$ $11\frac{1}{2}$	$6\times4\frac{3}{4}=n$ $28\frac{1}{2}$
4.	$9\times9\frac{7}{8}=n$ $88\frac{7}{8}$	$10\times5\frac{2}{7}=n$ $52\frac{6}{7}$
5.	$8\times14\frac{3}{5}=n$ $116\frac{4}{5}$	$6\times13\frac{7}{8}=n$ $83\frac{1}{4}$
6.	$17\frac{1}{5}\times7=n$ $120\frac{2}{5}$	$22\frac{3}{4}\times8=n$ 182
7.	$8\frac{7}{8}\times6=n$ $53\frac{1}{4}$	$15\frac{1}{2}\times10=n$ 155
8.	$5\frac{2}{3}\times9=n$ 51	$6\frac{7}{8}\times12=n$ $82\frac{1}{2}$
9.	$8\frac{1}{2}\times14=n$ 119	$15\times7\frac{2}{3}=n$ 115
10.	$25\times8\frac{2}{3}=n$ $216\frac{2}{3}$	$19\times8\frac{4}{5}=n$ $167\frac{1}{5}$
11.	$10\times9\frac{3}{4}=n$ $97\frac{1}{2}$	$4\frac{1}{2}\times10=n$ 45

Can you do this? Solve for n. Reminder: Associate, or regroup, the factors in the most convenient way.

	a	*b*
1.	$7\times5\times\frac{3}{5}=n$ 21	$\frac{5}{9}\times9\times6=n$ 30
2.	$8\times\frac{3}{4}\times6=n$ 36	$6\times\frac{2}{3}\times5=n$ 20
3.	$\frac{9}{10}\times5\times15=n$ $67\frac{1}{2}$	$7\times9\times\frac{7}{9}=n$ 49
4.	$36\times\frac{5}{12}\times10=n$ 150	$12\times4\times\frac{5}{6}=n$ 40
5.	$8\times7\times\frac{6}{7}=n$ 48	$15\times\frac{4}{5}\times8=n$ 96

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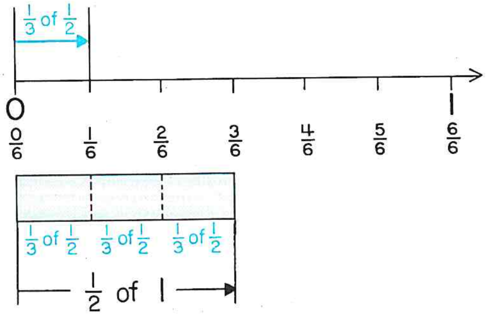
Multiplying Two Fractional Numbers

John had a piece of tape $\frac{1}{2}$ of a yard in length. He used $\frac{1}{3}$ of it in splicing 2 wires. What fractional part of a yard of tape did he use?

As you can see from the following diagram, dividing $\frac{1}{2}$ of 1 into thirds produces three $\frac{1}{6}$'s.


$\frac{1}{3}$ of $\frac{1}{2}$
0
1
$\frac{0}{6}$
$\frac{1}{6}$
$\frac{2}{6}$
$\frac{3}{6}$
$\frac{4}{6}$
$\frac{5}{6}$
$\frac{6}{6}$
$\frac{1}{3}$ of $\frac{1}{2}$
$\frac{1}{3}$ of $\frac{1}{2}$
$\frac{1}{3}$ of $\frac{1}{2}$
$\frac{1}{2}$ of 1

John used a piece of tape having a length of $\frac{1}{6}$ of a yard.

Notice that dividing $\frac{1}{2}$ of 1 into 3 segments of equal length increases the number of segments 3 times. Do you see why you can find the value of $\frac{1}{3}$ of $\frac{1}{2}$ by multiplying 3 and 2? Study the multiplication of $\frac{1}{2}$ and $\frac{1}{3}$ in A.

A

$\frac{1}{3}$ of $\frac{1}{2}=\frac{1}{3}\times\frac{1}{2}=\frac{1\times1}{3\times2}=\frac{1}{6}$

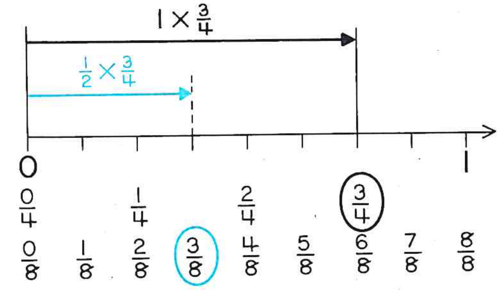
Suppose you were to divide $\frac{1}{2}$ of 1 into 4 segments of equal length, and were to find the length of each $\frac{1}{4}$ of $\frac{1}{2}$. You would proceed by finding $\frac{1}{4}$ of $\frac{1}{2}$ as shown in B.

B

$\frac{1}{4}$ of $\frac{1}{2}=\frac{1}{4}\times\frac{1}{2}=\frac{1\times1}{4\times2}=\frac{1}{8}$

Dividing $\frac{1}{2}$ of 1 into fourths increases the number of segments 4 times. Each segment formed is only $\frac{1}{4}$ the length of the original segment. On a ruler find $\frac{1}{4}$ of $\frac{1}{2}$ of an inch. Is its length $\frac{1}{8}$ of an inch?

Now suppose you were asked to find $\frac{1}{2}$ of $\frac{3}{4}$ of 1 yard of ribbon. You could use a diagram as shown below.



Do you see that multiplying $\frac{1}{2}$ and $\frac{3}{4}$ produces $\frac{3}{8}$? You have learned that $\frac{1}{4}\times\frac{1}{2}$ produces $\frac{1}{8}$. Would you expect $\frac{1}{2}\times\frac{1}{4}$ to also produce $\frac{1}{8}$? Then $\frac{1}{2}\times\frac{3}{4}$ should produce a result that is 3 times greater than $\frac{1}{2}\times\frac{1}{4}$. Thus $3\times(\frac{1}{2}\times\frac{1}{4})=3\times\frac{1}{8}=\frac{3}{8}$.

When you multiplied $\frac{1}{2}$ by $\frac{1}{3}$ you increased the number of units 3 times. When you multiplied $\frac{1}{2}$ by $\frac{1}{4}$ you increased the number of units 4 times. By how many times would you expect to increase the number of units if you multiplied $\frac{1}{3}$ by $\frac{1}{3}$? $\frac{1}{8}$ by $\frac{1}{6}$? $\frac{1}{9}$ by $\frac{1}{7}$? Try similar multiplications of fractions having numerators of 1 to convince yourself that the numerator of the product is 1 and that the denominator of the product names the product of the numbers named by the denominators.

Oral Tell how to construct a diagram to show that each statement below is true. See page 80.

1. $\frac{1}{2}$ of $\frac{1}{6}$ equals $\frac{1}{12}$.
2. $\frac{1}{3}$ of $\frac{1}{2}$ of 1 equals $\frac{1}{6}$.
3. $\frac{1}{3}$ of $\frac{2}{3}$ of 1 equals $\frac{2}{9}$.

Tell the denominator of the answer in each equation below.

	a	b
4.	$\frac{1}{4}\times\frac{1}{5}=n$ 20	$\frac{1}{7}\times\frac{1}{8}=n$ 56
5.	$\frac{1}{9}\times\frac{1}{6}=n$ 54	$\frac{1}{12}\times\frac{1}{15}=n$ 180

Written Copy. Solve for n.

	a	b
1.	$\frac{1}{3}\times\frac{1}{2}=n$ $\frac{1}{6}$	$\frac{1}{2}\times\frac{1}{5}=n$ $\frac{1}{10}$
2.	$\frac{1}{5}\times\frac{1}{6}=n$ $\frac{1}{30}$	$\frac{1}{2}\times\frac{1}{7}=n$ $\frac{1}{14}$
3.	$\frac{1}{2}\times\frac{1}{9}=n$ $\frac{1}{18}$	$\frac{1}{3}\times\frac{1}{5}=n$ $\frac{1}{15}$
4.	$\frac{1}{5}\times\frac{1}{4}=n$ $\frac{1}{20}$	$\frac{1}{2}\times\frac{1}{2}=n$ $\frac{1}{4}$
5.	$\frac{1}{5}\times\frac{1}{8}=n$ $\frac{1}{40}$	$\frac{1}{2}\times\frac{3}{5}=n$ $\frac{3}{10}$
6.	$3\times\frac{1}{6}=n$ $\frac{1}{2}$	$\frac{1}{4}\times\frac{3}{5}=n$ $\frac{3}{20}$
7.	$\frac{1}{4}\times\frac{2}{3}=n$ $\frac{1}{6}$	$\frac{1}{2}\times\frac{2}{5}=n$ $\frac{1}{5}$
8.	$\frac{1}{2}\times\frac{2}{9}=n$ $\frac{1}{9}$	$\frac{1}{3}\times\frac{3}{5}=n$ $\frac{1}{5}$
9.	$\frac{1}{4}\times\frac{2}{5}=n$ $\frac{1}{10}$	$\frac{1}{2}\times\frac{8}{9}=n$ $\frac{4}{9}$
10.	$\frac{1}{3}\times\frac{7}{8}=n$ $\frac{7}{24}$	$\frac{1}{7}\times\frac{5}{12}=n$ $\frac{5}{84}$

Solve these problems.

11. Dick had a string $\frac{3}{4}$ of a yard in length. He used $\frac{1}{3}$ of it to tie up a climbing rose. What fraction of a yard of string did he use?

$\frac{1}{3}\times\frac{3}{4}=n$ $\frac{1}{4}$ yd.

12. Sally lives $\frac{9}{10}$ of a mile from school. Her father gave her a ride for $\frac{1}{2}$ of the distance. How far did Sally ride with her father?

$\frac{1}{2}\times\frac{9}{10}=n$ $\frac{9}{20}$ mi.

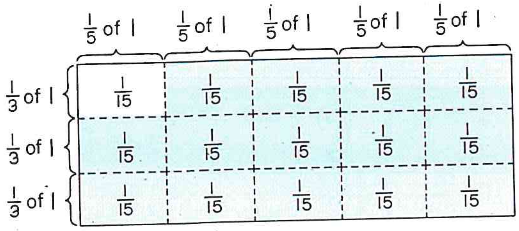
A quick review

	a	b
1.	$(3\times7)+n=36$ 15	$(\frac{2}{3}+\frac{1}{3})+n=6$ 5
2.	$n-(\frac{3}{4}+\frac{3}{4})=2$ $3\frac{1}{2}$	$(7-3\frac{1}{2})+3\frac{1}{2}=n$ 7
3.	$(\frac{1}{2}\times\frac{1}{2})+4=n$ $4\frac{1}{4}$	$n+9.6=10$ .4
4.	$n+(\frac{1}{2}\times3)=2$ $\frac{1}{2}$	$n-4.9=5.1$ 10
5.	$\frac{5}{6}+\frac{1}{2}+\frac{3}{4}=n$ $2\frac{1}{12}$	$7.6+n=9$ 1.4
6.	$9-(2\times\frac{1}{2})=n$ 8	$8.0-n=2.5$ 5.5
7.	$(3\times\frac{4}{5})+2=n$ $4\frac{2}{5}$	$n-7.1=9$ 16.1

Multiplying Fractional Numbers with Both Numerators Greater than 1

Diagram A illustrates a way of showing that $\frac{1}{3}\times\frac{1}{5}=\frac{1}{15}$.

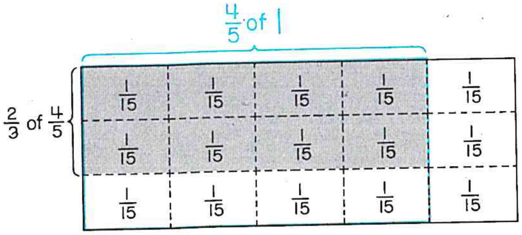
A



Do you see that multiplying the number of rows of rectangles (5) by the number of rectangles in a column (3) produces the total number of small rectangles in A? Each rectangle that results from the multiplication represents $\frac{1}{15}$ of 1.

Diagram B illustrates a way of showing that $\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$.

B



In diagram B, observe the vertical lines that divide the rectangle into fifths. Only $\frac{4}{5}$ of the $\frac{5}{5}$ are being considered. Then observe the horizontal lines that divide the rectangle into thirds. Only $\frac{2}{3}$ of the $\frac{3}{3}$ are being considered.

Look at all of the $\frac{1}{15}$'s that are colored gray. There are 8. Notice that taking $\frac{2}{3}$ of $\frac{4}{5}$ of 1 produces $\frac{8}{15}$.

Suppose you wanted to multiply $\frac{2}{3}$ and $\frac{4}{5}$ without using a diagram. The computation is shown.

$$\frac{2}{3}\times\frac{4}{5}=\frac{2\times4}{3\times5}=\frac{8}{15}$$

Observe that 3 and 5 are multiplied to produce the denominator of the answer. Also observe that 2 and 4 are multiplied to produce the numerator of the answer. Which two numbers would you expect to multiply to find the denominator of the answer when you multiply $\frac{2}{3}$ and $\frac{3}{4}$? Which two numbers would you multiply to find the numerator of the answer?

Sometimes the answer can be reduced, as in $\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}$. Then the answer should be expressed as $\frac{1}{2}$. How would you multiply $\frac{3}{5}$ and $\frac{5}{8}$? How would you express the answer?

Multiply two fractional numbers by multiplying the numbers named by their numerators (numerator of the product), and by multiplying the numbers named by their denominators (denominator of the product).

Oral Tell how you would construct diagrams to prove that each statement below is true. See page 82.

	a	*b*
1.	$\frac{3}{4}\times\frac{2}{5}=\frac{6}{20}$	$\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}$
2.	$\frac{2}{3}\times\frac{2}{5}=\frac{4}{15}$	$\frac{2}{5}\times\frac{2}{5}=\frac{4}{25}$

Tell the steps taken in solving each equation below. See page 82.

	a	*b*
3.	$n=\frac{3}{4}\times\frac{3}{5}$	$n=\frac{9}{10}\times\frac{7}{8}$
	$=\frac{3\times3}{4\times5}$	$=\frac{9\times7}{10\times8}$
	$=\frac{9}{20}$	$=\frac{63}{80}$
4.	$n=\frac{3}{5}\times\frac{5}{7}$	$n=\frac{3}{4}\times\frac{2}{5}$
	$=\frac{3\times5}{5\times7}$	$=\frac{3\times2}{4\times5}$
	$=\frac{15}{35}$	$=\frac{6}{20}$
	$=\frac{3}{7}$	$=\frac{3}{10}$

Written Copy. Solve for n.

	a	*b*
1.	$\frac{2}{5}\times\frac{1}{3}=n$ $\frac{2}{15}$	$\frac{3}{5}\times\frac{1}{4}=n$ $\frac{3}{20}$
2.	$\frac{5}{8}\times\frac{1}{5}=n$ $\frac{1}{8}$	$\frac{4}{7}\times\frac{1}{8}=n$ $\frac{1}{14}$
3.	$\frac{4}{5}\times\frac{1}{2}=n$ $\frac{2}{5}$	$\frac{2}{3}\times\frac{1}{4}=n$ $\frac{1}{6}$
4.	$\frac{2}{9}\times\frac{3}{8}=n$ $\frac{1}{12}$	$\frac{2}{3}\times\frac{2}{7}=n$ $\frac{4}{21}$
5.	$\frac{2}{3}\times\frac{3}{3}=n$ $\frac{2}{3}$	$\frac{2}{5}\times\frac{2}{4}=n$ $\frac{1}{5}$
6.	$\frac{2}{3}\times\frac{3}{4}=n$ $\frac{1}{2}$	$\frac{3}{2}\times\frac{3}{9}=n$ $\frac{1}{2}$
7.	$\frac{2}{4}\times\frac{7}{2}=n$ $1\frac{3}{4}$	$\frac{3}{4}\times\frac{5}{3}=n$ $1\frac{1}{4}$
8.	$\frac{7}{8}\times\frac{9}{8}=n$ $\frac{63}{64}$	$\frac{9}{10}\times\frac{9}{3}=n$ $2\frac{7}{10}$
9.	$\frac{5}{8}\times\frac{5}{3}=n$ $1\frac{1}{24}$	$\frac{4}{6}\times\frac{8}{3}=n$ $1\frac{7}{9}$
10.	$\frac{3}{4}\times\frac{7}{8}=n$ $\frac{21}{32}$	$\frac{9}{10}\times\frac{8}{2}=n$ $3\frac{3}{5}$

Translate each problem below into an equation. Solve for n. Read the problem again to see if your answer is reasonable. Write your answer.

11. Miss Carlson ordered 25 workbooks for her class. Each book weighed $\frac{1}{2}$ lb. What was the total weight of the package?

$25\times\frac{1}{2}=n$ $12\frac{1}{2}$ lb.

12. Miss Cox ordered 1 dozen jars of paint. Each jar weighed $\frac{1}{4}$ lb. What was the total weight?

$12\times\frac{1}{4}=n$ 3 lb.

13. Mary and each of her 3 friends bought $1\frac{1}{2}$ pounds of chocolates for their mothers for Mother's Day. How many pounds did they buy altogether?

$4\times1\frac{1}{2}=n$ 6 lb.

14. Sally bought a $2\frac{1}{2}$-pound box of candy. She repacked it in bags with $\frac{1}{4}$ pound of candy in each bag. How many bags of candy did she pack?

$2\frac{1}{2}\div\frac{1}{4}=n$ 10 bags

15. Bob needs 2 boards $18\frac{3}{16}$ inches long and 2 boards $9\frac{3}{4}$ inches long for a box he is making. How much lumber does he need?

$(2x18\frac{3}{16})+(2x9\frac{3}{4})=n$ $55\frac{7}{8}$ in.

Can you do this?

1. Make a diagram to show that $\frac{1}{2}$ of $\frac{1}{2}$ of 1 yard equals $\frac{1}{4}$ yard.

2. Make a rectangular drawing to show that $\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$.

MORE PRACTICE
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Using a Mixed Fraction in Multiplication

Ted's father mixes $\frac{1}{3}$ of a quart of oil with each gallon of gasoline he uses in his outboard motor. What fractional part of a quart of oil will he use in $2\frac{1}{2}$ gallons of gasoline?

You are to think of $\frac{1}{3}$ of a quart as being multiplied by $2\frac{1}{2}$ or of $2\frac{1}{2}\times\frac{1}{3}=n$, where n is the number you are seeking.

You may rename $2\frac{1}{2}$ as $\frac{5}{2}$, because $2\frac{1}{2}=2+\frac{1}{2}=\frac{4}{2}+\frac{1}{2}=\frac{5}{2}$. Then the equation is solved by multiplying two fractional numbers as shown in A.

A

$$2\frac{1}{2}\times\frac{1}{3}=\frac{5}{2}\times\frac{1}{3}=\frac{5\times1}{2\times3}=\frac{5}{6}$$

Ted's father will use $\frac{5}{6}$ of a quart of oil for $2\frac{1}{2}$ gallons of gasoline.

Suppose you were to multiply two numbers one of which is written as a fraction and the other as a mixed fraction as, for example, in $\frac{1}{2}\times1\frac{3}{4}=n$. Study the solution in B.

B

$$\frac{1}{2}\times1\frac{3}{4}=\frac{1}{2}\times\frac{7}{4}=\frac{1\times7}{2\times4}=\frac{7}{8}$$

How was $1\frac{3}{4}$ renamed? How is the solution in B the same as that in A?

Oral Tell the steps taken in each solution below. See column 1.

1. $1\frac{3}{5}\times\frac{1}{8}=\frac{8}{5}\times\frac{1}{8}=\frac{8\times1}{5\times8}=\frac{8}{40}=\frac{1}{5}$
2. $\frac{3}{4}\times5\frac{2}{3}=\frac{3}{4}\times\frac{17}{3}=\frac{51}{12}=4+\frac{3}{12}=4\frac{1}{4}$

Written Copy. Solve for n.

	a	*b*
1.	$6\frac{1}{2}\times\frac{5}{8}=n$ $4\frac{1}{16}$	$5\frac{2}{3}\times\frac{6}{7}=n$ $4\frac{6}{7}$
2.	$9\frac{1}{4}\times\frac{5}{6}=n$ $7\frac{17}{24}$	$9\frac{1}{2}\times\frac{5}{9}=n$ $5\frac{5}{18}$
3.	$8\frac{3}{4}\times\frac{1}{2}=n$ $4\frac{3}{8}$	$10\frac{5}{6}\times\frac{2}{3}=n$ $7\frac{2}{9}$
4.	$\frac{1}{5}\times1\frac{1}{3}=n$ $\frac{4}{15}$	$\frac{1}{2}\times1\frac{2}{5}=n$ $\frac{7}{10}$
5.	$3\frac{1}{4}\times\frac{5}{10}=n$ $1\frac{5}{8}$	$4\frac{1}{8}\times\frac{2}{3}=n$ $2\frac{3}{4}$
6.	$\frac{1}{3}\times2\frac{2}{3}=n$ $\frac{8}{9}$	$\frac{1}{6}\times2\frac{1}{3}=n$ $\frac{7}{18}$

Another way Study the solution of $2\frac{1}{2}\times\frac{1}{3}=n$ shown below.

$$\begin{aligned}2\tfrac{1}{2}\times\tfrac{1}{3}&=(2+\tfrac{1}{2})\times\tfrac{1}{3}=(2\times\tfrac{1}{3})+(\tfrac{1}{2}\times\tfrac{1}{3})\\&=\tfrac{2\times1}{1\times3}+\tfrac{1\times1}{2\times3}\\&=\tfrac{2}{3}+\tfrac{1}{6}\\&=\tfrac{4}{6}+\tfrac{1}{6}=\tfrac{5}{6}\end{aligned}$$

After the mixed fraction was named as a sum, the multiplication was done by multiplying $\frac{1}{3}$ by the fractional number $\frac{1}{2}$ and the whole number 2. Then the partial products were added.

Use the above method as a check of your computation in rows 1 through 6 above.

Multiplying Two Numbers Given as Mixed Fractions

The members of a hiking club hiked $2\frac{3}{4}$ miles the first hour of their hike. At this rate, how far would they hike in $3\frac{1}{2}$ hours?

The problem can be expressed by the equation $3\frac{1}{2}\times2\frac{3}{4}=n$. Why? How would you rename $3\frac{1}{2}$ as halves? $2\frac{3}{4}$ as fourths?

After $3\frac{1}{2}$ is renamed as $\frac{7}{2}$ and $2\frac{3}{4}$ as $\frac{11}{4}$, the equation is solved by multiplying fractional numbers. Study the solution.

$$3\frac{1}{2}\times2\frac{3}{4}=\frac{7}{2}\times\frac{11}{4}=\frac{7\times11}{2\times4}=\frac{77}{8}=77\div8=9\frac{5}{8}$$

The answer to the problem is $9\frac{5}{8}$ miles.

If you can rename mixed fractions as fractional numbers, you should have little difficulty in multiplying numbers named by mixed fractions.

Oral Tell each step you would take in finding the numeral to replace the letter n. See above.

	a	*b*	*c*
1.	$4\frac{1}{2}=\frac{n}{2}$ 9	$3\frac{1}{4}=\frac{n}{4}$ 13	$3\frac{1}{3}=\frac{n}{3}$ 10
2.	$2\frac{3}{4}=\frac{n}{4}$ 11	$6\frac{3}{4}=\frac{n}{4}$ 27	$7\frac{1}{3}=\frac{n}{3}$ 22
3.	$12\frac{2}{3}=\frac{n}{3}$ 38	$20\frac{1}{2}=\frac{n}{2}$ 41	$30\frac{1}{8}=\frac{n}{8}$ 241
4.	$5\frac{1}{2}=\frac{n}{2}$ 11	$6\frac{5}{6}=\frac{n}{6}$ 41	$40\frac{5}{8}=\frac{n}{8}$ 325
5.	$2\frac{1}{2}=\frac{n}{2}$ 5	$4\frac{1}{4}=\frac{n}{4}$ 17	$8\frac{3}{8}=\frac{n}{8}$ 67

Written Solve for n. Check for accuracy.

	a		*b*	
1.	$4\frac{1}{2}\times3\frac{1}{4}=n$	$14\frac{5}{8}$	$3\frac{1}{3}\times5\frac{1}{2}=n$	$18\frac{1}{3}$
2.	$2\frac{3}{4}\times6\frac{3}{4}=n$	$18\frac{9}{16}$	$7\frac{1}{3}\times6\frac{5}{6}=n$	$50\frac{1}{9}$
3.	$1\frac{2}{3}\times20\frac{1}{2}=n$	$34\frac{1}{6}$	$3\frac{1}{8}\times24\frac{1}{2}=n$	$76\frac{9}{16}$
4.	$40\frac{5}{8}\times30\frac{2}{3}=n$	$1245\frac{5}{6}$	$72\frac{3}{4}\times84\frac{2}{3}=n$	$6159\frac{1}{2}$
Cindy **5.**	$70\frac{5}{8}\times2\frac{3}{4}=n$	$194\frac{7}{32}$	$6\frac{3}{5}\times70\frac{1}{2}=n$	$465\frac{3}{10}$ Randy
Kathy **6.**	$6\frac{2}{3}\times9\frac{3}{4}=n$	65	$5\frac{5}{8}\times4\frac{3}{8}=n$	$24\frac{39}{64}$ Ricky
Carlton **7.**	$12\frac{2}{3}\times3\frac{1}{4}=n$	$41\frac{1}{6}$	$19\frac{1}{2}\times16\frac{7}{8}=n$	$329\frac{1}{16}$ Anita
Lajuana **8.**	$7\frac{3}{8}\times4\frac{5}{16}=n$	$31\frac{103}{128}$	$20\frac{2}{3}\times10\frac{1}{2}=n$	217 Rhonda
Gary **9.**	$n=2\frac{1}{3}\times6\frac{1}{8}$	$14\frac{7}{24}$	$n=17\frac{1}{10}\times4\frac{3}{5}$	$78\frac{33}{50}$ Jane
Mike **10.**	$n=1\frac{1}{4}\times3\frac{1}{6}$	$3\frac{23}{24}$	$n=3\frac{9}{10}\times14\frac{1}{2}$	$56\frac{11}{20}$ Wanda

Another way It is also possible to work in column form. Then $3\frac{1}{2}\times4\frac{3}{4}=n$ may be solved as shown.

$$\begin{array}{lr}
 & 4\frac{3}{4} \\
 & \underline{\times3\frac{1}{2}} \\
\frac{1}{2}\times\frac{3}{4}=\frac{1\times3}{2\times4}= & \frac{3}{8} \\
\frac{1}{2}\times4=\frac{1\times4}{2\times1}=\frac{4}{2}= & 2 \\
3\times\frac{3}{4}=\frac{3\times3}{1\times4}=\frac{9}{4}=2\frac{1}{4}= & 2\frac{2}{8} \\
3\times4= & \underline{12} \\
 & 16\frac{5}{8}
\end{array}$$

Use the way suggested above in checking your computation in rows 1 through 10 above.

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Solve the equation $\frac{4}{5}\times\frac{3}{8} = n$.

Step 1	$n = \frac{4}{5}\times\frac{3}{8} = \frac{4\times 3}{5\times 8}$	
Step 2	$= \frac{4\times 3}{8\times 5}$	Use the commutative property of multiplication
Step 3	$= \frac{4}{8}\times\frac{3}{5}$	Rewrite as two separate fractions
Step 4	$= \frac{1}{2}\times\frac{3}{5}$	Reduce $\frac{4}{8}$ to $\frac{1}{2}$
Step 5	$= \frac{1\times 3}{2\times 5}$	Rewrite as one fraction
Step 6	$= \frac{3}{10}$	Perform the operation

A Different Way of Multiplying Fractional Numbers

So far you have been reducing fractions to lowest terms *after* multiplying fractional numbers. The completed work on the board shows that often you can reduce *before* you multiply.

Step 1 is the same as the step you have always used. Simply rewrite the numerators and the denominators as an indicated product.

You may remember that multiplication has the commutative property and that you can therefore change the order of the numbers you are multiplying without changing the result. In step 2, the denominators have been interchanged to help you see that the fraction $(\frac{4}{8})$ can be reduced. In step 3 the fractions have been rewritten separately to permit reducing $\frac{4}{8}$ to $\frac{1}{2}$, as shown in step 4. Since $\frac{4}{8}=\frac{1}{2}$, would you expect $\frac{1}{2}\times\frac{3}{5}$ to produce the same result as $\frac{4}{8}\times\frac{3}{5}$? Steps 5 and 6 are the same as steps taken in multiplying fractional numbers in previous lessons. Observe that $\frac{4}{5}\times\frac{3}{8}=\frac{12}{40}=\frac{3}{10}$ and $\frac{1}{2}\times\frac{3}{5}=\frac{3}{10}$. Which is easier for you to do, reduce before or after multiplying the fractional numbers?

Sometimes you can reduce both of the fractions that are used as factors, as when you multiply $\frac{2}{3}$ and $\frac{3}{4}$. Observe the solution below.

$n=\frac{2}{3}\times\frac{3}{4}=\frac{2\times 3}{3\times 4}$	
$=\frac{2\times 3}{4\times 3}$	Interchange 3 and 4
$=\frac{2}{4}\times\frac{3}{3}$	Rewrite
$=\frac{1}{2}\times 1$	Reduce $\frac{2}{4}$
$=\frac{1}{2}$	Multiply $\frac{1}{2}\times 1$

You may be able to do the reducing without using pencil and paper. In any event, write only as many of the steps as you need to. For example, in $\frac{2}{3}\times\frac{3}{4}=n$, you may be able to see that $\frac{2}{3}\times\frac{3}{4}=\frac{2}{4}\times\frac{3}{3}=\frac{1}{2}$ without doing any writing at all. You simply use the commutative property mentally.

Oral Tell the reason for each step used in each solution in rows 1 and 2.
See tint block, page 86.

	a	*b*
1.	$6\times\frac{1}{2}=\frac{6}{1}\times\frac{1}{2}$ $=\frac{6\times1}{1\times2}$ $=\frac{6\times1}{2\times1}$ $=\frac{6}{2}\times\frac{1}{1}$ $=3\times1$ $=3$	$1\frac{1}{5}\times15=\frac{6}{5}\times\frac{15}{1}$ $=\frac{6\times15}{5\times1}$ $=\frac{6\times3\times5}{1\times5}$ $=6\times3\times\frac{5}{5}$ $=6\times3\times1$ $=18$
2.	$2\frac{4}{5}\times1\frac{1}{4}=\frac{14\times5}{5\times4}$ $=\frac{14\times5}{4\times5}$ $=\frac{14}{4}\times\frac{1}{1}$ $=\frac{7}{2}$ $=3\frac{1}{2}$	$5\frac{5}{8}\times6\frac{2}{5}=\frac{45\times32}{8\times5}$ $=\frac{45\times32}{5\times8}$ $=\frac{45}{5}\times\frac{32}{8}$ $=\frac{9}{1}\times\frac{4}{1}$ $=36$

Written Solve for n.

	a	*b*
1.	$\frac{5}{8}\times\frac{4}{5}=n$ $\frac{1}{2}$	$\frac{2}{3}\times\frac{9}{14}=n$ $\frac{3}{7}$
2.	$\frac{3}{4}\times\frac{8}{9}=n$ $\frac{2}{3}$	$\frac{2}{5}\times\frac{5}{8}=n$ $\frac{1}{4}$
3.	$\frac{3}{5}\times\frac{5}{18}=n$ $\frac{1}{6}$	$\frac{3}{5}\times\frac{5}{9}=n$ $\frac{1}{3}$
4.	$\frac{3}{4}\times\frac{2}{7}=n$ $\frac{3}{14}$	$\frac{5}{8}\times\frac{9}{10}=n$ $\frac{9}{16}$
5.	$\frac{7}{12}\times\frac{6}{7}=n$ $\frac{1}{2}$	$\frac{2}{3}\times\frac{3}{16}=n$ $\frac{1}{8}$
6.	$\frac{4}{5}\times\frac{7}{12}=n$ $\frac{7}{15}$	$\frac{4}{5}\times\frac{9}{16}=n$ $\frac{9}{20}$
7.	$9\times\frac{5}{6}=n$ $7\frac{1}{2}$	$\frac{5}{9}\times27=n$ 15
8.	$3\frac{1}{3}\times\frac{3}{5}=n$ 2	$1\frac{7}{8}\times\frac{4}{9}=n$ $\frac{5}{6}$
9.	$\frac{2}{3}\times4\frac{1}{2}=n$ 3	$\frac{9}{10}\times1\frac{2}{3}=n$ $1\frac{1}{2}$

Carefully study each problem by reading it several times. Decide upon the operation to use in solving each one. Then express each problem as an equation and solve it. Remember: The signs $ and . are not part of an equation.

10. Mrs. Parker had $15\frac{1}{2}$ yards of drapery material. She used $6\frac{1}{2}$ yards for one pair of drapes and $5\frac{3}{4}$ yards for another pair. How much material did she have left?
$15\frac{1}{2}-(6\frac{1}{2}+5\frac{3}{4})=n$ $3\frac{1}{4}$ yd.

11. Kathy bought $2\frac{1}{2}$ dozen eggs at 60¢ a dozen. How much change should she get from $5.00?
$500-(2\frac{1}{2}\times60)=n$ $3.50

12. Four boys shared the cost of $\frac{1}{2}$ pound of candy at 80¢ a pound. Find each boy's share of the cost.
$(\frac{1}{2}\times80)\div4=n$ 10¢

13. Alice had $10\frac{1}{2}$ yards of ribbon on a spool. She used $7\frac{1}{3}$ yards of ribbon for a costume. How many yards of ribbon should be left on the spool?
$10\frac{1}{2}-7\frac{1}{3}=n$ $3\frac{1}{6}$ yd.

14. Judy bought $1\frac{1}{8}$ yards of gingham at $.64 a yard. How much did she spend for gingham?
$1\frac{1}{8}\times64=n$ $.72

15. Nancy bought $2\frac{1}{2}$ yards of percale and Sally bought $3\frac{1}{3}$ yards of percale. How many yards of material did the girls buy altogether?
$2\frac{1}{2}+3\frac{1}{3}=n$ $5\frac{5}{6}$ yd.

16. Betty had $7\frac{1}{3}$ yards of lace for a costume. She needed 12 yards. How much lace did Betty need to buy? $12-7\frac{1}{3}=n$ $4\frac{2}{3}$ yd.

MORE PRACTICE PAGE 303

Finding the Product of Three Fractional Numbers

You may remember that multiplication of whole numbers has a property called the *associative property*. This property makes it possible to regroup the factors without changing the product. Thus $(29\times5)\times2$ represents the same product as $29\times(5\times2)$. Do you suppose that the associative property also applies to the multiplication of fractional numbers? See what happens when you solve $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=n$ by associating the factors in two different ways.

$$
\begin{array}{ll}
(\frac{1}{2}\times\frac{2}{3})\times\frac{3}{4}=n & \frac{1}{2}\times(\frac{2}{3}\times\frac{3}{4})=n \\
\frac{1\times2}{2\times3}\times\frac{3}{4}=n & \frac{1}{2}\times\frac{2\times3}{3\times4}=n \\
\frac{1\times2}{3\times2}\times\frac{3}{4}=n & \frac{1}{2}\times\frac{2\times3}{4\times3}=n \\
(\frac{1}{3}\times1)\times\frac{3}{4}=n & \frac{1}{2}\times(\frac{1}{2}\times1)=n \\
\frac{1}{3}\times\frac{3}{4}=n & \frac{1}{2}\times\frac{1}{2}=n \\
\frac{1\times3}{3\times4}=n & \frac{1\times1}{2\times2}=n \\
\frac{1\times3}{4\times3}=n & \frac{1}{4}=n \\
\frac{1}{4}\times1=n & \\
\frac{1}{4}=n &
\end{array}
$$

The above solutions seem to indicate that the associative property of multiplication applies also to the multiplication of fractional numbers.

When finding the product of three fractional numbers, do the work without pencil and paper if possible. Suppose, for example, that you are solving the equation $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=n$. Do you suppose that you can find the numerator of the answer by associating the numerators and then multiplying them? Do you suppose that you can find the denominator of the answer by associating the denominators and multiplying them?

Study the solution of $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=n$.

$$
\begin{aligned}
n&=\tfrac{1}{2}\times\tfrac{2}{3}\times\tfrac{3}{4} & \\
&=\frac{1\times(2\times3)}{(2\times3)\times4} & \text{Rewrite as 1 fraction} \\
&=\frac{1\times(2\times3)}{4\times(2\times3)} & \text{Associate factors} \\
&=\tfrac{1}{4}\times1 & \text{Multiply} \\
&=\tfrac{1}{4} &
\end{aligned}
$$

Observe that you use steps similar to those taken in multiplying two fractional numbers. The only additional steps are those of associating and commuting the numbers to be multiplied. These steps are necessary because the multiplication operation is a *binary operation*. You may remember that a binary operation is one that can be done on only *two* numbers at a time.

Choose any three fractional factors. Regroup them in any way you wish. Do you obtain the same result regardless of how you associate them?

Why might you sometimes want to associate one pair of numbers rather than another when you are finding the product of three whole numbers? When you are finding the product of three fractional numbers?

Oral Tell the reasons for the steps taken in solving these equations.
See page 88.

	a	b
1.	$n=\frac{2}{3}\times\frac{4}{5}\times\frac{7}{8}$	$n=\frac{1}{2}\times\frac{3}{4}\times\frac{2}{3}$
	$=\frac{(2\times4)\times7}{(3\times5)\times8}$	$=\frac{1\times(3\times2)}{(2\times4)\times3}$
	$=\frac{(2\times4)\times7}{8\times(3\times5)}$	$=\frac{1\times(2\times3)}{(4\times2)\times3}$
	$=\frac{8}{8}\times\frac{7}{15}$	$=\frac{1\times(2\times3)}{4\times(2\times3)}$
	$=1\times\frac{7}{15}$	$=\frac{1}{4}\times1$
	$=\frac{7}{15}$	$=\frac{1}{4}$

Tell the steps you would take in solving each equation below. You may combine as many of the steps as you can. Solutions only.

	a	b
2.	$\frac{3}{8}\times\frac{1}{4}\times\frac{2}{3}=n$ $\frac{1}{16}$	$n=\frac{3}{4}\times\frac{3}{5}\times\frac{1}{4}$ $\frac{9}{80}$
3.	$\frac{5}{8}\times\frac{4}{5}\times\frac{3}{4}=n$ $\frac{3}{8}$	$n=\frac{5}{12}\times\frac{4}{5}\times\frac{3}{5}$ $\frac{1}{5}$

Written Solve for n. How would you associate the numerators and the denominators in solving each equation below?

	a	b
1.	$\frac{5}{8}\times\frac{1}{2}\times\frac{2}{3}=n$ $\frac{5}{24}$	$\frac{1}{4}\times\frac{3}{5}\times\frac{1}{4}=n$ $\frac{3}{80}$
2.	$\frac{5}{6}\times\frac{4}{5}\times\frac{3}{4}=n$ $\frac{1}{2}$	$\frac{5}{12}\times\frac{1}{5}\times\frac{3}{5}=n$ $\frac{1}{20}$
3.	$\frac{3}{10}\times\frac{2}{3}\times\frac{5}{6}=n$ $\frac{1}{6}$	$\frac{5}{8}\times\frac{3}{5}\times\frac{5}{16}=n$ $\frac{15}{128}$
4.	$\frac{1}{2}\times\frac{3}{4}\times\frac{2}{3}=n$ $\frac{1}{4}$	$\frac{1}{3}\times\frac{2}{5}\times\frac{3}{7}=n$ $\frac{2}{35}$
	Kathy	Anita

	a	b
	Cindy	Diane
5.	$\frac{7}{8}\times\frac{5}{6}\times\frac{1}{3}=n$ $\frac{35}{144}$	$\frac{5}{6}\times\frac{3}{4}\times\frac{3}{4}=n$ $\frac{15}{32}$
Lajuana 6.	$\frac{1}{3}\times\frac{1}{2}\times\frac{1}{4}=n$ $\frac{1}{24}$	$\frac{1}{6}\times\frac{1}{7}\times\frac{9}{16}=n$ $\frac{3}{224}$ Rhonda
7.	$\frac{3}{4}\times\frac{3}{4}\times\frac{1}{2}=n$ $\frac{9}{32}$	$\frac{3}{5}\times\frac{1}{5}\times\frac{1}{4}=n$ $\frac{3}{100}$
8.	$\frac{2}{3}\times\frac{3}{5}\times\frac{1}{2}=n$ $\frac{1}{5}$	$\frac{3}{4}\times\frac{3}{5}\times\frac{7}{8}=n$ $\frac{63}{160}$
9.	$\frac{1}{7}\times\frac{4}{7}\times\frac{5}{6}=n$ $\frac{10}{147}$	$\frac{2}{3}\times\frac{3}{4}\times\frac{9}{10}=n$ $\frac{9}{20}$
10.	$\frac{7}{8}\times\frac{1}{2}\times\frac{5}{6}=n$ $\frac{35}{96}$	$\frac{3}{7}\times\frac{5}{6}\times\frac{1}{2}=n$ $\frac{5}{28}$
11.	$\frac{1}{7}\times\frac{5}{7}\times\frac{3}{8}=n$ $\frac{15}{392}$	$n=\frac{2}{3}\times\frac{5}{8}\times\frac{2}{3}$ $\frac{5}{18}$
12.	$n=\frac{3}{16}\times\frac{5}{16}\times\frac{1}{2}$ $\frac{15}{512}$	$n=\frac{19}{24}\times\frac{10}{11}\times\frac{10}{21}$ $\frac{475}{1386}$

Can you do this? Replace each dot with =, >, or <, whichever makes the statements true. Remember: > is read *is greater than,* and < is read *is less than.*

	a	b
1.	$\frac{2}{3}\times\frac{3}{4}\overset{=}{\bullet}\frac{3}{4}\times\frac{2}{3}$	$1\frac{1}{2}\times3\frac{1}{4}\overset{=}{\bullet}3\frac{1}{4}\times1\frac{1}{2}$
2.	$\frac{3}{5}+\frac{4}{5}\overset{=}{\bullet}\frac{4}{5}+\frac{3}{5}$	$\frac{4}{5}+\frac{3}{5}\overset{=}{\bullet}\frac{3}{5}+\frac{4}{5}$
3.	$\frac{7}{8}-\frac{3}{8}\overset{<}{\bullet}\frac{3}{8}+\frac{1}{4}$	$\frac{9}{10}-\frac{3}{10}\overset{>}{\bullet}\frac{9}{10}-\frac{1}{3}$
4.	$\frac{1}{4}\times\frac{2}{3}\overset{<}{\bullet}\frac{1}{3}\times\frac{2}{3}$	$\frac{1}{3}\times\frac{3}{4}\overset{<}{\bullet}\frac{2}{3}\times\frac{3}{4}$
5.	$\frac{3}{4}\times\frac{2}{3}\overset{<}{\bullet}\frac{3}{4}\times\frac{3}{4}$	$\frac{5}{8}\times\frac{3}{8}\overset{>}{\bullet}\frac{3}{8}\times\frac{5}{16}$

Supply the names of operations to make each statement below true.

Addition multiplication
6. ____and____have the commutative property.

Division subtraction
7. ____and____do not have the commutative property.

MORE PRACTICE PAGE 303

You should be able to do the work on this page without any help.

Part 1 Copy. Perform each operation as indicated by the operational sign.

	a	*b*
1.	94r23 93)8765	75r444 461)35019
2.	1432r40 65)93120	659r162 729)480573
3.	89321 46503 70968 3472 +89000 299264	476534 27474 389543 7640 +937654 1838845
4.	495210 −189795 305415	9876453 −7563879 2312574
5.	39257 ×48 1884336	59328 ×758 44970624

Part 2 Solve for n.

	a	*b*
1.	$30\frac{1}{2}+14\frac{3}{4}=n$ $45\frac{1}{4}$	$83\frac{5}{6}+29\frac{7}{8}=n$ $113\frac{17}{24}$
2.	$42\frac{1}{4}+39\frac{6}{8}=n$ 82	$64\frac{1}{5}+33\frac{1}{3}=n$ $97\frac{8}{15}$
3.	$23\frac{1}{16}-5\frac{7}{16}=n$ $17\frac{5}{8}$	$32-12\frac{3}{8}=n$ $19\frac{5}{8}$
4.	$62\frac{3}{4}-4\frac{1}{2}=n$ $58\frac{1}{4}$	$70\frac{1}{8}-19\frac{3}{5}=n$ $50\frac{21}{40}$
5.	$9\times3\frac{1}{2}=n$ $31\frac{1}{2}$	$17\times5\frac{1}{4}=n$ $89\frac{1}{4}$
6.	$1\frac{11}{24}$ $n=2\frac{1}{3}\times\frac{5}{8}$	$9\frac{5}{24}$ $n=3\frac{1}{4}\times2\frac{5}{6}$

Part 3 Express each problem as an equation. Solve for n. Write the answer for each problem.

1. Jane and Judy made $2\frac{1}{2}$ dozen coconut cookies and $1\frac{3}{4}$ dozen brownies for the cooky sale. How many dozen cookies did they make in all? $2\frac{1}{2}+1\frac{3}{4}=n$ $4\frac{1}{4}$ doz.

2. Ted's mother made $3\frac{1}{2}$ dozen sugar cookies for the sale. If they were sold at 30¢ a dozen, how much money was received for all of them?

$3\frac{1}{2}\times30=n$ \$1.05

3. Rachel had 3 pieces of ribbon. One was $12\frac{1}{2}$ in. long, one was $27\frac{3}{4}$ in. long, and one $18\frac{1}{2}$ in. long. How many yards of ribbon did she have?

$(12\frac{1}{2}+27\frac{3}{4}+18\frac{1}{2})\div36=n$ 1 yd. $22\frac{3}{4}$ in

4. An airplane flew 2370 miles in 6 hours. What was its average speed per hour? $2370\div6=n$ 395 mph

5. Mr. Rogers has \$25 to spend for books that cost \$2.98 each. How many books can he buy and how much of his money will be left?

$2500\div298=n$ 8 books, \$1.16 left

6. Jerry has a board that is $5\frac{1}{8}$ inches wide. He needs a $6\frac{3}{4}$-inch board for a birdhouse. By how much is his board too narrow? $6\frac{3}{4}-5\frac{1}{8}=n$

$1\frac{5}{8}$ in.

7. Laura bought $3\frac{1}{4}$ yards of broadcloth and Joanne bought $2\frac{5}{8}$ yards of broadcloth. How many more yards of material were bought by Laura than by Joanne? $3\frac{1}{4}-2\frac{5}{8}=n$

$\frac{5}{8}$ yd.

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. The same properties govern the multiplication of whole numbers and of fractional numbers. (73, 75, 77, 88)

2. To multiply two fractional numbers, multiply the numbers named by their numerators and multiply the numbers named by their denominators. (82)

3. Fractions may be reduced either *before* or *after* multiplication. (86)

4. Three or more factors may be associated, or grouped, in any order before multiplying them. (88)

Questions to Discuss

See T91 for answers.

1. How could you use repeated addition in solving the equation $3\times\frac{1}{6}=n$? (73)

2. How could you use the commutative property of multiplication in solving the equation $\frac{1}{4}\times3=n$? (75)

3. How do you determine the operation when solving problems? (76)

4. What are two different ways of solving $9\times4\frac{1}{6}=n$? (77)

5. What is the advantage of changing a mixed fraction to an improper fraction before multiplying? (78)

Written Practice

Solve for n.

	a	*b*	
1.	$2\times\frac{4}{7}=n$ $1\frac{1}{7}$	$n=2\times\frac{3}{8}$ $\frac{3}{4}$	(74)
2.	$6\times\frac{4}{15}=n$ $1\frac{3}{5}$	$n=5\times\frac{7}{12}$ $2\frac{11}{12}$	(74)
3.	$\frac{1}{4}\times8=n$ 2	$\frac{1}{5}\times13=n$ $2\frac{3}{5}$	(75)
4.	$2\times7\frac{2}{5}=n$ $14\frac{4}{5}$	$n=4\times2\frac{1}{8}$ $8\frac{1}{2}$	(77)
5.	$10\frac{1}{2}\times15=n$ $157\frac{1}{2}$	$n=10\times5\frac{1}{2}$ 55	(79)
6.	$\frac{1}{8}\times\frac{1}{5}=n$ $\frac{1}{40}$	$\frac{1}{2}\times\frac{3}{4}=n$ $\frac{3}{8}$	(80)
7.	$\frac{3}{5}\times\frac{3}{4}=n$ $\frac{9}{20}$	$\frac{3}{8}\times\frac{2}{9}=n$ $\frac{1}{12}$	(82)
8.	$4\frac{1}{2}\times3\frac{4}{5}=n$ $17\frac{1}{10}$	$n=2\frac{3}{4}\times5\frac{1}{6}$ $14\frac{5}{24}$	(85)
9.	$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{5}=n$ $\frac{1}{5}$	$n=\frac{3}{8}\times\frac{2}{7}\times\frac{4}{5}$ $\frac{3}{35}$	(88)

Solve these problems.

10. Bill had 5 bottles of ammonia in $4\frac{3}{4}$ oz. bottles. How much ammonia did he have? (77)

$5\times4\frac{3}{4}=n$ $23\frac{3}{4}$ oz.

11. Mrs. Jones filled $15\frac{2}{3}$ boxes of holiday cookies. She had planned to put 2 pounds of cookies in each box. How many pounds of cookies had she actually made? (77)

$15\frac{2}{3}\times2=n$ $31\frac{1}{3}$ lb.

12. 35 women worked for $7\frac{1}{4}$ hours each at a bazaar. How many hours did they work altogether? (79)

$35\times7\frac{1}{4}=n$ $253\frac{3}{4}$ hr.

Self-Evaluation

Part 1 Copy. Perform each operation as indicated by the operational sign.

	a	*b*
1.	198r321 485)96351	1357r558 586)795760
2.	3927 257 4009 3710 +195 12098	49321 27543 6912 89350 +90000 263126
3.	9875003 −2754934 7120069	\$3542.00 −1756.75 \$1785.25
4.	39560 ×729 $28839\underset{2}{1}40$	41050 ×950 38997500

Part 2 Write the numeral of each statement below. After the numeral of each statement, write T if it is true and F if it is false.

T **1.** The numerator of $\frac{2}{3}$ is 2.

T **2.** The denominator of $\frac{12}{2}$ is 2.

F **3.** The numeral $3\frac{2}{5}$ is called an improper fraction.

T **4.** A mixed fraction represents the addition of a whole number and a fractional number.

F **5.** The answer for an addition is called the product.

F **6.** The word dividend should cause you to think of multiplication.

Part 3 Solve for n.

	a	*b*
1.	$\frac{2}{3}$ $n=2\times\frac{1}{3}$	$\frac{3}{4}$ $n=2\times\frac{3}{8}$
2.	$6\frac{6}{7}$ $3\times2\frac{2}{7}=n$	$13\frac{1}{2}$ $6\times2\frac{1}{4}=n$
3.	3 $\frac{1}{3}\times9=n$	9 $n=\frac{1}{8}\times72$
4.	27 $\frac{3}{4}\times36=n$	32 $48\times\frac{2}{3}=n$
5.	$\frac{1}{10}$ $\frac{1}{2}\times\frac{1}{5}=n$	$\frac{1}{7}$ $\frac{1}{3}\times\frac{3}{7}=n$
6.	$\frac{5}{24}$ $\frac{1}{4}\times\frac{5}{6}=n$	$\frac{2}{7}$ $\frac{1}{2}\times\frac{4}{7}=n$
7.	$\frac{7}{10}$ $\frac{1}{2}\times1\frac{2}{5}=n$	$\frac{13}{24}$ $n=\frac{1}{8}\times4\frac{1}{3}$
8.	$\frac{8}{15}$ $1\frac{1}{3}\times\frac{2}{5}=n$	$\frac{5}{7}$ $n=1\frac{2}{3}\times\frac{3}{7}$
9.	20 $n=2\frac{1}{2}\times8$	$13\frac{1}{3}$ $n=3\frac{1}{3}\times4$
10.	$5\frac{5}{6}$ $2\frac{1}{2}\times2\frac{1}{3}=n$	$3\frac{2}{3}$ $n=1\frac{1}{3}\times2\frac{3}{4}$
11.	$14\frac{3}{10}$ $3\frac{1}{4}\times4\frac{2}{5}=n$	$44\frac{17}{32}$ $n=7\frac{1}{8}\times6\frac{1}{4}$

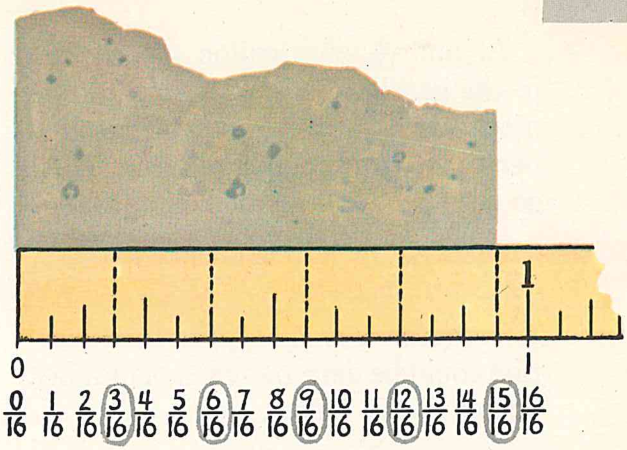
Part 4 Solve these problems.

1. John figured that on the average, each of the 25 pupils in his room had brought $75\frac{1}{2}$ pounds of old newspaper to school to sell. How many pounds of paper had been collected by John's room?

$25\text{x}75\frac{1}{2}=n$ $1887\frac{1}{2}$ lb.

2. A Scout leader formed patrols of 8 Scouts each. Enough boys had reported for scouting to permit forming $5\frac{1}{2}$ patrols. How many boys reported for scouting?

$5\frac{1}{2}\text{x}8=n$ 44 boys



Dividing a Fractional Number

Bob wants strips of cork $\frac{3}{16}$ of an inch in width. How many strips can he cut from a piece of cork that has a width of $\frac{15}{16}$ of an inch?

Let n represent the number of strips he can cut. Then the problem can be translated into the equation $n\times\frac{3}{16}=\frac{15}{16}$. From the number line you can see that $5\times\frac{3}{16}=\frac{15}{16}$; so $n=5$. Bob can cut 5 strips of cork.

You can also use division to find the answer to Bob's problem, because division is the inverse operation of multiplication.

Then $n\times\frac{3}{16}=\frac{15}{16}$ becomes $n=\frac{15}{16}\div\frac{3}{16}$

We want the division of fractional numbers to agree as much as possible with that of whole numbers. You know that $15\div3=5$ and that $1\div1=1$.

Then $\frac{15}{1}\div\frac{3}{1}=\frac{15\div3}{1\div1}=\frac{5}{1}=5$

A procedure that is correct for denominators of 1 should also work for any fractions with like denominators. Study the solution below.

$$\frac{15}{16}\div\frac{3}{16}=\frac{15\div3}{16\div16}=\frac{5}{1}=5$$

This division worked out very well because 15 is divisible by 3 and, also, 16 is divisible by 16. Such divisions always have a quotient which is a whole number.

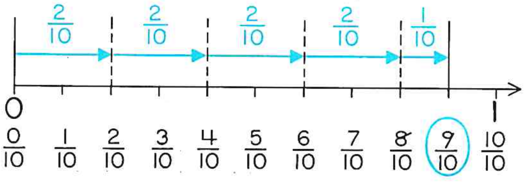
Oral Tell the steps you would take in solving for n. See column 1 and above.

	a	*b*
1.	$\overset{2}{n}\times\frac{2}{3}=\frac{4}{3}$	$\overset{2}{n}=\frac{4}{3}\div\frac{2}{3}$
2.	$\overset{3}{n}\times\frac{2}{8}=\frac{6}{8}$	$\overset{3}{n}=\frac{6}{8}\div\frac{2}{8}$
3.	$\overset{2}{n}\times\frac{5}{12}=\frac{10}{12}$	$\overset{2}{n}=\frac{10}{12}\div\frac{5}{12}$
4.	$\overset{4}{n}\times\frac{2}{9}=\frac{8}{9}$	$\overset{4}{n}=\frac{8}{9}\div\frac{2}{9}$
5.	$\overset{3}{n}\times\frac{3}{12}=\frac{9}{12}$	$\overset{3}{n}=\frac{9}{12}\div\frac{3}{12}$

Practice in Dividing Fractional Numbers

Al put $\frac{9}{10}$ of a gallon of gasoline in the gasoline tank of a motor. The motor used $\frac{2}{10}$ of a gallon of gasoline per mile. How many miles could Al go with the $\frac{9}{10}$ of a gallon of gasoline?

You are to find how many times $\frac{2}{10}$ is contained in $\frac{9}{10}$; so you divide. The equation is $\frac{9}{10} \div \frac{2}{10} = n$. The solution could be determined on a number line as shown below.



How many times is the segment of length $\frac{2}{10}$ contained in the segment of length $\frac{9}{10}$? Is $\frac{1}{10}$ exactly $\frac{1}{2}$ of $\frac{2}{10}$? Is $\frac{2}{10}$ contained in $\frac{9}{10}$ exactly $4\frac{1}{2}$ times? Check your answer by multiplying $\frac{2}{10}$ by $4\frac{1}{2}$. Does $4\frac{1}{2} \times \frac{2}{10} = \frac{9}{10}$? Al can go $4\frac{1}{2}$ miles on the gasoline.

You would probably solve the equation $\frac{9}{10} \div \frac{2}{10} = n$ by using numerals rather than a number line. Observe the solution below.

$n = \frac{9}{10} \div \frac{2}{10}$	Rewrite as one fraction
$= \frac{9 \div 2}{10 \div 10}$	$10 \div 10 = 1$
$= \frac{9 \div 2}{1}$	Divide by 1
$= 9 \div 2$	Rename $9 \div 2$
$= 4\frac{1}{2}$	

MORE PRACTICE PAGE 304

You may remember that dividing any number by 1 leaves the number unchanged. That is why dividing the number $(9 \div 2)$ by 1 produces $9 \div 2$, or $4\frac{1}{2}$.

Oral Tell the reason for each step taken in solving each equation below. See column 1.

a	*b*
$n = \frac{15}{16} \div \frac{7}{16}$	$n = \frac{11}{12} \div \frac{3}{12}$
$= \frac{15 \div 7}{16 \div 16}$	$= \frac{11 \div 3}{12 \div 12}$
$= \frac{15 \div 7}{1}$	$= \frac{11 \div 3}{1}$
$= 15 \div 7$	$= 11 \div 3$
$= 2\frac{1}{7}$	$= 3\frac{2}{3}$

In the above solutions, which steps could you have taken without using pencil and paper? Answers will vary.

Written Solve for n.

	a	*b*
1.	$\frac{5}{8} \div \frac{3}{8} = n$ $1\frac{2}{3}$	$1\frac{1}{2}$ $n = \frac{3}{5} \div \frac{2}{5}$
2.	$\frac{5}{9} \div \frac{2}{9} = n$ $2\frac{1}{2}$	$1\frac{1}{2}$ $n = \frac{6}{11} \div \frac{4}{11}$
3.	$\frac{14}{15} \div \frac{8}{15} = n$ $1\frac{3}{4}$	$1\frac{2}{3}$ $n = \frac{15}{16} \div \frac{9}{16}$
4.	$\frac{17}{32} \div \frac{3}{32} = n$ $5\frac{2}{3}$	$6\frac{1}{3}$ $n = \frac{19}{24} \div \frac{3}{24}$
5.	4 $n \times \frac{3}{5} = \frac{12}{5}$	$\frac{12}{5} \div \frac{3}{5} = n$ 4
6.	5 $n \times \frac{2}{3} = \frac{10}{3}$	$\frac{10}{3} \div \frac{2}{3} = n$ 5
7.	9 $n \times \frac{3}{4} = \frac{27}{4}$	$\frac{27}{4} \div \frac{3}{4} = n$ 9
8.	19 $n = \frac{57}{64} \div \frac{3}{64}$	$4\frac{1}{11}$ $n = \frac{45}{64} \div \frac{11}{64}$

Mixed Fractions in Division

Mary had $1\frac{2}{3}$ cups of sugar. She needed $\frac{2}{3}$ of a cup of sugar for making each batch of cookies. How many batches of cookies could she make?

You are to find how many times $\frac{2}{3}$ is contained in $1\frac{2}{3}$; so you divide. The equation is $1\frac{2}{3} \div \frac{2}{3} = n$.

A good way to begin the solution is to rename $1\frac{2}{3}$. Recall that $1\frac{2}{3} = 1 + \frac{2}{3} = \frac{3}{3} + \frac{2}{3} = \frac{3+2}{3} = \frac{5}{3}$. Then $1\frac{2}{3} \div \frac{2}{3} = n$ becomes $\frac{5}{3} \div \frac{2}{3} = n$. How many times is $\frac{2}{3}$ contained in $\frac{5}{3}$?

Mary could make $2\frac{1}{2}$ batches of cookies. Of course, she would probably make 2 batches. Then how much sugar would be left? Reminder: $\frac{5}{3} = (\frac{2}{3} + \frac{2}{3}) + \frac{1}{3} = (2 \times \frac{2}{3}) + \frac{1}{3}$. She would have $\frac{1}{3}$ of a cup of sugar left.

In solving $1\frac{2}{3} \div \frac{2}{3} = n$, you may see directly that $\frac{5}{3} \div \frac{2}{3}$ produces the same result as $5 \div 2$. Why?

Oral Tell the reason for each step taken in solving each equation below. Why might you omit the third and fourth steps shown? See above.

a	*b*
$n = 2\frac{1}{4} \div \frac{3}{4}$	$n = 3\frac{2}{3} \div \frac{2}{3}$
$= \frac{9}{4} \div \frac{3}{4}$	$= \frac{11}{3} \div \frac{2}{3}$
$= \frac{9 \div 3}{4 \div 4}$	$= \frac{11 \div 2}{3 \div 3}$
$= (9 \div 3) \div 1$	$= (11 \div 2) \div 1$
$= 3$	$= 5\frac{1}{2}$

Written Solve for n.

	a	*b*
1.	$4\frac{1}{6} \div \frac{5}{6} = n$ 5	$3\frac{5}{9} \div \frac{8}{9} = n$ 4
2.	$2\frac{1}{5} \div \frac{2}{5} = n$ $5\frac{1}{2}$	$3\frac{4}{5} \div \frac{3}{5} = n$ $6\frac{1}{3}$
3.	$3\frac{2}{3} \div \frac{2}{3} = n$ $5\frac{1}{2}$	$1\frac{3}{7} \div \frac{3}{7} = n$ $3\frac{1}{3}$
4.	$2\frac{3}{5} \div \frac{4}{5} = n$ $3\frac{1}{4}$	$1\frac{2}{9} \div \frac{5}{9} = n$ $2\frac{1}{5}$
5.	$2\frac{5}{6} \div \frac{5}{6} = n$ $3\frac{2}{5}$	$1\frac{1}{8} \div \frac{7}{8} = n$ $1\frac{2}{7}$
6.	$1\frac{8}{9} \div \frac{7}{9} = n$ $2\frac{3}{7}$	$2\frac{7}{8} \div \frac{3}{8} = n$ $7\frac{2}{3}$
7.	$7\frac{1}{6} \div \frac{5}{6} = n$ $8\frac{3}{5}$	$2\frac{3}{8} \div \frac{5}{8} = n$ $3\frac{4}{5}$
8.	$2\frac{2}{9} \div \frac{8}{9} = n$ $2\frac{1}{2}$	$3\frac{3}{5} \div \frac{4}{5} = n$ $4\frac{1}{2}$
9.	$3\frac{1}{7} \div \frac{4}{7} = n$ $5\frac{1}{2}$	$1\frac{5}{9} \div \frac{8}{9} = n$ $1\frac{3}{4}$
10.	$2\frac{4}{5} \div \frac{4}{5} = n$ $3\frac{1}{2}$	$2\frac{4}{9} \div \frac{4}{9} = n$ $5\frac{1}{2}$
11.	$2\frac{6}{7} \div \frac{6}{7} = n$ $3\frac{1}{3}$	$4\frac{2}{7} \div \frac{4}{7} = n$ $7\frac{1}{2}$

Using Two Mixed Fractions in Division

Betty had $5\frac{1}{4}$ pounds of candy. She put it in jars that held $1\frac{3}{4}$ pounds each. How many jars did she fill?

You are to think of $5\frac{1}{4}$ pounds as being separated into jars each containing $1\frac{3}{4}$ pounds and are to find the number of jars; so you divide. The equation is $5\frac{1}{4} \div 1\frac{3}{4} = n$.

After renaming $5\frac{1}{4}$ as $\frac{21}{4}$ and $1\frac{3}{4}$ as $\frac{7}{4}$, the equation becomes $\frac{21}{4} \div \frac{7}{4} = n$. Then n has a value of 3. Betty can fill 3 jars.

Whenever you have two mixed fractions in division, you will save time if you first rename them as improper fractions as in A and B.

A

$$n = 2\frac{1}{4} \div 1\frac{3}{4} = \frac{9}{4} \div \frac{7}{4} = \frac{9 \div 7}{4 \div 4}$$

B

$$n = 2\frac{4}{5} \div 1\frac{3}{5} = \frac{14}{5} \div \frac{8}{5} = \frac{14 \div 8}{5 \div 5}$$

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If you complete the solution of A, you will divide 4 by 4 in the denominator to produce 1. Then you will divide 9 by 7 in the numerator to find that $n = 1\frac{2}{7}$.

If you complete the solution in B, you will find that n has a value of $\frac{14}{8}$ or $1\frac{6}{8}$, or in reduced form, $1\frac{3}{4}$.

Oral Tell the steps you would take in solving each equation below.

See column 1.

	a	*b*
1.	$3\frac{3}{4} \div 1\frac{1}{4} = n$ 3	3 $n = 4\frac{1}{2} \div 1\frac{1}{2}$
2.	$2\frac{4}{7} \div 1\frac{2}{7} = n$ 2	3 $n = 3\frac{3}{8} \div 1\frac{1}{8}$

Written Solve for n in rows 1 through 8.

	a	*b*
1.	$2\frac{2}{3} \div 1\frac{1}{3} = n$ 2	$2\frac{1}{4} \div 1\frac{3}{4} = n$ $1\frac{2}{7}$
2.	$2\frac{1}{4} \div 1\frac{1}{4} = n$ $1\frac{4}{5}$	$1\frac{6}{7} \div 1\frac{2}{7} = n$ $1\frac{4}{9}$
3.	$1\frac{3}{8} \div 1\frac{1}{8} = n$ $1\frac{2}{9}$	$2\frac{3}{5} \div 1\frac{3}{5} = n$ $1\frac{5}{8}$
4.	$5\frac{5}{8} \div 1\frac{1}{8} = n$ 5	$2\frac{2}{9} \div 1\frac{1}{9} = n$ 2
5.	$3\frac{5}{7} \div 1\frac{1}{7} = n$ $3\frac{1}{4}$	$2\frac{2}{3} \div 1\frac{1}{3} = n$ 2
6.	$5\frac{2}{5} \div 1\frac{4}{5} = n$ 3	$6\frac{3}{4} \div 1\frac{1}{4} = n$ $5\frac{2}{5}$
7.	$5\frac{1}{2} \div 1\frac{1}{2} = n$ $3\frac{2}{3}$	$2\frac{1}{5} \div 1\frac{3}{5} = n$ $1\frac{3}{8}$
8.	$6\frac{1}{3} \div 2\frac{1}{3} = n$ $2\frac{5}{7}$	$4\frac{1}{4} \div 1\frac{1}{4} = n$ $3\frac{2}{5}$

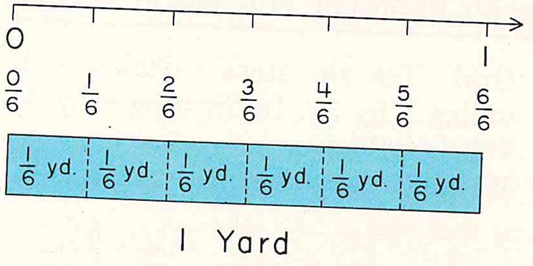
Can you do this? Make each of the following statements true by completing it with <, >, or =.

1. $3 \times (\frac{2}{3} + \frac{1}{2}) \underline{=} (3 \times \frac{2}{3}) + (3 \times \frac{1}{2})$
2. $(\frac{3}{4} + \frac{1}{8}) + \frac{2}{3} \underline{=} \frac{3}{4} + (\frac{1}{8} + \frac{2}{3})$
3. $\frac{13}{16} \div \frac{10}{16} \underline{>} \frac{10}{16} \div \frac{13}{16}$
4. $4\frac{3}{8} \div 1\frac{1}{8} \underline{>} 1\frac{1}{8} \div 4\frac{3}{8}$
5. $5 \times (4\frac{5}{8} \div 1\frac{3}{8}) \underline{>} 5 \times (4\frac{5}{8} \div 1\frac{5}{8})$

Dividing a Whole Number by a Fractional Number

June had 1 yard of ribbon to cut into pieces $\frac{1}{6}$ of a yard long. How many pieces could she have cut?

You are to find how many times $\frac{1}{6}$ is contained in 1. The diagram below shows one way of finding the answer.



Using numerals is a faster way to proceed. The equation to solve is $1 \div \frac{1}{6} = n$. Perhaps you can see in the diagram that $6 \times \frac{1}{6} = \frac{6}{6} = 1$. Thus $1 \div \frac{1}{6} = 6$.

There is also a direct approach to the solution by using division. You rename 1 as $\frac{6}{6}$. Why is $\frac{6}{6}$ a good name to choose?

Observe the solution in abbreviated form in A.

A $n = 1 \div \frac{1}{6} = \frac{6}{6} \div \frac{1}{6} = \frac{6 \div 1}{6 \div 6} = (6 \div 1) \div 1 = 6$

Suppose you were to divide 4 by $\frac{1}{2}$. You might be able to see that $8 \times \frac{1}{2} = 4$; so $4 \div \frac{1}{2} = 8$. In equation form an abbreviated solution is shown in B.

B $n = 4 \div \frac{1}{2} = \frac{8}{2} \div \frac{1}{2} = \frac{8 \div 1}{2 \div 2} = 8$

Now study the solutions in C and D.

C $n = 3 \div \frac{2}{3} = \frac{9}{3} \div \frac{2}{3} = \frac{9 \div 2}{3 \div 3}$
$= (9 \div 2) \div 1 = \frac{9}{2} = 4\frac{1}{2}$

D $n = 2 \div \frac{4}{15} = \frac{30}{15} \div \frac{4}{15} = \frac{30 \div 4}{15 \div 15}$
$= (30 \div 4) \div 1 = \frac{30}{4} = \frac{15}{2} = 7\frac{1}{2}$

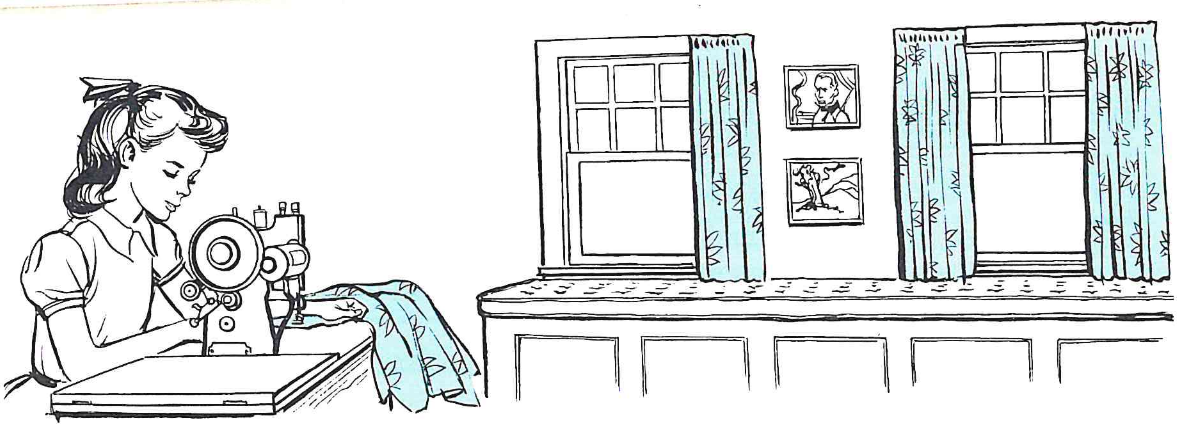
Why was the whole number 3 renamed as $\frac{9}{3}$ in C? Why was the whole number 2 renamed as $\frac{30}{15}$ in D? In D there is one more step than in C. What is the additional step?

Oral Tell the steps you would take in solving for n. `Quotients only.`

	a		*b*	
1.	14 $7 \div \frac{1}{2} = n$	$\frac{14}{2} \div \frac{1}{2} = 14$	8 $n = 6 \div \frac{3}{4}$	$\frac{24}{4} \div \frac{3}{4} = 8$
2.	$13\frac{1}{2}$ $9 \div \frac{2}{3} = n$	$\frac{27}{3} \div \frac{2}{3} = 13\frac{1}{2}$	$3\frac{1}{5}$ $n = 2 \div \frac{5}{8}$	$\frac{16}{8} \div \frac{5}{8} = 3\frac{1}{5}$
3.	$13\frac{1}{2}$ $6 \div \frac{4}{9} = n$	$\frac{54}{9} \div \frac{4}{9} = 13\frac{2}{4}$	$9\frac{1}{3}$ $n = 7 \div \frac{3}{4}$	$\frac{28}{4} \div \frac{3}{4} = 9\frac{1}{3}$

Written Solve for n in rows 1 through 9.

	a	*b*
4.	20 $5 \div \frac{1}{4} = n$	9 $n = 6 \div \frac{2}{3}$
5.	18 $4 \div \frac{2}{9} = n$	21 $n = 6 \div \frac{2}{7}$
6.	12 $8 \div \frac{2}{3} = n$	16 $n = 6 \div \frac{3}{8}$
7.	8 $3 \div \frac{3}{8} = n$	15 $n = 9 \div \frac{3}{5}$
8.	20 $6 \div \frac{3}{10} = n$	8 $n = 5 \div \frac{5}{8}$
9.	12 $9 \div \frac{3}{4} = n$	$11\frac{1}{5}$ $n = 7 \div \frac{5}{8}$



Using an Improper Fraction in Division

Ann bought 9 yards of material for drapes. She needs $2\frac{1}{4}$ yards for each of the drapes. How many drapes can she make?

You are to think of 9 yards as separated into pieces $2\frac{1}{4}$ yd. long and are to find how many pieces there are; so you divide 9 by $2\frac{1}{4}$. What would be a reasonable answer?

Observe the solution below.

$n = 9 \div 2\frac{1}{4}$ Rename $2\frac{1}{4}$

$= 9 \div \frac{9}{4}$ Rename 9

$= \frac{36}{4} \div \frac{9}{4}$ Rewrite

$= \frac{36 \div 9}{4 \div 4}$ Divide

$= (36 \div 9) \div 1$

$= 4$

By renaming $2\frac{1}{4}$ as $\frac{9}{4}$, you changed the problem into one of dividing a whole number by a fractional number.

Ann can make 4 drapes. Does the answer seem reasonable?

Oral Tell the steps to take in dividing 7 by $3\frac{1}{2}$. In dividing 8 by $3\frac{1}{5}$.
See column 1.

Written Solve for n.

	a	*b*
1.	$5 \div 1\frac{2}{3} = n$ 3	5 $n = 8 \div 1\frac{3}{5}$
2.	$6 \div 2\frac{1}{2} = n$ $2\frac{2}{5}$	$4\frac{4}{9}$ $n = 10 \div 2\frac{1}{4}$
3.	$6 \div 1\frac{3}{4} = n$ $3\frac{3}{7}$	$3\frac{3}{5}$ $n = 12 \div 3\frac{1}{3}$
4.	$3 \div 1\frac{4}{5} = n$ $1\frac{2}{3}$	$2\frac{2}{3}$ $n = 6 \div 2\frac{1}{4}$
5.	$7 \div 2\frac{5}{8} = n$ $2\frac{2}{3}$	$7\frac{1}{5}$ $n = 8 \div 1\frac{1}{9}$
6.	$4 \div 3\frac{1}{3} = n$ $1\frac{1}{5}$	$2\frac{2}{3}$ $n = 14 \div 5\frac{1}{4}$
7.	$4 \div 1\frac{1}{9} = n$ $3\frac{3}{5}$	$1\frac{2}{3}$ $n = 7 \div 4\frac{1}{5}$
8.	$12 \div 3\frac{1}{3} = n$ $3\frac{3}{5}$	$5\frac{1}{3}$ $n = 12 \div 2\frac{1}{4}$
9.	$8 \div 1\frac{7}{9} = n$ $4\frac{1}{2}$	$2\frac{2}{3}$ $n = 6 \div 2\frac{1}{4}$
10.	$4 \div 1\frac{5}{9} = n$ $2\frac{4}{7}$	$1\frac{1}{3}$ $n = 3 \div 2\frac{1}{4}$
11.	$4 \div 1\frac{5}{6} = n$ $2\frac{2}{11}$	$1\frac{9}{11}$ $n = 5 \div 2\frac{3}{4}$
12.	$3 \div 1\frac{1}{8} = n$ $2\frac{2}{3}$	$1\frac{1}{2}$ $n = 2 \div 1\frac{1}{3}$

$\frac{12}{3} \div \frac{10}{3} = 1\frac{2}{10} = 1\frac{1}{5}$

$\frac{6}{3} \div \frac{4}{3} = 1\frac{2}{4} = 1$

Dividing Fractional Numbers by Whole Numbers

Al worked a total of $9\frac{1}{4}$ hours in 4 days. What was the average number of hours he worked each day?

About how many times is 4 contained in $9\frac{1}{4}$? The problem may be expressed by the equation $9\frac{1}{4} \div 4 = n$. Do you suppose that you can solve it by renaming both $9\frac{1}{4}$ and 4 as fourths?

Observe the solution below.

$$n = 9\tfrac{1}{4} \div 4 = \tfrac{37}{4} \div 4 = \tfrac{37}{4} \div \tfrac{16}{4}$$
$$= \tfrac{37 \div 16}{4 \div 4}$$
$$= (37 \div 16) \div 1$$
$$= \tfrac{37}{16} \text{ or } 2\tfrac{5}{16}$$

Al worked an average of $2\frac{5}{16}$ hours each day.

Oral Tell the steps you would take in dividing $24\frac{2}{3}$ by 2. See above.

Written Solve for n.

	a	*b*
1.	$4\frac{3}{8} \div 2 = n$ $2\frac{3}{16}$	$1\frac{5}{6}$ $n = 3\frac{2}{3} \div 2$
2.	$2\frac{1}{5} \div 2 = n$ $1\frac{1}{10}$	$2\frac{1}{14}$ $n = 4\frac{1}{7} \div 2$
3.	$3\frac{1}{8} \div 3 = n$ $1\frac{1}{24}$	$1\frac{1}{8}$ $n = 2\frac{1}{4} \div 2$
4.	$6\frac{2}{5} \div 4 = n$ $1\frac{3}{5}$	$1\frac{3}{4}$ $n = 5\frac{1}{4} \div 3$
5.	$7\frac{1}{7} \div 5 = n$ $1\frac{3}{7}$	$1\frac{5}{8}$ $n = 3\frac{1}{4} \div 2$
6.	$7\frac{1}{2} \div 5 = n$ $1\frac{1}{2}$	$1\frac{2}{3}$ $n = 8\frac{1}{3} \div 5$
7.	$3\frac{5}{7} \div 2 = n$ $1\frac{6}{7}$	$1\frac{1}{8}$ $n = 5\frac{5}{8} \div 5$
8.	$9\frac{3}{5} \div 3 = n$ $3\frac{1}{5}$	$1\frac{1}{4}$ $n = 3\frac{3}{4} \div 3$
9.	$6\frac{3}{8} \div 3 = n$ $2\frac{1}{8}$	$1\frac{4}{9}$ $n = 7\frac{2}{9} \div 5$

Another way When you multiplied numbers in the form of mixed fractions by whole numbers, you learned that you could multiply the fractional numbers and the whole numbers separately and add the result. Thus $2 \times 24\frac{2}{3} = 2 \times (24 + \frac{2}{3}) = (2 \times 24) + (2 \times \frac{2}{3}) = 48 + \frac{4}{3} = 48 + 1\frac{1}{3} = 49\frac{1}{3}$.

Do you suppose you could divide $24\frac{2}{3}$ by 2, by dividing 24 by 2 and $\frac{2}{3}$ by 2 and adding the results?

Observe the division below.

$$24\tfrac{2}{3} \div 2 = (24 + \tfrac{2}{3}) \div 2$$
$$= (24 \div 2) + (\tfrac{2}{3} \div 2)$$
$$= 12 + (\tfrac{2}{3} \div \tfrac{6}{3})$$
$$= 12 + \tfrac{2}{6}$$
$$= 12 + \tfrac{1}{3} \text{ or } 12\tfrac{1}{3}$$

This method will always work well if you rename the whole number part of the mixed fraction so that it is directly divisible by the whole number divisor. For example in $9\frac{1}{2} \div 4 = n$, rename $9\frac{1}{2}$ as $8\frac{3}{2}$.

Solve each equation for n.

	a	*b*
1.	$48\frac{1}{4} \div 4 = n$ $12\frac{1}{16}$	$2\frac{3}{8}$ $n = 9\frac{1}{2} \div 4$
2.	$125\frac{1}{2} \div 25 = n$ $5\frac{1}{50}$	$4\frac{2}{45}$ $n = 60\frac{2}{3} \div 15$

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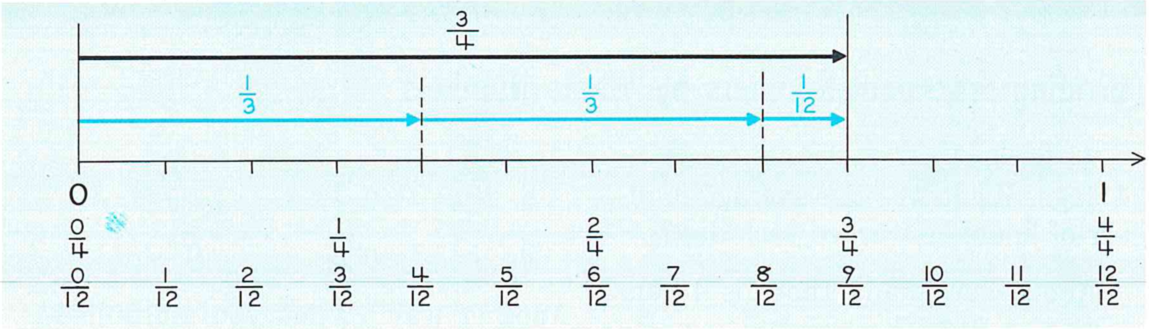

Dividing Fractional Numbers with Different Denominators

Suppose that you are to find how many $\frac{1}{3}$'s are contained in $\frac{3}{4}$. One way to find the answer is to work with a number line as shown above. The number line shows that $\frac{3}{4}=\frac{9}{12}$ and that $\frac{1}{3}=\frac{4}{12}$. It also shows that 2 segments of length $\frac{1}{3}$ plus 1 segment of length $\frac{1}{12}$ are contained in 1 segment of length $\frac{3}{4}$.

Look again at length $\frac{1}{12}$. Does $\frac{1}{12}$ equal $\frac{1}{4}$ of $\frac{1}{3}$? Does $\frac{4}{12}=\frac{1}{3}$? Then length $\frac{1}{12}$ is $\frac{1}{4}$ of $\frac{1}{3}$.

Do you see that 2 lengths of $\frac{1}{3}$ plus 1 length that is $\frac{1}{4}$ of $\frac{1}{3}$ are contained in length $\frac{9}{12}$?

Then $\frac{3}{4}\div\frac{1}{3}=2\frac{1}{4}$. Does $2\frac{1}{4}\times\frac{1}{3}=\frac{3}{4}$?

Look at the number line again. Its scale is twelfths. The denominators of $\frac{3}{4}$ and $\frac{1}{3}$ are 4 and 3. Does $12=4\times3$? Do you suppose that you could always determine the scale of the solution on the number line by multiplying the numbers named by the denominators of the fractions? Choose another division, for example $\frac{1}{2}\div\frac{1}{3}$. Draw a number line to convince yourself that the scale of the number line will be sixths. Use the number line to solve for n in $\frac{1}{2}\div\frac{1}{3}=n$.

You can always use a number line to solve such equations as $\frac{3}{4}\div\frac{1}{3}=n$. Although this method works, it is long and tedious. Fortunately there is a direct method in which you use equations. In this method you will use these two facts.

a. 1 is the identity number for multiplication of fractional numbers. Thus $\frac{3}{4}\times1=\frac{3}{4}$.

b. 1 can be written in many different ways as a fraction. Thus $1=\frac{1}{1}=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\cdot\ \cdot\ \cdot\ \cdot$

See what happens when you try to solve $n=\frac{3}{4}\div\frac{1}{3}$ directly.

$$\begin{aligned} n&=\tfrac{3}{4}\div\tfrac{1}{3}\\ &=\tfrac{3\div1}{4\div3}\end{aligned}$$

You cannot continue, because the denominator of this fraction is not 1. One way out of the difficulty is to make the denominator equal to 1 by the following method:

$$\begin{aligned} n &= \tfrac{3}{4} \div \tfrac{1}{3} \\ &= (\tfrac{3}{4} \times 1) \div (1 \times \tfrac{1}{3}) \\ &= (\tfrac{3}{4} \times \tfrac{3}{3}) \div (\tfrac{4}{4} \times \tfrac{1}{3}) \\ &= \tfrac{3\times 3}{12} \div \tfrac{4\times 1}{12} \\ &= \tfrac{(3\times 3)\div(4\times 1)}{1} \\ &= \tfrac{3\times 3}{4\times 1} = \tfrac{9}{4} \text{ or } 2\tfrac{1}{4} \end{aligned}$$

Renaming 1 as $\frac{3}{3}$ in $(\frac{3}{4}\times 1)$ and as $\frac{4}{4}$ in $(1\times\frac{1}{3})$ produced fractions with the denominator 12. Does $12\div 12=1$?

Now compare $n=\frac{3}{4}\div\frac{1}{3}$ with $n=\frac{3}{4}\times\frac{3}{1}$. What do you observe? Is the operational sign different in the two equations? Does multiplying $\frac{3}{4}$ by $\frac{1}{3}$ produce the same result as dividing $\frac{3}{4}$ by $\frac{3}{1}$?

Do you suppose that you can solve any division equation by using multiplication? Try solving another equation, for example $n=\frac{4}{5}\div\frac{3}{7}$. If you can use multiplication, $n=\frac{4}{5}\div\frac{3}{7}$ should become $n=\frac{4}{5}\times\frac{7}{3}$. Test the method as shown below.

$$\begin{aligned} n &= \tfrac{4}{5} \div \tfrac{3}{7} \\ &= (\tfrac{4}{5} \times 1) \div (1 \times \tfrac{3}{7}) \\ &= (\tfrac{4}{5} \times \tfrac{7}{7}) \div (\tfrac{5}{5} \times \tfrac{3}{7}) \\ &= (\tfrac{4\times 7}{35}) \div (\tfrac{5\times 3}{35}) \\ &= \tfrac{(4\times 7)\div(5\times 3)}{1} \\ &= \tfrac{4\times 7}{5\times 3} = \tfrac{4}{5} \times \tfrac{7}{3} \end{aligned}$$

As you can see, $n=\frac{4}{5}\div\frac{3}{7}$ becomes $n=\frac{4}{5}\times\frac{7}{3}$. The method works!

Oral Discuss each step in the solution of $n=\frac{3}{4}\div\frac{1}{3}$ and $n=\frac{4}{5}\div\frac{3}{7}$. Then tell the reason for the steps you would take in solving the equations below, using the multiplication method. See page 100 and column 1, this page.

	a	*b*
1.	$\frac{1}{2}\div\frac{1}{4}=n$ 2	$7\frac{1}{2}$ $n=\frac{15}{8}\div\frac{1}{4}$
2.	$\frac{4}{5}\div\frac{2}{10}=n$ 4	12 $n=\frac{4}{3}\div\frac{1}{9}$
3.	$\frac{17}{16}\div\frac{1}{2}=n$ $2\frac{1}{8}$	6 $n=\frac{9}{5}\div\frac{3}{10}$
4.	$\frac{8}{3}\div\frac{1}{6}=n$ 16	$2\frac{3}{4}$ $n=\frac{11}{12}\div\frac{1}{3}$
5.	$\frac{9}{2}\div\frac{1}{6}=n$ 27	$2\frac{1}{9}$ $n=\frac{19}{12}\div\frac{3}{4}$
6.	$\frac{11}{3}\div\frac{4}{15}=n$ $13\frac{3}{4}$	$1\frac{1}{10}$ $n=\frac{11}{12}\div\frac{5}{6}$
7.	$\frac{9}{5}\div\frac{4}{15}=n$ $6\frac{3}{4}$	$3\frac{1}{4}$ $n=\frac{13}{12}\div\frac{1}{3}$

Written Solve for n in rows 1 through 7 after rewriting the fractions with a common denominator. Use the multiplication method as a check. Then solve for n in rows 8 through 14. See above. Quotients only.

	a	*b*
8.	$\frac{2}{3}\div\frac{2}{9}=n$ 3	$2\frac{2}{5}$ $n=\frac{3}{4}\div\frac{5}{16}$
9.	$\frac{8}{9}\div\frac{7}{18}=n$ $2\frac{2}{7}$	$1\frac{1}{2}$ $n=\frac{9}{16}\div\frac{3}{8}$
10.	$\frac{2}{5}\div\frac{4}{15}=n$ $1\frac{1}{2}$	2 $n=\frac{3}{4}\div\frac{3}{8}$
11.	$\frac{3}{8}\div\frac{1}{4}=n$ $1\frac{1}{2}$	$1\frac{2}{3}$ $n=\frac{5}{6}\div\frac{1}{2}$
12.	$\frac{5}{8}\div\frac{1}{2}=n$ $1\frac{1}{4}$	4 $n=\frac{1}{4}\div\frac{1}{16}$
13.	$\frac{3}{4}\div\frac{3}{8}=n$ 2	$2\frac{3}{4}$ $n=\frac{11}{12}\div\frac{1}{3}$
14.	$\frac{9}{10}\div\frac{1}{3}=n$ $2\frac{7}{10}$	$3\frac{3}{4}$ $n=\frac{15}{16}\div\frac{1}{4}$

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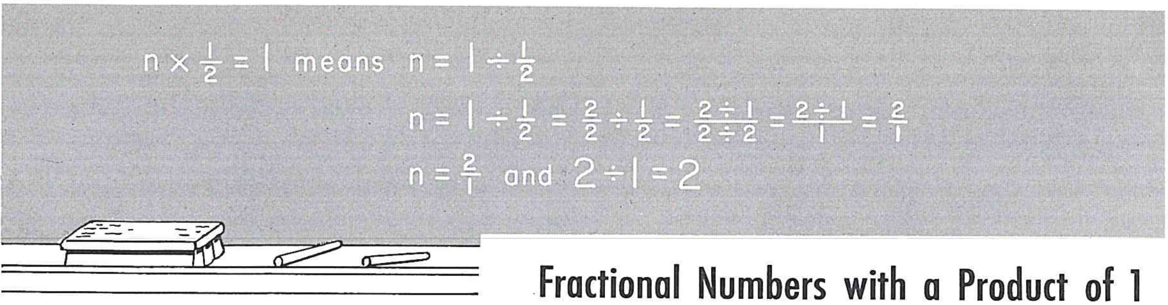
$n\times\frac{1}{2}=1$ means $n=1\div\frac{1}{2}$
$n=1\div\frac{1}{2}=\frac{2}{2}\div\frac{1}{2}=\frac{2\div1}{2\div2}=\frac{2\div1}{1}=\frac{2}{1}$
$n=\frac{2}{1}$ and $2\div1=2$

Fractional Numbers with a Product of 1

Study the work on the board. Do you see that $\frac{2}{2}$ was chosen as a name for 1 as the first step in solving the equation $n=1\div\frac{1}{2}$? In solving the equation $n\times\frac{1}{2}=1$, you found a multiplier which gave a product of 1. Do you suppose that it is possible to find a multiplier for other fractional numbers to make a product of 1? Study the example below.

$$n\times\tfrac{2}{3}=1 \text{ means } n=1\div\tfrac{2}{3}$$
$$n=1\div\tfrac{2}{3}=\tfrac{3}{3}\div\tfrac{2}{3}=\tfrac{3\div2}{3\div3}$$
$$=\tfrac{3\div2}{1}=3\div2=\tfrac{3}{2}$$

Use $\frac{3}{2}$ as a multiplier. Does $\frac{3}{2}\times\frac{2}{3}=1$?

Try the above procedure in solving other equations in which n, when multiplied by any number you choose, except 0, equals 1. For example, take $n\times\frac{1}{4}=1$, $n\times\frac{3}{4}=1$, $n\times\frac{2}{1}=1$, $n\times\frac{4}{3}=1$, and $n\times\frac{5}{2}=1$. Convince yourself that you can find a multiplier for any fractional number to make a product of 1. Do you see that in the above examples, the whole number 2 was expressed as $\frac{2}{1}$?

Two fractional numbers whose product is 1 are called **reciprocals.** We say that $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$ and that $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$. All fractional numbers, except numbers like $\frac{0}{1}$, $\frac{0}{2}$, $\frac{0}{3}$ and so on, have reciprocals. Therefore, for any fractional number other than 0, you simply multiply a number by its reciprocal to obtain 1.

Oral Tell the steps to take in solving for n. Quotients only.

	a	b
1.	8 $n\times\frac{1}{8}=1$	$\frac{8}{3}$ $n\times\frac{3}{8}=1$
2.	$\frac{3}{8}$ $n\times\frac{8}{3}=1$	$\frac{1}{4}$ $n\times\frac{4}{1}=1$
3.	$\frac{1}{2}$ $n\times2=1$	$\frac{1}{3}$ $n\times3=1$
4.	$\frac{6}{5}$ $n\times\frac{5}{6}=1$	$\frac{2}{7}$ $n\times\frac{7}{2}=1$

Tell the reciprocal for each of the following numbers.

	a	b	c	d
5.	$\frac{1}{2}$ $\frac{2}{1}$	$\frac{3}{4}$ $\frac{4}{3}$	2 $\frac{1}{2}$	$\frac{5}{6}$ $\frac{6}{5}$
6.	3 $\frac{1}{3}$	$\frac{7}{3}$ $\frac{3}{7}$	10 $\frac{1}{10}$	$\frac{9}{3}$ $\frac{3}{9}$
7.	$\frac{7}{2}$ $\frac{2}{7}$	$\frac{9}{2}$ $\frac{2}{9}$	$\frac{43}{8}$ $\frac{8}{43}$	$\frac{0}{5}$ Non

Written Solve for n in rows 1 through 4 above. Then write the reciprocal of each number in rows 5, 6, and 7.

Using Reciprocals in Division

Suppose you are to solve for n in the equation $n=\frac{3}{5}\div\frac{2}{3}$. You may recall that $n=\frac{3}{5}\div\frac{2}{3}$ may be solved as $n=\frac{3}{5}\times\frac{3}{2}$. This is possible because $\frac{3}{2}$ is the reciprocal of the divisor $\frac{2}{3}$ and dividing by a fractional number is the same as multiplying by its reciprocal. The solution below will help you see why.

	$n=\frac{3}{5}\div\frac{2}{3}$
Inverse	$n\times\frac{2}{3}=\frac{3}{5}$
Identity number, 1	$n\times\frac{2}{3}=\frac{3}{5}\times 1$
Use $\frac{2}{3}$ and its reciprocal for 1	$n\times\frac{2}{3}=\frac{3}{5}\times(\frac{3}{2}\times\frac{2}{3})$
Associative property	$n\times\frac{2}{3}=(\frac{3}{5}\times\frac{3}{2})\times\frac{2}{3}$

In the final step above, what is the multiplier of $\frac{2}{3}$ on the left? Is it n? Is $(\frac{3}{5}\times\frac{3}{2})$ the multiplier of $\frac{2}{3}$ on the right? Since the two sides of the equation must be equal, the two multipliers n and $(\frac{3}{5}\times\frac{3}{2})$ must be equal.

$$n=\frac{3}{5}\times\frac{3}{2}$$

Compare this result with the original equation. Do you see that $n=\frac{3}{5}\div\frac{2}{3}$ and $n=\frac{3}{5}\times\frac{3}{2}$?

See if you can use the steps suggested above to discover why $n=\frac{7}{8}\div\frac{1}{3}$ may be solved as $n=\frac{7}{8}\times\frac{3}{1}$. Be ready to give the reason for each step that you use.

Oral Tell which number is the divisor in each equation below. Tell its reciprocal.

	a	*b*
1.	$\frac{2}{3}\div\underline{\frac{1}{6}}=\overset{4}{n}$ $\frac{6}{1}$	$\overset{4}{n}=\frac{3}{4}\div\underline{\frac{3}{16}}$ $\frac{16}{3}$
2.	$\frac{3}{5}\div\underline{\frac{3}{10}}=\overset{2}{n}$ $\frac{10}{3}$	$\overset{2}{n}=\frac{1}{2}\div\underline{\frac{1}{4}}$ $\frac{4}{1}$
3.	$\overset{4}{n}=\frac{1}{4}\div\underline{\frac{1}{16}}$ $\frac{16}{1}$	$\overset{2}{n}=\frac{5}{8}\div\underline{\frac{5}{16}}$ $\frac{16}{5}$
4.	$\frac{5}{6}\div\underline{\frac{2}{3}}=\overset{1\frac{1}{4}}{n}$ $\frac{3}{2}$	$\frac{3}{4}\div\underline{\frac{1}{2}}=\overset{1\frac{1}{2}}{n}$ $\frac{2}{1}$
5.	$\frac{7}{9}\div\underline{\frac{5}{18}}=\overset{2\frac{4}{5}}{n}$ $\frac{18}{5}$	$\frac{8}{9}\div\underline{\frac{2}{3}}=\overset{1\frac{1}{3}}{n}$ $\frac{3}{2}$
6.	$\overset{1\frac{1}{2}}{n}=\frac{9}{10}\div\underline{\frac{3}{5}}$ $\frac{5}{3}$	$\overset{1\frac{1}{3}}{n}=\frac{6}{7}\div\underline{\frac{9}{14}}$ $\frac{14}{9}$
7.	$\frac{3}{4}\div\underline{\frac{9}{20}}=\overset{1\frac{2}{3}}{n}$ $\frac{20}{9}$	$\overset{1\frac{1}{2}}{n}=\frac{4}{5}\div\underline{\frac{8}{15}}$ $\frac{15}{8}$
8.	$\frac{6}{7}\div\underline{\frac{4}{21}}=\overset{4\frac{1}{2}}{n}$ $\frac{21}{4}$	$\overset{2\frac{1}{2}}{n}=\frac{2}{3}\div\underline{\frac{4}{15}}$ $\frac{15}{4}$

Written Solve for n in rows 1 through 8. Solve the problems.

9. Betty had $\frac{3}{4}$ of a pound of candy. She put it into bags with 2 ounces ($\frac{1}{8}$ of a pound) in each bag. How many bags did she fill? $\frac{3}{4}\div\frac{1}{8}=n$
6 bags

10. Mary had a piece of material 30 inches ($\frac{5}{6}$ of a yard) wide. She cut it into strips each $\frac{1}{4}$ of a yard wide. How many strips did she have?
$\frac{5}{6}\div\frac{1}{4}=n$ $3\frac{1}{3}$ strips

11. An indoor track had a total length of $\frac{3}{5}$ of a mile. It was marked off in sections each $\frac{1}{10}$ of a mile long. Into how many sections was the track divided? $\frac{3}{5}\div\frac{1}{10}=n$ 6 sections

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Practice in Dividing Fractional Numbers

You should be able to solve each equation on this page without further help. First determine which of the two numbers in each equation is the divisor. Then find its reciprocal. After taking these two steps, you solve for n by using the multiplication method. `Quotients only.`

Oral Tell the steps to take in solving for n.

	a	*b*
1.	$\frac{1}{4} \div \frac{1}{6} = n$ $1\frac{1}{2}$	$\frac{3}{8} \div \frac{1}{5} = n$ $1\frac{7}{8}$
2.	$\frac{4}{5} \div \frac{1}{3} = n$ $2\frac{2}{5}$	$\frac{5}{9} \div \frac{2}{5} = n$ $1\frac{7}{18}$
3.	$\frac{3}{8} \div \frac{1}{5} = n$ $1\frac{7}{8}$	$1\frac{7}{18}$ $n = \frac{5}{9} \div \frac{2}{5}$
4.	$\frac{7}{8} \div \frac{1}{5} = n$ $4\frac{3}{8}$	$1\frac{1}{8}$ $n = \frac{3}{4} \div \frac{2}{3}$
5.	$\frac{3}{4} \div \frac{3}{5} = n$ $1\frac{1}{4}$	$1\frac{1}{3}$ $n = \frac{5}{6} \div \frac{5}{8}$
6.	$\frac{8}{9} \div \frac{4}{7} = n$ $1\frac{5}{9}$	$1\frac{3}{5}$ $n = \frac{3}{5} \div \frac{3}{8}$
7.	$\frac{5}{6} \div \frac{5}{8} = n$ $1\frac{1}{3}$	$1\frac{1}{5}$ $n = \frac{4}{5} \div \frac{2}{3}$
8.	$\frac{5}{6} \div \frac{5}{9} = n$ $1\frac{1}{2}$	$1\frac{5}{9}$ $n = \frac{4}{9} \div \frac{2}{7}$
9.	$\frac{6}{7} \div \frac{2}{3} = n$ $1\frac{2}{7}$	$1\frac{2}{3}$ $n = \frac{2}{3} \div \frac{2}{5}$
10.	$\frac{5}{7} \div \frac{5}{9} = n$ $1\frac{2}{7}$	$1\frac{1}{5}$ $n = \frac{4}{15} \div \frac{2}{9}$
11.	$\frac{9}{16} \div \frac{1}{3} = n$ $1\frac{11}{16}$	$1\frac{1}{8}$ $n = \frac{27}{32} \div \frac{3}{4}$
12.	$\frac{9}{10} \div \frac{2}{6} = n$ $2\frac{7}{10}$	$1\frac{24}{25}$ $n = \frac{49}{50} \div \frac{1}{2}$

Written Solve for n in rows 1 through 12 above.

Another way Suppose you wished to use a fraction to represent $\frac{3}{4} \div \frac{2}{3}$. You would write a complex fraction as shown at the right. The division sign has been replaced by a heavy fraction line. You can use complex fractions in solving such questions as $\frac{3}{4} \div \frac{2}{3}$.

$$\dfrac{\frac{3}{4}}{\frac{2}{3}}$$

Division of fractional numbers by means of complex fractions is based upon two principles.

a. It is always possible to change the denominator to 1 by using its reciprocal.

b. A fractional number is not changed by multiplying both numbers (named by the numerator and denominator) by the same number.

See how these principles apply in the solution below.

$n = \dfrac{\frac{3}{4}}{\frac{2}{3}}$ Think: $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$.

$n = \dfrac{\frac{3}{4} \times \frac{3}{2}}{\frac{2}{3} \times \frac{3}{2}}$ Tell the purpose of this step.

$n = \dfrac{\frac{3}{4} \times \frac{3}{2}}{1}$ or $n = \frac{3}{4} \times \frac{3}{2}$

$n = \frac{9}{8}$ or $1\frac{1}{8}$

Use complex fractions in solving the equations in column 1.

Solving Problems

You determine the operations to be used in solving problems with fractional numbers in a way similar to that you have used for whole numbers. Reminders:

a. When you have to find the sum or total of two or more different numbers, you *add*.

b. When you have to find the sum or total of two or more equal numbers, you may either *add* or *multiply*.

c. When you have to find the difference between two numbers, you *subtract*.

d. When you have to find how many times one number is contained in another, you *divide*.

Write the equation for each problem below. Be ready to tell why you decided upon the equation you wrote.

1. Bob wrapped a package that held 4 boxes of candy. Each box held $10\frac{1}{2}$ ounces of candy. What was the weight of the candy in the package?
$10\frac{1}{2} \times 4 = n$ 42 ounces

2. Nancy played her violin for $\frac{3}{4}$ hour on Friday, $1\frac{1}{2}$ hours on Saturday, and $\frac{2}{3}$ hour on Sunday. On the average, how much time did she spend in practice on each day?
$(\frac{3}{4} + 1\frac{1}{2} + \frac{2}{3}) \div 3 = n$ $\frac{35}{36}$ hr.

3. Sally took the last piece of ribbon from a spool that held $10\frac{1}{2}$ yards of ribbon. Her piece of ribbon had a length of $1\frac{5}{6}$ yards. How many yards of ribbon had others taken from the spool? $10\frac{1}{2} - 1\frac{5}{6} = n$ $8\frac{2}{3}$ yd.

4. Doris took a piece of crepe paper that was $1\frac{2}{3}$ feet wide and cut it into 6 strips of equal width. What was the width in inches of each strip?
$(1\frac{2}{3} \times 12) \div 6 = n$ $3\frac{1}{3}$ in.

5. Bob practices shooting baskets for $\frac{1}{2}$ hour a day on every day of the week except Sunday. How many hours does he practice each week?
$6 \times \frac{1}{2} = n$ 3 hr.

6. Betty picked $4\frac{1}{2}$ pails of berries. Each pail held $\frac{3}{4}$ of a quart of berries. How many quarts of berries did she pick altogether? $4\frac{1}{2} \times \frac{3}{4} = n$ $3\frac{3}{8}$ qt.

7. The Carlsons have a floor that is $12\frac{3}{4}$ feet wide. Mr. Carlson plans to lay rows of tile on the floor. If each tile is 9 inches wide ($\frac{3}{4}$ ft.), how many rows of tile must he lay?
$12\frac{3}{4} \div \frac{3}{4} = n$ 17 rows

8. John needed a board $36\frac{1}{4}$ inches long. He had a piece that was $33\frac{7}{8}$ inches long. How much too short was the piece that he had?
$36\frac{1}{4} - 33\frac{7}{8} = n$ $2\frac{3}{8}$ in.

Oral Explain why you decided upon the equation you wrote for each of the problems. See column 1.

Written Solve each equation you wrote for problems 1 through 8.

Using Mixed Fractions in Division

Mary had an apron pattern that required $1\frac{1}{8}$ yards of material. She had a $3\frac{1}{2}$-yard piece of material. How many aprons could Mary make and how much material would be left?

Would Mary be able to make more than 2 aprons? More than 3 aprons? How did you decide?

The equation for the problem is $3\frac{1}{2} \div 1\frac{1}{8} = n$. How would you express $3\frac{1}{2}$ and $1\frac{1}{8}$ as improper fractions?

Think of the equation as $\frac{7}{2} \div \frac{9}{8} = n$. Study the solution.

$$\begin{aligned} n &= 3\tfrac{1}{2} \div 1\tfrac{1}{8} \\ &= \tfrac{7}{2} \div \tfrac{9}{8} \\ &= \tfrac{7}{2} \times \tfrac{8}{9} \\ &= \tfrac{7 \times 8}{9 \times 2} \text{ or } \tfrac{7 \times 4}{9 \times 1} \\ &= \tfrac{28}{9} = 3\tfrac{1}{9} \end{aligned}$$

Mary could make 3 aprons. She would have $\frac{1}{9}$ of a $1\frac{1}{8}$-yard length of material left.

Observe that $\frac{1}{9} \times 1\frac{1}{8} = \frac{1}{9} \times \frac{9}{8} = \frac{1 \times 9}{8 \times 9} = \frac{1}{8} \times 1 = \frac{1}{8}$. Therefore the remainder would be $\frac{1}{8}$ of a yard in length.

Oral Express each dividend and divisor as an improper fraction.

	a	*b*
1.	$4\frac{1}{5} \div 2\frac{1}{10} = n$ $\frac{21}{5} \div \frac{21}{10} = 2$	$n = 2\frac{5}{6} \div 1\frac{5}{12}$ $2 = \frac{17}{6} \div \frac{17}{12}$
2.	$2\frac{3}{4} \div 1\frac{1}{2} = n$ $\frac{11}{4} \div \frac{3}{2} = 1\frac{5}{6}$	$n = 1\frac{3}{4} \div 1\frac{1}{8}$ $1\frac{5}{9} = \frac{7}{4} \div \frac{9}{8}$
3.	$1\frac{4}{9} \div 1\frac{1}{3} = n$ $\frac{13}{9} \div \frac{4}{3} = 1\frac{1}{12}$	$n = 3\frac{1}{8} \div 1\frac{1}{4}$ $2\frac{1}{2} = \frac{25}{8} \div \frac{5}{4}$
4.	$4\frac{1}{6} \div 2\frac{1}{2} = n$ $\frac{25}{6} \div \frac{5}{2} = 1\frac{2}{3}$	$n = 5\frac{1}{3} \div 2\frac{2}{9}$ $2\frac{2}{5} = \frac{16}{3} \div \frac{20}{9}$
5.	$2\frac{4}{5} \div 1\frac{1}{10} = n$ $\frac{14}{5} \div \frac{11}{10} = 2\frac{6}{11}$	$n = 5\frac{5}{8} \div 2\frac{1}{4}$ $2\frac{1}{2} = \frac{45}{8} \div \frac{9}{4}$

Written Solve for n in rows 1 through 5 above. Then solve for n in rows 6 through 20.

	a	*b*
6.	$7\frac{5}{6} \div 1\frac{3}{18} = n$ $6\frac{5}{7}$	$8\frac{27}{31}$ $n = 18\frac{1}{3} \div 2\frac{1}{15}$
7.	$19\frac{5}{8} \div 3\frac{7}{16} = n$ $5\frac{39}{55}$	$12\frac{4}{5}$ $n = 26\frac{2}{3} \div 2\frac{1}{12}$
8.	$15\frac{4}{9} \div 1\frac{2}{3} = n$ $9\frac{4}{15}$	$7\frac{1}{18}$ $n = 15\frac{7}{8} \div 2\frac{1}{4}$
9.	$36\frac{2}{3} \div 2\frac{5}{6} = n$ $12\frac{16}{17}$	$9\frac{1}{17}$ $n = 19\frac{1}{4} \div 2\frac{1}{8}$
10.	$21\frac{1}{8} \div 3\frac{1}{4} = n$ $6\frac{1}{2}$	$10\frac{7}{18}$ $n = 35\frac{1}{16} \div 3\frac{3}{8}$
11.	$1\frac{5}{8} \div \frac{3}{4} = n$ $2\frac{1}{6}$	$4\frac{1}{6}$ $n = 2\frac{1}{2} \div \frac{3}{5}$
12.	$1\frac{3}{5} \div \frac{3}{10} = n$ $5\frac{1}{3}$	$6\frac{2}{3}$ $n = 5\frac{5}{6} \div \frac{7}{8}$
13.	$1\frac{1}{6} \div \frac{7}{9} = n$ $1\frac{1}{2}$	$4\frac{3}{8}$ $n = 2\frac{5}{8} \div \frac{3}{5}$
14.	$3\frac{1}{3} \div \frac{2}{5} = n$ $8\frac{1}{3}$	$5\frac{5}{7}$ $n = 2\frac{2}{7} \div \frac{2}{5}$
15.	$2\frac{2}{7} \div \frac{2}{9} = n$ $10\frac{2}{7}$	$4\frac{3}{8}$ $n = 2\frac{5}{8} \div \frac{3}{5}$
16.	$1\frac{1}{9} \div \frac{2}{7} = n$ $3\frac{8}{9}$	$9\frac{4}{5}$ $n = 2\frac{4}{5} \div \frac{2}{7}$
17.	$1\frac{1}{2} \div 1\frac{1}{3} = n$ $1\frac{1}{8}$	$1\frac{3}{5}$ $n = 2\frac{4}{5} \div 1\frac{3}{4}$
18.	$1\frac{5}{6} \div 1\frac{1}{4} = n$ $1\frac{7}{15}$	$2\frac{1}{10}$ $n = 3\frac{1}{2} \div 1\frac{2}{3}$
19.	$3\frac{3}{7} \div 1\frac{1}{2} = n$ $2\frac{2}{7}$	$1\frac{7}{9}$ $n = 2\frac{2}{9} \div 1\frac{1}{4}$
20.	$2\frac{1}{3} \div 1\frac{1}{2} = n$ $1\frac{5}{9}$	$2\frac{2}{5}$ $n = 2\frac{4}{5} \div 1\frac{1}{6}$

Using a Divisor Larger than the Dividend

Bob added $\frac{3}{4}$ of a quart of oil to $2\frac{1}{2}$ gallons of gasoline in the tank of his motor. At this rate what fractional part of a quart of oil should he add to each gallon of gasoline?

You are to think of $\frac{3}{4}$ as separated into $2\frac{1}{2}$ parts of equal size; so you divide. The equation is $\frac{3}{4}\div 2\frac{1}{2}=n$.

Think of $2\frac{1}{2}$ as $\frac{5}{2}$. Then the equation is $\frac{3}{4}\div\frac{5}{2}=n$. A solution follows.

$$\begin{aligned} n &= \tfrac{3}{4}\div 2\tfrac{1}{2} \\ &= \tfrac{3}{4}\div\tfrac{5}{2} \\ &= \tfrac{3}{4}\times\tfrac{2}{5} \\ &= \tfrac{3\times 2}{4\times 5} \text{ or } \tfrac{3\times 2}{5\times 4} \text{ or } \tfrac{3}{5}\times\tfrac{1}{2} \\ &= \tfrac{3}{10} \end{aligned}$$

Bob needs $\frac{3}{10}$ of a quart of oil for each gallon of gasoline.

In problems such as that above, the divisor is a number greater than the dividend. As you can see from the above solution, you use the same steps as those used in dividing any fractional numbers.

Oral Tell the steps to take in solving for n. `Quotients only.`

	a	*b*
1.	$\frac{5}{8}\div\frac{5}{6}=n$ $\frac{3}{4}$	$n=\frac{2}{3}\div 4$ $\frac{1}{6}$
2.	$n=\frac{5}{9}\div 1\frac{2}{3}$ $\frac{1}{3}$	$n=1\frac{1}{3}\div 2\frac{2}{5}$ $\frac{5}{9}$

Written Copy. Solve for n.

	a	*b*
1.	$\frac{1}{3}\div\frac{1}{2}=n$ $\frac{2}{3}$	$n=\frac{2}{5}\div\frac{4}{5}$ $\frac{1}{2}$
2.	$\frac{2}{9}\div\frac{2}{3}=n$ $\frac{1}{3}$	$n=\frac{1}{3}\div 2$ $\frac{1}{6}$
3.	$3\div 5\frac{1}{4}=n$ $\frac{4}{7}$	$n=\frac{1}{4}\div 1\frac{1}{2}$ $\frac{1}{6}$
4.	$\frac{1}{2}\div 1\frac{1}{8}=n$ $\frac{4}{9}$	$n=\frac{1}{3}\div 1\frac{1}{6}$ $\frac{2}{7}$
5.	$\frac{1}{3}\div 1\frac{1}{4}=n$ $\frac{4}{15}$	$n=\frac{1}{4}\div 1\frac{1}{3}$ $\frac{3}{16}$
6.	$4\frac{3}{4}\div 6\frac{1}{8}=n$ $\frac{38}{49}$	$n=9\frac{5}{6}\div 2\frac{3}{4}$ $3\frac{19}{33}$
7.	$8\frac{7}{8}\div 3\frac{2}{3}=n$ $2\frac{37}{88}$	$n=6\frac{5}{6}\div 2\frac{2}{7}$ $2\frac{95}{96}$
8.	$7\frac{2}{5}\div 4\frac{9}{10}=n$ $1\frac{25}{49}$	$n=6\frac{4}{8}\div 3\frac{1}{2}$ $1\frac{6}{7}$
9.	$8\frac{4}{5}\div 2\frac{6}{15}=n$ $3\frac{2}{3}$	$n=7\frac{8}{8}\div 5\frac{1}{9}$ $1\frac{13}{23}$
10.	$5\frac{8}{9}\div 2\frac{1}{3}=n$ $2\frac{11}{21}$	$n=7\frac{1}{4}\div 2\frac{1}{2}$ $2\frac{9}{10}$

A quick review

1. $(n+3\frac{1}{2})-3\frac{1}{2}=7$ 7

2. $(27\frac{3}{4}+18\frac{1}{4})-n=4$ 42

3. $n=(3\times 2\frac{1}{3})+(2\times 1)$ 9

4. $(\frac{7}{8}\div\frac{1}{8})+n=15$ 8

5. $(n-5)+(2\frac{1}{2}+3)=5\frac{1}{2}$ 5

6. $(10\times 12\frac{1}{2})+(10\times 12\frac{1}{2})=n$ 250

7. $(n\times 15\frac{3}{4})\div 15\frac{3}{4}=10$ 10

8. $(n\div 3)+2\frac{1}{8}=11\frac{1}{8}$ 27

9. $n=(14\frac{3}{4}\times 1)-(\frac{2}{3}\times\frac{0}{3})$ $14\frac{3}{4}$

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Solving Equations

Solve each equation without any help. Check each multiplication by interchanging the factors and multiplying again. Check each division by using methods other than those used in your first computation.

Written Solve for n. Check.

a *b*

1. $3\frac{1}{2} \times 7\frac{1}{4} = n$ $25\frac{3}{8}$ $n = 9\frac{5}{6} \times 6\frac{2}{3}$ $65\frac{5}{9}$
2. $8\frac{4}{5} \times 2\frac{3}{10} = n$ $20\frac{6}{25}$ $n = 5\frac{5}{6} \times 4\frac{7}{12}$ $26\frac{53}{72}$
3. $9\frac{3}{4} \times 2\frac{3}{8} = n$ $23\frac{5}{32}$ $n = 5\frac{7}{8} \times 4\frac{3}{4}$ $27\frac{29}{32}$
4. $6\frac{2}{3} \times 9\frac{1}{6} = n$ $61\frac{1}{9}$ $n = 7\frac{3}{5} \times 8\frac{9}{10}$ $67\frac{16}{25}$
5. $8\frac{5}{8} \times 4\frac{5}{12} = n$ $38\frac{3}{32}$ $n = 9\frac{2}{5} \times 6\frac{3}{7}$ $60\frac{3}{7}$
6. $10\frac{3}{10} \times 15\frac{4}{5} = n$ $162\frac{37}{50}$ $n = 15\frac{4}{9} \times 23\frac{2}{3}$ $365\frac{14}{27}$
7. $56\frac{3}{4} \times 98\frac{1}{2} = n$ $5589\frac{7}{8}$ $n = 87\frac{5}{7} \times 29\frac{9}{14}$ $2600\frac{5}{49}$
8. $10\frac{3}{4} \div 2\frac{1}{2} = n$ $4\frac{3}{10}$ $n = 20\frac{1}{2} \div 10\frac{2}{3}$ $1\frac{59}{64}$
9. $35\frac{1}{6} \div 7\frac{1}{4} = n$ $4\frac{74}{87}$ $n = 56\frac{2}{7} \div 9\frac{3}{4}$ $5\frac{211}{273}$
10. $25\frac{1}{5} \div 5\frac{5}{6} = n$ $4\frac{8}{25}$ $n = 63\frac{4}{9} \div 9\frac{1}{3}$ $6\frac{67}{84}$
11. $72\frac{8}{9} \div 8\frac{6}{7} = n$ $8\frac{64}{279}$ $n = 58\frac{4}{6} \div 3\frac{2}{3}$ 16
12. $75\frac{7}{8} \div 5\frac{3}{4} = n$ $13\frac{9}{46}$ $n = 63\frac{4}{5} \div 8\frac{2}{3}$ $7\frac{47}{130}$

Express each problem as an equation. To illustrate, problem 13 is expressed as $n = 100 - (4\frac{2}{3} \times 13\frac{1}{2})$. Solve for n. Reminder: The signs $ and . are used in the final answer but not in the equation. Try to solve each problem without any help.

13. Mrs. Hamilton bought $4\frac{2}{3}$ pounds of peaches at $13\frac{1}{2}$¢ a pound. How much change did she receive from $1.00? $100-(4\frac{2}{3} \text{x} 13\frac{1}{2})=n$ 37¢

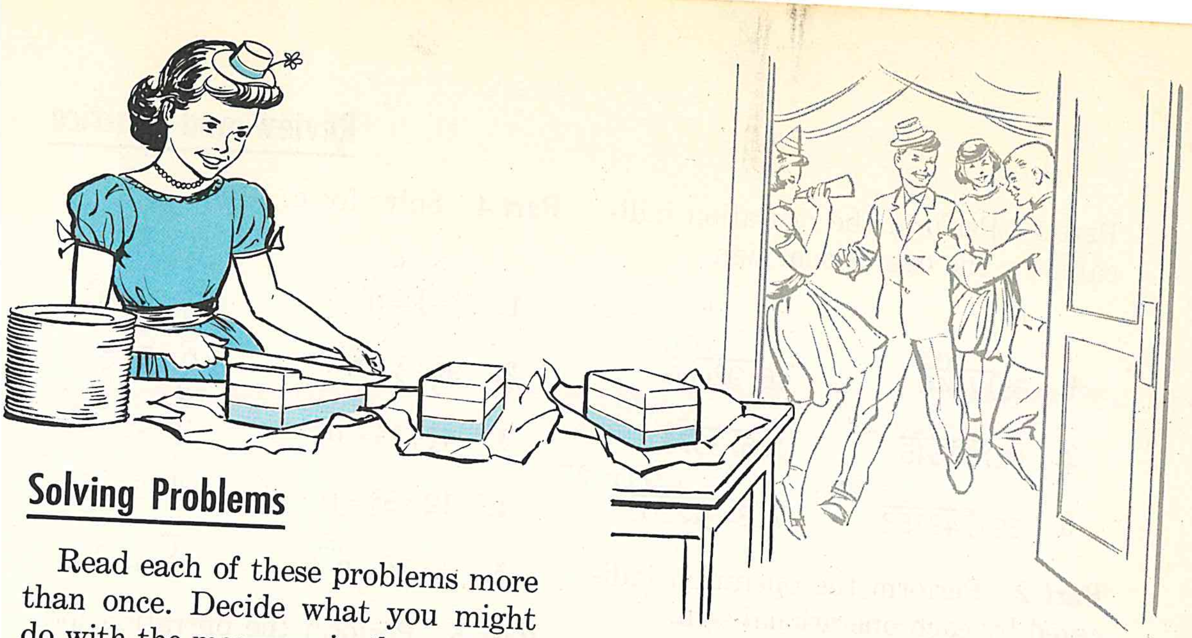
14. Mrs. Reynolds bought $1\frac{1}{4}$ pounds of green beans at 20¢ a pound, and $2\frac{1}{2}$ pounds of onions at 8¢ a pound. How much change did she receive from $5.00? $500-[(20\text{x}1\frac{1}{4})+(2\frac{1}{2}\text{x}8)] =n$ $4.55

15. Mrs. Hanks bought $\frac{3}{4}$ pound of seedless grapes at 24¢ a pound, and 10 pounds of new red potatoes at $9\frac{9}{10}$¢ a pound. How much change did she receive from $2.00? $200-[(\frac{3}{4}\text{x}24)+(9\frac{9}{10}\text{x}10)] =n$ 83¢

16. Mrs. Zorn bought $5\frac{1}{2}$ yards of material at 98¢ a yard, $3\frac{1}{4}$ yards of trimming at 59¢ a yard, a pattern for $1.25, and 6 buttons at 42¢ apiece. How much change did she receive from $20.00? $2000-[(5\frac{1}{2}\text{x}98)+(3\frac{1}{4}\text{x}59)+125+(42\text{x}6)] =n$ $8.92

Can you do this? Replace each ● with an operational sign to make each statement true.

1. $38 \overset{-}{\bullet} (6 \overset{\text{x}}{\bullet} 3) = 20$
2. $(3\frac{1}{2} \overset{-}{\bullet} 1\frac{1}{2}) \overset{+}{\bullet} 5\frac{1}{2} = 7\frac{1}{2}$
3. $10 = (8\frac{5}{8} \overset{-}{\bullet} 3\frac{5}{8}) \overset{+}{\bullet} 5$
4. $3\frac{1}{3} = (6\frac{2}{3} \overset{-}{\bullet} 4\frac{1}{3}) \overset{+}{\bullet} 1$
5. $100 = (3 \overset{\text{x}}{\bullet} 25) \overset{+}{\bullet} 25$



Solving Problems

Read each of these problems more than once. Decide what you might do with the measures in the problem to help you answer the question. Use your decision in helping you decide upon an equation for each problem.

Write the equation to use in solving each problem. Be ready to tell why you decided upon it.

1. Nancy has 3 quarts of ice cream. How many guests can she serve if each serving is $\frac{1}{6}$ of a quart? $3 \div \frac{1}{6}$
$6 \times 3 = n$ 18 guests

2. How many $\frac{1}{2}$-bushel baskets can be filled from $2\frac{3}{4}$ bushels of apples?
$2\frac{3}{4} \div \frac{1}{2} = n$ $5\frac{1}{2}$ baskets

3. Ann has $\frac{5}{6}$ of a yard of ribbon to se in making 2 bows. What fraction f a yard can she use for each?
$\div 2 = n$ $\frac{5}{12}$ yd.

4. Mary paid $.90 for a $1\frac{1}{2}$-pound ox of candy. What was the price er pound? $90 \div 1\frac{1}{2} = n$ 60¢

5. Judy uses $\frac{1}{6}$ of a yard of ribbon make a badge. She has $2\frac{1}{4}$ yards of ribbon. How many badges can she make? $2\frac{1}{4} \div \frac{1}{6} = n$ $n = 13\frac{1}{2}$ 13 badges

6. Bob cut 4 pieces each $\frac{3}{4}$ of a foot long from a 6-foot board. How much of the board was left?
$6 - (4 \times \frac{3}{4}) = n$ 3 ft.

7. Janet grew $1\frac{1}{2}$ inches in 8 months. What was the average growth per month? $1\frac{1}{2} \div 8 = n$ $\frac{3}{16}$ in.

8. Joe walked $4\frac{1}{2}$ miles in $1\frac{1}{3}$ hours. What was his average speed per hour?
$4\frac{1}{2} \div 1\frac{1}{3} = n$ $3\frac{3}{8}$ mph

9. Bob and his father drove 27 miles in $\frac{3}{4}$ of an hour. What was their average rate per hour? $27 \div \frac{3}{4} = n$
36 mph

Oral Tell why you decided upon each equation you wrote.

Written Solve each of your equations for n. Read the problems again to see whether your answers are reasonable. If so, write your answers.

Review and Practice

Part 1 Perform the operation indicated by the operational sign.

	a	*b*
1.	$35\overline{)7140}$ 204	$46\overline{)92464}$ 2010r4
2.	$82\overline{)83645}$ 1020r5	$135\overline{)3976}$ 29r61
3.	$391\overline{)49153}$ 125r278	$718\overline{)913205}$ 1271r627

Part 2 Perform the operation indicated by each operational sign.

	a	*b*	*c*
1.	478653 243798 312587 +469358 1504396	931289 693152 974212 +865301 3463954	\$7603.58 5821.93 3440.16 +2168.72 \$19034.39
2.	506843 −237921 268922	940609 −317909 622700	\$9004.00 −3975.63 \$5028.37
3.	300750 ×75 22556250	425720 ×87 37037640	\$3695.42 ×256 \$946027.52

Part 3 Solve for n.

	a	*b*
1.	$3\frac{1}{3}+4\frac{5}{6}=n$ $8\frac{1}{6}$	$16\frac{1}{8}$ $n=9\frac{3}{8}+6\frac{3}{4}$
2.	$3\frac{1}{2}+7\frac{2}{5}=n$ $10\frac{9}{10}$	$10\frac{7}{12}$ $n=7\frac{3}{4}+2\frac{5}{6}$
3.	$8\frac{7}{10}-4\frac{2}{5}=n$ $4\frac{3}{10}$	$5\frac{3}{8}$ $n=9\frac{1}{8}-3\frac{3}{4}$
4.	$2\frac{1}{3}-1\frac{1}{2}=n$ $\frac{5}{6}$	$3\frac{11}{12}$ $n=7\frac{3}{4}-3\frac{5}{6}$
5.	$\frac{2}{3}\times\frac{3}{4}=n$ $\frac{1}{2}$	6 $n=\frac{2}{5}\times 15$
6.	$\frac{1}{5}\times 1\frac{1}{4}=n$ $\frac{1}{4}$	$20\frac{1}{4}$ $n=5\frac{2}{5}\times 3\frac{3}{4}$

Part 4 Solve for n.

	a	*b*
1.	$\frac{3}{4}\div\frac{1}{4}=n$ 3	$2\frac{1}{3}$ $n=\frac{7}{16}\div\frac{3}{16}$
2.	$9\frac{1}{2}\div\frac{1}{2}=n$ 19	$6\frac{2}{3}$ $n=5\div\frac{3}{4}$
3.	$4\frac{2}{3}\div 1\frac{1}{3}=n$ $3\frac{1}{2}$	$2\frac{5}{9}$ $n=5\frac{3}{4}\div 2\frac{1}{4}$
4.	$12\div 3\frac{3}{4}=n$ $3\frac{1}{5}$	$7\frac{5}{9}$ $n=5\frac{2}{3}\div\frac{3}{4}$
5.	$\frac{1}{2}\div 2\frac{2}{3}=n$ $\frac{3}{16}$	$\frac{9}{16}$ $n=\frac{3}{4}\div 1\frac{1}{3}$

Part 5 Perform the operation indicated by each operational sign.

	a	*b*	*c*
1.	$24\frac{1}{2}$ $14\frac{2}{3}$ $+20\frac{1}{8}$ $59\frac{7}{24}$	$59\frac{3}{4}$ $27\frac{2}{7}$ $+83\frac{9}{14}$ $170\frac{19}{28}$	$348\frac{3}{4}$ $296\frac{4}{5}$ $+101\frac{9}{10}$ $747\frac{9}{20}$
2.	$19\frac{5}{6}$ $26\frac{2}{3}$ $+55\frac{7}{8}$ $102\frac{3}{8}$	$35\frac{5}{8}$ $24\frac{4}{6}$ $+76\frac{3}{4}$ $137\frac{1}{24}$	$701\frac{2}{4}$ $54\frac{1}{8}$ $+159\frac{5}{6}$ $915\frac{11}{24}$
3.	$237\frac{7}{8}$ $-116\frac{3}{4}$ $121\frac{1}{8}$	$817\frac{1}{3}$ $-459\frac{3}{4}$ $357\frac{7}{12}$	$506\frac{3}{10}$ $-295\frac{5}{6}$ $210\frac{7}{15}$
4.	$407\frac{2}{5}$ $-398\frac{5}{7}$ $8\frac{24}{35}$	$730\frac{5}{16}$ $-475\frac{1}{5}$ $255\frac{9}{80}$	$912\frac{3}{32}$ $-730\frac{5}{6}$ $181\frac{25}{96}$
5.	$118\frac{1}{2}$ $\times 28$ 3318	$378\frac{2}{3}$ $\times 39$ 14768	$563\frac{7}{8}$ $\times 27$ $15224\frac{5}{8}$
6.	735 $\times 43\frac{1}{5}$ 31752	620 $\times 91\frac{3}{4}$ 56885	925 $\times 50\frac{1}{2}$ $46712\frac{1}{2}$

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. You divide fractional numbers as you divide whole numbers. (93)

2. When the denominators of the divisor and dividend are the same, the denominator of the quotient will always be 1. (94)

3. In the common denominator method, divide the numbers named by the denominators, then divide those named by the numerators. (94)

4. Multiplying both the divisor and the dividend by any number other than 0 leaves the quotient unchanged. (102)

5. In the multiplication method of dividing fractions, rewrite the equation, using the reciprocal of the divisor. Then multiply. (103)

Words to Know

1. Reciprocal (102)

2. Complex fractions (104)

Questions to Discuss

See T111 for answers.

1. How can you express a whole umber in fractional form? (93)

2. What is the first step in dividng numbers expressed as mixed actions? (95, 99)

3. How should you express the number written as the mixed fraction $1\frac{2}{3}$ or the whole number 2 before you divide either one by $\frac{2}{3}$? (95, 97)

4. How do you divide 2 fractional numbers having unlike denominators when you do not wish to change the denominators to like denominators? (102-103)

Written Practice

Solve each equation for n.

	a	*b*	
1.	$n \times \frac{3}{4} = \frac{9}{4}$ 3	$\frac{9}{4} \div \frac{3}{4} = n$ 3	(93)
2.	$n = \frac{25}{32} \div \frac{5}{32}$ 5	$n = \frac{11}{16} \div \frac{3}{16}$ $3\frac{2}{3}$	(94)
3.	$2\frac{3}{5} \div \frac{4}{5} = n$ $3\frac{1}{4}$	$n = 4\frac{1}{3} \div \frac{2}{3}$ $6\frac{1}{2}$	(95)
4.	$9\frac{1}{4} \div 3\frac{3}{4} = n$ $2\frac{7}{15}$	$n = 7\frac{5}{6} \div 2\frac{5}{6}$ $2\frac{13}{17}$	(96)
5.	$8 \div \frac{1}{4} = n$ 32	$n = 9 \div \frac{2}{3}$ $13\frac{1}{2}$	(97)
6.	$10 \div 1\frac{2}{5} = n$ $7\frac{1}{7}$	$8 \div 2\frac{7}{9} = n$ $2\frac{22}{25}$	(98)
7.	$n = 6\frac{2}{3} \div 4$ $1\frac{2}{3}$	$15\frac{3}{4} \div 3 = n$ $5\frac{1}{4}$	(99)
8.	$n = \frac{9}{10} \div \frac{3}{5}$ $1\frac{1}{2}$	$\frac{18}{12} \div \frac{3}{4} = n$ 2	(100)
9.	$n = \frac{15}{16} \div \frac{2}{12}$ $5\frac{5}{8}$	$n = \frac{17}{24} \div \frac{3}{10}$ $2\frac{13}{36}$	(104)
10.	$4\frac{4}{5} \div 2\frac{1}{7} = n$ $2\frac{6}{25}$	$7\frac{3}{4} \div 2\frac{1}{3} = n$ $3\frac{9}{28}$	(106)
11.	$n = \frac{1}{3} \div \frac{3}{4}$ $\frac{4}{9}$	$n = 5\frac{5}{6} \div 9\frac{2}{3}$ $\frac{35}{58}$	(107)

Self-Evaluation

Part 1 Copy. Solve for n.

	a	*b*
1.	$n=\frac{7}{12}\div\frac{5}{12}$ $1\frac{2}{5}$	$\frac{8}{9}\div\frac{4}{9}=n$ 2
2.	$6\frac{3}{8}\div\frac{5}{8}=n$ $10\frac{1}{5}$	$14\div\frac{2}{5}=n$ 35
3.	$3\frac{3}{4}\div1\frac{2}{3}=n$ $2\frac{1}{4}$	$n=24\frac{3}{4}\div8\frac{1}{2}$ $2\frac{31}{34}$
4.	$n=35\frac{1}{7}\div6\frac{1}{3}$ $5\frac{73}{133}$	$9\frac{6}{7}\div3\frac{1}{4}=n$ $3\frac{3}{91}$
5.	$n=9\frac{4}{5}\div6\frac{2}{3}$ $1\frac{47}{100}$	$1\frac{1}{8}\div5\frac{2}{3}=n$ $\frac{27}{136}$

Part 2 Decide whether each of the following statements is true or false. Write T if the statement is true. Write F if the statement is false. Reminder: ≠ means *is not equal to*, > means *is greater than*, and < means *is less than*.

F **1.** $5\times\frac{2}{3}=\frac{5\times2}{5\times3}$

T **2.** $3\times1\frac{1}{2}=3\frac{3}{2}$

$3X = 3 + 3/2$.
$3X\frac{1}{2} = 3/2$

F **3.** $(2\frac{1}{2}+2\frac{1}{2})\div3\neq(2\frac{1}{2}\div3)+(2\frac{1}{2}\div3)$

T **4.** $(7\frac{5}{16}\div3\frac{3}{5})>2$

T **5.** $\frac{7}{8}\div\frac{9}{10}>\frac{7}{8}$

F **6.** $(\frac{9}{10}\div\frac{3}{5})+(\frac{9}{10}\div\frac{3}{5})\neq\frac{18}{10}\div\frac{3}{5}$

T **7.** $37\frac{1}{15}\div19\frac{5}{16}\neq19\frac{5}{16}\div37\frac{1}{15}$

F **8.** $(12\frac{1}{3}\div2\frac{1}{2})\div3=(\frac{37}{3}\times\frac{5}{1})\div1$

T **9.** $7\frac{3}{8}\div\frac{2}{3}=\frac{59}{8}\times\frac{3}{2}$

T **10.** $(25\frac{1}{2}\div3\frac{1}{8})>8$

Part 3 Express each problem as an equation. Solve for n. Write each answer.

1. Marilyn weighs $91\frac{1}{4}$ pounds. Her friend weighs $79\frac{1}{2}$ pounds. How much heavier is Marilyn than her friend?
$91\frac{1}{4}-79\frac{1}{2}=n$ $11\frac{3}{4}$ lb.

2. A new road will cut $1\frac{9}{10}$ miles off the present $12\frac{3}{10}$ miles of driving distance between two cities. How long will the new driving distance be?
$12\frac{3}{10}-1\frac{9}{10}=n$ $10\frac{2}{5}$ mi.

3. Tom earned $1.26 in $2\frac{1}{3}$ hours. How much did he earn per hour?
$126\div2\frac{1}{3}=n$ 54¢

4. Jane cut $10\frac{1}{2}$ yards of ribbon into 3 ribbons of equal length. What was the length of each piece?
$10\frac{1}{2}\div3=n$ $3\frac{1}{2}$ yd.

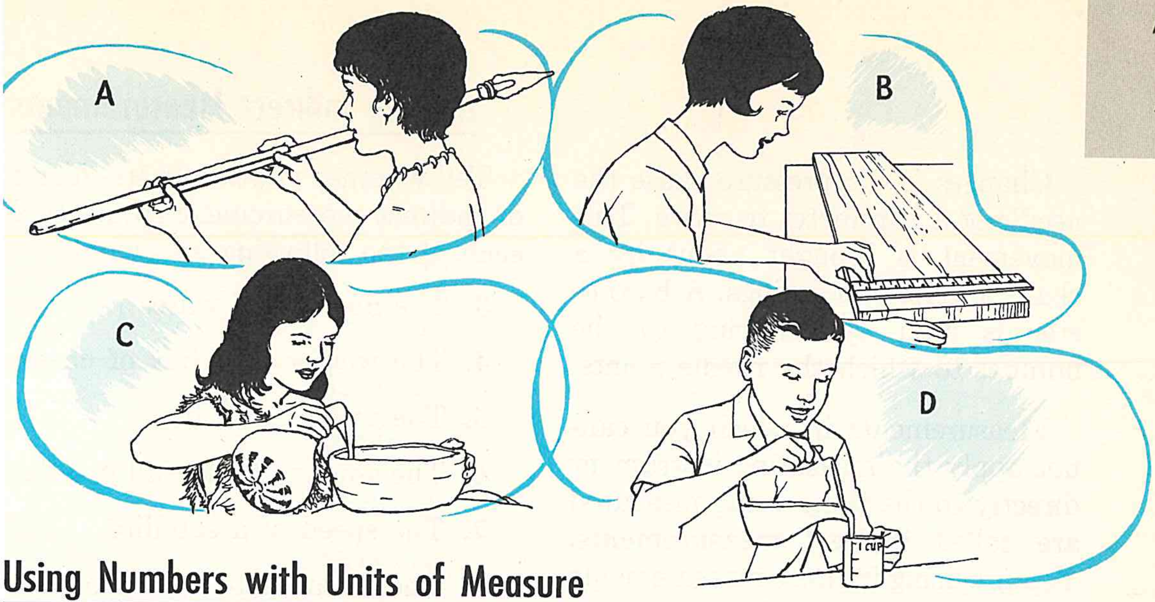
5. Betty practiced her piano lesson for $1\frac{1}{4}$ hr. on Friday, $\frac{3}{4}$ hr. on Saturday, and $\frac{2}{3}$ hr. on Sunday. How long did she practice in all?
$1\frac{1}{4}+\frac{3}{4}+\frac{2}{3}=n$ $2\frac{2}{3}$ hr.

6. The girls made 120 cookies. How many boxes of $1\frac{1}{4}$ dozen each can they fill? $(120\div12)\div1\frac{1}{4}=n$
8 boxes

7. The boys hiked $5\frac{1}{2}$ miles in $1\frac{3}{4}$ hours. How far did they walk per hour? $5\frac{1}{2}\div1\frac{3}{4}=n$ $3\frac{1}{7}$ mph

8. At $\frac{1}{2}$¢ a square inch, how much will 96 square inches of leather cost?
$\frac{1}{2}x96=n$ 48¢

9. Frances used $\frac{3}{4}$ of a pound of butter in making $2\frac{1}{2}$ pounds of candy. At this rate, how much butter was used in each pound of candy?
$\frac{3}{4}\div2\frac{1}{2}=n$ $\frac{3}{10}$ lb.



Using Numbers with Units of Measure

The boy in A is using a stick as his unit of measure. The length of the stick is contained in the length of the spear 4 times. The girl in B is using an inch as her unit of measure. The length of the inch is contained in the length of the board 11 times. Is the gourd the unit of measure in C? What is the unit of measure in D? Which number was obtained as the result of the measur-ng in A? In B?

Measurements such as those in A through D are made by comparing a unit of measure and an object to determine how many times the unit s contained in the object. Measurements of this kind are called **direct measurements.**

Whenever you measure something, you obtain a number. This number may be a whole number or a fractional number and is called a *measure.* A *measurement* consists of a number (the measure) together with a unit of measure, as shown in the following table.

Object Measured	Measurement	
	Number	Units of Measure
Spear	4	Measuring sticks
Board	11	Inches
Jar	10	Gourds
Carton	8	Cups

Oral

1. What other examples of direct measurement can you give?

2. Which number and which unit of measure would you use to express the length of your room after measuring it?

Making Indirect Measurements

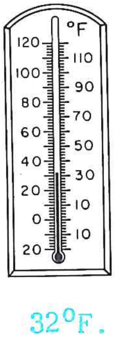
Changes in air pressure cause the needle of a barometer to move. This movement is brought about by a chain, levers, and springs. A barometer is read by referring to the number to which the needle points.

Measurements in which you cannot apply the measuring instrument directly to the thing being measured are called **indirect measurements.** You are using indirect measurements when you measure time, temperature, speed, and distances that are too great to be measured directly. Perhaps you can think of other indirect measurements.

You always obtain a number as the result of indirect measurement. This number is usually obtained by reading the scale of a measuring instrument. You express the result of the measurement by using a numeral and also the unit of measure.

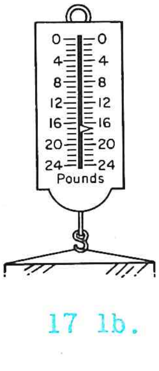
Oral Read the scale of each instrument below.

1.



32°F.

2.



17 lb.

Tell whether you would use direct or indirect measurement to measure each of the following and why.

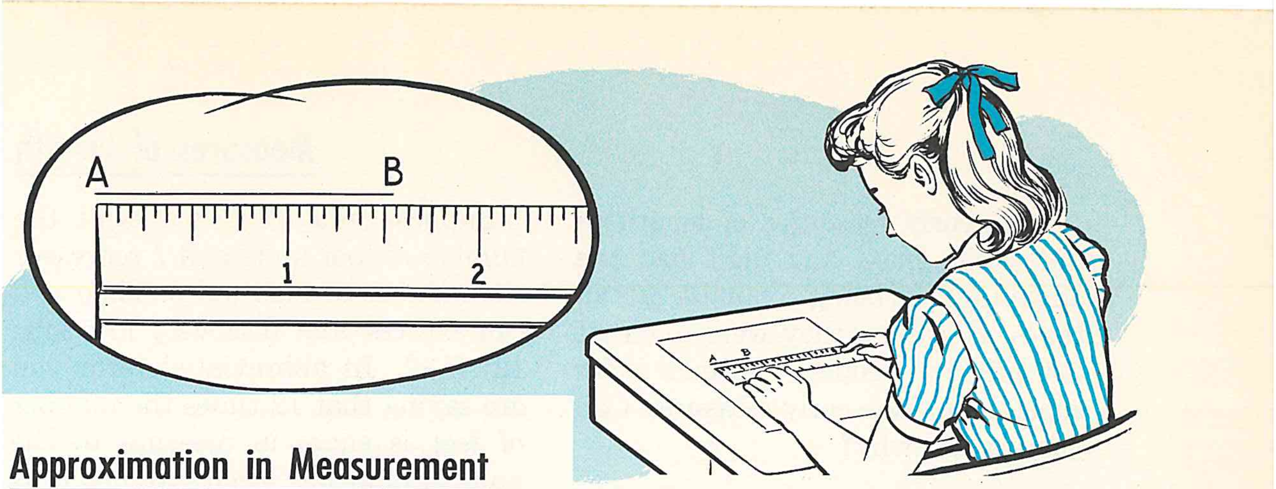
3. The amount of water in a can
direct
4. The weight of a box of cheese
indirect
5. The weight of a boy
indirect
6. The amount of rainfall in a day
direct
7. The speed of a satellite
indirect
8. The distance to the moon
indirect

Written Make two columns on your paper. Head one *Direct Measurement* and the other *Indirect Measurement.* In the appropriate column, write the name of each instrument below.

	a	*b*
1.	clock indirect	barometer indirect
2.	yardstick direct	pint direct
3.	tape measure direct	spring scale indirect
4.	speedometer indirect	battery tester indirect
5.	gallon direct	thermometer indirect

Something to do Try to find out what is measured with each of the following instruments.

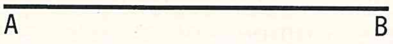
	a	*b*
1.	voltmeter electricity	altimeter altitude
2.	light meter light	ammeter electricity



Approximation in Measurement

Betty is using a ruler with a scale **graduated,** or marked, in sixteenths of an inch. Look at the line segment that she is measuring. Its length falls between $1\frac{8}{16}$ and $1\frac{9}{16}$ on the scale, but is closer to $1\frac{9}{16}$ than to $1\frac{8}{16}$. Thus the length of the segment may be expressed as $1\frac{9}{16}$ in. The possible error is $\frac{1}{2}$ the unit of measure used. Since the measure used is $\frac{1}{16}$ inch, the possible error is $\frac{1}{2}\times\frac{1}{16}$ or $\frac{1}{32}$ in. in either direction. The measurement, if carefully made, is said to be accurate to the nearest sixteenth of an inch.

Now try an experiment. Begin with a line segment AB.



Find the length of AB to the nearest inch, half inch, quarter inch, and eighth inch.

In no case will the endpoints of AB match the scale of your ruler exactly. Thus there is always an error of measurement. The **error of measurement** is the difference between the actual length of AB and the length you recorded. Do you see that the error of measurement is reduced by using measures graduated to smaller and smaller units?

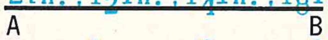
Oral

1. How exactly could you express temperature using a thermometer with a scale graduated in degrees?
Possible error of $\frac{1}{2}$ degree.

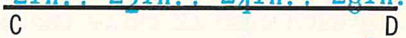
2. How exactly could you express weight using a weighing scale graduated in quarter pounds?
Possible error of 2 oz. ($\frac{1}{8}$ lb).

Written Express the length of each line segment below to the nearest inch, half inch, quarter inch, and eighth inch.

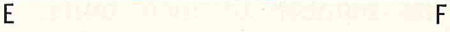
1. 2 in.; $1\frac{1}{2}$ in.; $1\frac{3}{4}$ in.; $1\frac{5}{8}$ in.



2. 2 in.; $2\frac{0}{2}$ in.; $2\frac{0}{4}$ in.; $2\frac{0}{8}$ in.



3. 2 in.; $2\frac{1}{2}$ in.; $2\frac{1}{4}$ in.; $2\frac{3}{8}$ in.



Such early measures of length as the cubit, span, and digit had the advantage of being available at all times. However, they were different for different people. The inch is far superior to these early measures. Can you explain why?

The length of an inch is defined by law in terms of a unit called the **meter.** The meter is a standard unit of linear measure in a system of measures called the **metric system.** All of our units of linear measure are related to the meter. Our yard is $\frac{3600}{3937}$ of a meter, our foot is $\frac{1}{3}$ of a yard, and our inch is $\frac{1}{12}$ of a foot.

Suppose you wanted to discover the numerical relationship between feet and inches. You might try this experiment. Cut a strip of cardboard 1 foot long and another strip 1 inch long. Use both strips to make the same measurements.

You will find that 12 inches are equal in measure to 1 foot. Thus 12 inches = 1 foot. Observe the use of = for *equal in measure.* How many inches are equal in measure to 2 feet? To 3 feet? To 4 feet?

In each case 12 times the number of foot units is equal in measure to the number of inch units. Why? Suppose you let F represent the number of foot units and I represent the number of inch units. Then you can express your discovery like this: $12 \times F = I$. In abbreviated form you are saying that 12 times the number of feet is equal in measure to the number of inches. Such statements as $12 \times F = I$ are called **formulas.** A formula states a general fact or rule in the language of mathematics.

From the formula $12 \times F = I$, you can develop a formula to help you express in inches a measurement made in feet. You know that $12 \times F = I$ can be rewritten as $F \times 12 = I$, because multiplication is commutative.

Study the steps taken in expressing $3 \times 4 = 12$ as $3 = \frac{12}{4}$ and $F \times 12 = I$ as $F = \frac{I}{12}$. The steps are identical.

$$\begin{array}{ll} 3 \times 4 = 12 & F \times 12 = I \\ 3 = 12 \div 4 & F = I \div 12 \\ 3 = \frac{12}{4} & F = \frac{I}{12} \end{array}$$

The formula $F = \frac{I}{12}$ means that the number of feet is equal in measure to the number of inches divided by 12.

See whether you can develop a formula which relates yards to feet.

Observe how you can use formulas to help you express relationships between measures. Suppose, for example, that you wanted to express 13 feet as inches. You know that 12 times the number of feet equals the number of inches, or $12 \times F = I$. Then proceed as below.

$$12 \times F = I$$
$$12 \times 13 = I$$
$$156 = I$$

Since $I = 156$, a length of 13 feet is equal in measure to 156 inches.

Observe how to use the formula $F = \frac{I}{12}$ in helping you express 79 inches as feet.

$$F = \frac{I}{12}$$
$$= \frac{79}{12}$$
$$= 6\tfrac{7}{12}$$

A length of 79 inches is equal in measure to $6\frac{7}{12}$ ft. or to 6 feet 7 inches.

Now study the relationships between linear measures expressed in the following table of equivalences. How many of the relationships do you already know?

12 inches (in.) = 1 foot (ft.)
36 in. = 1 yard (yd.)
3 ft. = 1 yd.
$16\frac{1}{2}$ ft. = 1 rod (rd.)
$5\frac{1}{2}$ yd. = 1 rd.
5280 ft. = 1 mile (mi.)
1760 yd. = 1 mi.
320 rd. = 1 mi.

Oral Use sentences to tell the meaning of each formula below. See page 116 and column 1.

	a	b
1.	$36 \times Y = I$	$Y = I \div 36$
2.	$16\frac{1}{2} \times R = F$	$R = F \div 16\frac{1}{2}$
3.	$5280 \times M = F$	$M = F \div 5280$

Tell the formula or formulas you would use in finding a numeral to use in making each statement true.

4. 14 ft. = 168 in. 12xF=I

5. 18 ft. = 6 yd. $\frac{F}{3}=Y$

6. 8 ft. 3 in. = 99 in. (12xF)+3=I

Written Copy. Write a numeral in each blank that makes the statement true.

1. 5 ft. = 60 in.

2. 59 ft. = 19 yd. 24 in.

3. 9 ft. 3 in. = 111 in.

4. 6 yd. 2 ft. = 20 ft.

5. 9 yd. 1 ft. 8 in. = 344 in.

6. 7 mi. = 2240 rd.

7. 850 yd. = 2550 ft.

8. 7 rd. = $38\frac{1}{2}$ yd. = $115\frac{1}{2}$ ft.

9. 6 mi. = 10560 yd. = 31680 ft.

10. 15,840 ft. = 3 mi.

Liquid and Dry Measure

The pint, quart, gallon, peck, and bushel are some examples of measures used in making direct measurements. You can use such measures to determine how many times the unit of measure is contained in the quantity being measured.

Suppose you used a pint and then a quart to measure the same liquid. You could keep a record of your data as follows.

Number of quart units	1	2	3
Number of pint units	2	4	6

From your table you might develop the formulas: $2 \times Q = P$, which is read *two times the number of quarts equals the number of pints,* and $Q = \frac{P}{2}$, which is read *the number of quarts equals the number of pints divided by 2.* How might you develop formulas to show the relationship between cups and pints? Between quarts and gallons?

The following relationships among the units of liquid measure are given below for review.

16 fluid ounces (fl. oz.) = 1 pint (pt.) ✓
2 cups (c.) = 1 pt. ✓
2 pt. = 1 quart (qt.)
4 qt. = 1 gallon (gal.) ✓

Below is a table showing the relationships between the common units of dry measure.

2 pints (pt.) = 1 quart (qt.)
8 qt. = 1 peck (pk.)
4 pk. = 1 bushel (bu.)

In the United States the dry quart is a slightly larger measure than the liquid quart.

Oral Read each formula below. For 1 *a* say: "Two times the number of pints is equal in measure to the number of cups." Observe the use of *C* for cup, *P* for pint, *Q* for quart, and *G* for gallon. See column 1.

	a	b
1.	$2 \times P = C$ (no. of pints; no. of cups)	$\frac{C}{2} = P$
2.	$4 \times G = Q$ (no. of gallons; no. of quarts)	$\frac{Q}{4} = G$
3.	$2 \times Q = P$ (no. of quarts; no. of pints)	$\frac{P}{2} = Q$ the no. of qts. is eq. to the no. pints

Written Use a formula to help you complete each statement below.

	a	b
1.	5 gal. = 20 qt.	14 pt. = 7 qt.
2.	3 gal. = 24 pt.	4 qt. = 8 pt.
3.	27 pk. = 216 qt.	8 bu. = 256 qt.
4.	24 qt. = 6 gal.	18 pt. = 9 qt.

Measures of Weight

A balance scale can be used in making direct measurements. A balance scale is often used by scientists in their experiments.

Most measurement of weight, however, is done by using indirect measurement. The instrument used is a weighing scale, usually called a scale. Weight is read directly from the number scale of the instrument.

The table of weights is given below.

16 ounces (oz.) = 1 pound (lb.)
100 lb. = 1 hundredweight (cwt.)
2000 lb. = 1 ton (T.)

The formulas below are helpful in finding equivalent measures of weight. Observe the use of *O* for ounces, *P* for pound, *H* for hundredweight, and *T* for ton. Read each formula.

$16 \times P = O$ $\qquad \frac{O}{16} = P$

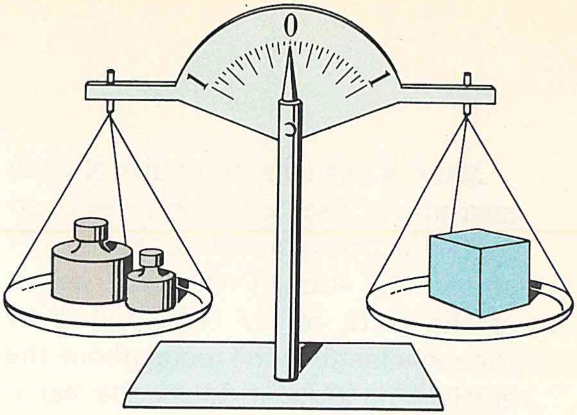
$100 \times H = P$ $\qquad \frac{P}{100} = H$

$2000 \times T = P$ $\qquad \frac{P}{2000} = T$

Oral

1. Where have you seen scales being used?

2. What information other than weight can be obtained from weighing scales in a supermarket?



Written Fill each blank with the numeral that makes the statement true.

1. $5\frac{1}{2}$ lb. = 88 oz. $16 \times P = O$

2. 128 oz. = 8 lb. $\frac{O}{16} = P$

3. 175 oz. = 10 lb. and 15 oz. $\frac{O}{16} = P$

4. 9 tons = 18000 lb. $2000 \times T = P$

5. 9535 lb. = 4 T. and 1535 lb. $\frac{P}{2000} = P$

6. 19 cwt. = 1900 lb. $19 \times H = P$

Solve these problems.

7. Last year the Johnsons used 9 tons of coal. How many pounds of coal did they use?

9x2000=n 18000 lb.

8. Alice's mother bought a piece of meat that weighed 4 pounds and another that weighed 2 pounds. How many ounces of meat did she buy?

(4+2)x16=n 96 oz.

9. Mary helped her uncle pack 10 pounds of chocolates into boxes holding 8 ounces each. How many boxes did they pack?

(10x16)÷8=n 20 boxes

Measures of Time

Most of the common units of time measure in use today were derived from the movements of the earth among the stars, from the rotation of the earth on its axis, and from the movement of the moon about the earth. The time it takes the earth to complete one orbit around the sun is called a **mean solar year,** or just a *year*. The time it takes the earth to complete one rotation on its axis is called a *day*.

Until fairly recent times, the above-named measures were adequate. However, modern science has replaced these crude measures of time by high-precision instruments. Why do you think this change was necessary?

The common units of time, the year and day, were used in developing the equivalence table below.

60 seconds (sec.) = 1 minute (min.)
60 min. = 1 hour (hr.)
24 hr. = 1 day (da.)
7 da. = 1 week (wk.)
12 mo. = 1 yr.
10 yr. = 1 decade
100 yr. = 1 century

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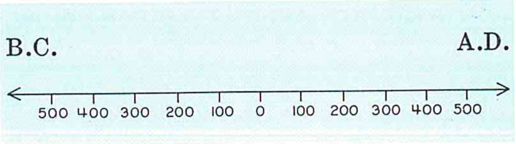
For general purposes of reference, but not for measurement, the following statements are often found in a table of equivalences for measures.

365 da. = 1 yr.
30 da. = 1 mo.
52 wk. = 1 yr.

You should find it easy to show that the above statements are only approximations. For example, what is $\frac{1}{12}$ of 365, 12×30, and 7×52?

Observe how centuries are indicated on the time line below.

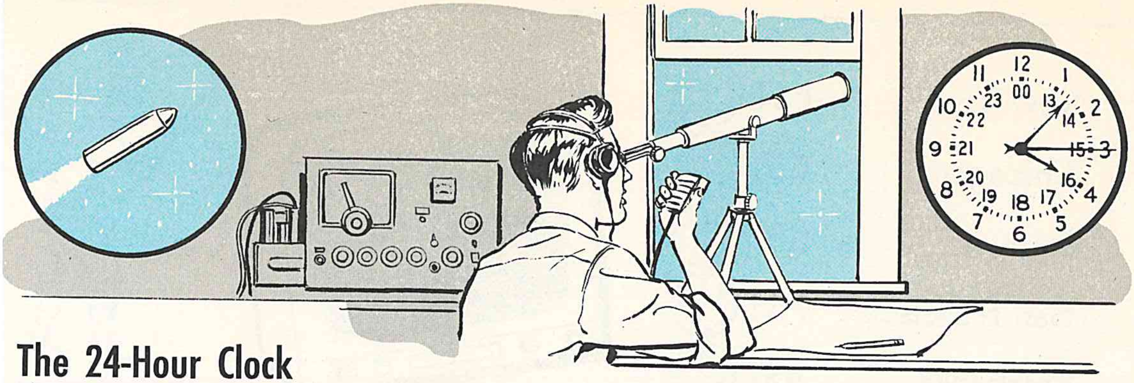


The point 0 indicates the birth of Christ. Time previous to His birth is called B.C., and the time after His birth is called A.D. How much time passed between 100 B.C. and A.D. 100 inclusive?

Oral What formulas might you use in determining equivalent measures of time?

Written Fill each blank with the numeral that makes the statement true.

1. 19 hr. = 68400 sec.
2. 8 min. 42 sec. = 522 sec.
3. $9\frac{1}{4}$ hr. = 555 min.
4. 60 mo. = 5 yr.



The 24-Hour Clock

If someone says he is coming to visit you at 8:00, you must find out whether he means 8:00 a.m. or 8:00 p.m. This problem would not arise if you were using a clock with a scale beginning at 0 and ending at 24. The people in military service, in radio communications, and in certain countries have adopted such a clock. Time is always expressed in 4 digits, like this:

0000	midnight
0001	1 min. after midnight
0010	10 min. after midnight
0100	1 hr. 00 min. after midnight
1000	10 hr. 00 min. after midnight
1500	15 hr. 00 min. after midnight

The two digits at the right indicate minutes and the two digits at the left indicate hours *after midnight.*

Oral Answer these questions about expressing time on a 24-hour clock.
See below.

1. How would you express 2 minutes after midnight? 15 minutes after midnight? 45 minutes after midnight? 2 hours past midnight?

1. 0002; 0015; 0045; 0200
2. 0145; 1530
3. 1415; 0430

2. How would you express 1 hour and 45 minutes after midnight? 15 hours and 30 minutes after midnight?

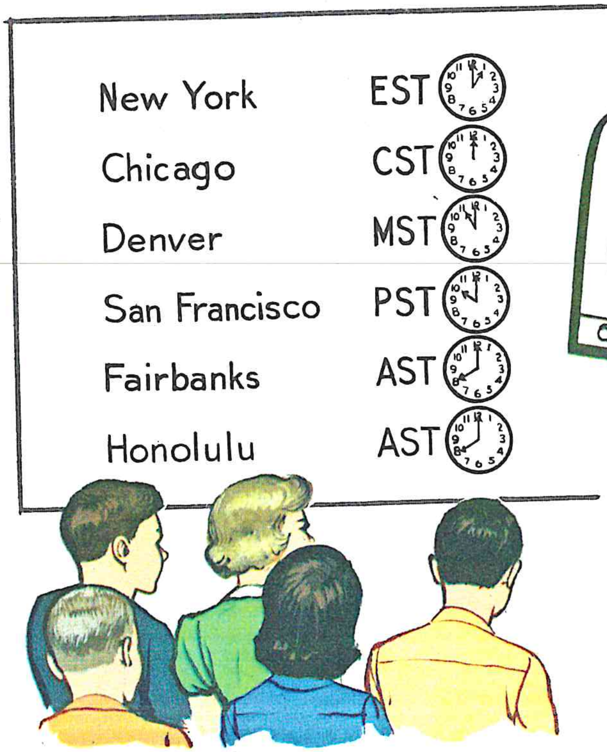
3. On a 24-hour clock, how would you express 2:15 p.m.? 4:30 a.m.?

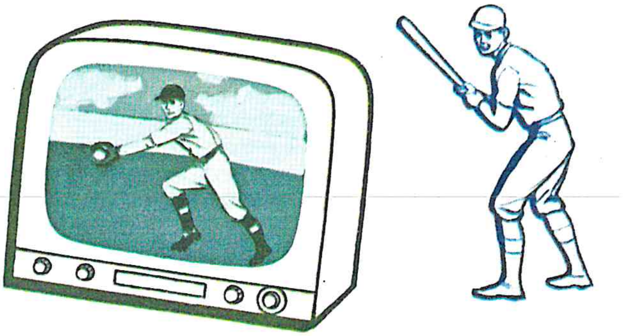
Written Express as time on a 24-hour clock.

	a		*b*	
1.	8:45 a.m.	0845	12:30 a.m.	0030
2.	12:00 noon	1200	3:35 a.m.	0335
3.	3:04 p.m.	1504	3:15 p.m.	1515
4.	8:45 p.m.	2045	10:15 p.m.	2215

Express as time on a 12-hour clock the following 24-hour clock times.

	a	*b*
5.	1100 11:00 a.m.	1155 11:55 a.m.
6.	0126 1:26 a.m.	0517 5:17 a.m.
7.	1045 10:45 a.m.	1300 1:00 p.m.
8.	1520 3:20 p.m.	1545 3:45 p.m.
9.	1720 5:20 p.m.	2037 8:37 p.m.





Time Zones

Suppose you read that the World Series game was to begin at 1:00 p.m. EST, and that you turned on your television set at 1:00 p.m. your time to watch it. Would you be seeing the very beginning of the game? Unless you live in the same time zone as New York, your answer is *no*. To avoid confusion that arises when each community sets its own time, time zones have been established by international agreement.

Within each zone the time, called standard time, is the same. The letters E for Eastern, C for Central, M for Mountain, P for Pacific, and A for Alaska are used in naming the standard time zones of the United States. What time zone is represented by the letters EST? CST? MST? PST? AST?

The earth rotates from west to east. For that reason the sun rises an hour earlier in the time zone east of you than in your time zone. Thus when it is 12:00 CST, it is 1:00 p.m. EST and 11:00 a.m. MST.

The time zones are roughly the same width throughout. However, their boundaries vary from north to south to avoid dividing states or even large cities into separate zones.

Although zone boundaries are set by an agency of the government, local areas are not required to use the time of their zone. That is why many communities feel free to use daylight saving time. Daylight saving time is normally 1 hour earlier than standard time. Why does confusion arise when people in one part of a time zone use daylight saving time and people in other parts of the same zone use standard time?

Oral

1. What is the time as you read this page? What time is it in the zone to the east of you?

2. Would time zones be of greater importance in a big country or in a small country? Why?

3. As you travel from New York to Hawaii, how many times should you reset a watch if you reset it each time you enter a time zone?

Written Replace each blank with the numeral or letters that make each statement true.

1. 2 p.m. MST corresponds to 3 p.m. CST.

2. 2 p.m. MST corresponds to 1 p.m. PST.

3. 12 p.m. CST corresponds to 11 a.m. MST.

4. 11 a.m. MST corresponds to 8 a.m. AST.

5. 1:00 p.m. PST corresponds to 4 p.m. EST.

6. 3 p.m. EST corresponds to 12 p.m. PST.

Answer these questions.

7. If you travel from New York (EST) to San Francisco (PST), how many times would you reset your watch? Would you set it ahead or move it back? `3 times; back`

8. It is 9 a.m. daylight saving time in Chicago. What is the standard time in Seattle? (PST) In Denver? (MST) In Chicago? (CST)
`6:00 a.m.; 7:00 a.m.; 8:00 a.m.`

9. An airplane flight leaves Chicago at 3:25 p.m. (CST) and arrives in Detroit at 5:25 p.m. (EST) Does this flight take 2 hours? Why or why not? How long does the flight take?
`No; time change; 1 hour`

A quick review Replace each blank with a numeral to make each statement true.

	a	*b*
1.	$\frac{1}{3}$ ft. = 4 in.	$\frac{3}{8}$ lb. = 6 oz.
2.	$\frac{1}{4}$ T. = 500 lb.	$\frac{5}{6}$ yd. = 30 in.
3.	$\frac{5}{6}$ ft. = 10 in.	$\frac{5}{16}$ lb. = 5 oz.
4.	$\frac{3}{5}$ T. = 1200 lb.	$\frac{7}{16}$ lb. = 7 oz.
5.	$\frac{3}{4}$ yd. = 27 in.	$\frac{7}{8}$ lb. = 14 oz.
6.	$\frac{1}{4}$ mi. = 1320 ft.	$\frac{3}{8}$ mi. = 1980 ft.
7.	5 pt. = $\frac{5}{8}$ gal.	8 in. = $\frac{2}{3}$ ft.
8.	2 oz. = $\frac{1}{8}$ lb.	20 in. = $1\frac{2}{3}$ ft.
9.	2 ft. = $\frac{2}{3}$ yd.	5 ft. = $1\frac{2}{3}$ yd.
10.	9 oz. = $\frac{9}{16}$ lb.	33 qt. = $8\frac{1}{4}$ gal.
11.	17 in. = $\frac{17}{36}$ yd.	55 sec. = $\frac{11}{12}$ min.

John needs 1 board 6 feet 6 inches long for the sides of a box and 1 board 3 feet 2 inches long for the ends. What is the total length of the boards?

You may remember that the signs \$ and. are never part of an equation. Other units of measure are not part of an equation either.

Before you can express the problem as an equation, you may think of the measurements as fractional numbers. 6 ft. 6 in. is $6\frac{6}{12}$ ft., and 3 ft. 2 in. is $3\frac{2}{12}$ ft. Then the problem may be expressed in equation form as in A.

A
$$6\tfrac{6}{12}+3\tfrac{2}{12}=n$$
$$9\tfrac{8}{12}=n$$

The total length of the boards is $9\frac{8}{12}$ feet or 9 ft. 8 in.

The computation can also be done in column form as shown in B.

B
$$\begin{array}{r} 6\text{ ft. }6\text{ in.}\\ +3\text{ ft. }2\text{ in.}\\ \hline 9\text{ ft. }8\text{ in.}\end{array}$$

The numbers indicating feet and inches are added separately.

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Suppose that you worked at one task for 1 hr. and 45 min. and another for 1 hr. and 30 min. Also suppose that you wanted to find the number of hours and minutes you worked. You could work in column form as in C.

C
$$\begin{array}{r} 1\text{ hr. }45\text{ min.}\\ +1\text{ hr. }30\text{ min.}\\ \hline 2\text{ hr. }75\text{ min.}\end{array}$$

How many hours and minutes are equivalent to 75 minutes? Then 2 hr. 75 min. = 3 hr. 15 min. +

associative property

Solve the equation $1\frac{45}{60}+1\frac{30}{60}=n$. Does $2\frac{75}{60}$ hr. $=3\frac{15}{60}$ hr. = 3 hr. 15 min.?

Oral Tell how you would convert each answer below to larger units. See above.

	a	*b*
1.	3 ft. 13 in. 4' 1"	5 yd. 4 ft. 6 yds 1 ft
2.	9 gal. 6 qt. 5 g. 2 q.	12 bu. 9 pk. 13 bu 1 pk.

Written Find each sum. Convert each answer to larger units. Larger units only.

1. *a* 10 4 (2 gls.)
$$\begin{array}{r} 9\text{ qt. }3\text{ pt.}\\ +1\text{ qt. }1\text{ pt.}\\ \hline 12\text{qt.}\end{array}$$

b 9 5 (1 gal 1 qt)
$$\begin{array}{r} 5\text{ gal. }2\text{ qt.}\\ +3\text{ gal. }3\text{ qt.}\\ \hline 9\text{gal. }1\text{qt.}\end{array}$$

2. *a*
$$\begin{array}{r} 8\text{ ft. }11\text{ in.}\\ +9\text{ ft. }\ 9\text{ in.}\\ \hline 18\text{ft. }8\text{in.}\end{array}$$
17 20 (18 ft 5")

b
$$\begin{array}{r} 5\text{ wk. }4\text{ da.}\\ +8\text{ wk. }6\text{ da.}\\ \hline 14\text{wk. }3\text{da.}\end{array}$$
13 10 1 wk 3 da

8 ft. 6 in − 5 ft. 8 in = ____ ft. and ____ in.

A

$8\frac{6}{12}$ ft. = $7\frac{18}{12}$ ft.

$-5\frac{8}{12}$ ft. = $5\frac{8}{12}$ ft.

$2\frac{10}{12}$ ft. = 2 ft. 10 in.

B

$\overset{7}{\cancel{8}}$ ft. $\overset{18}{\cancel{6}}$ in.

5 ft. 8 in.

2 ft. 10 in.

C

$8\frac{6}{12} - 5\frac{8}{12} = n$

$7\frac{18}{12} - 5\frac{8}{12} = n$

$2\frac{10}{12} = n$

Subtracting Measures

John had a board 8 feet 6 inches long and cut off a piece 5 feet 8 inches long. To the nearest inch, what was the length of the board that was left? what operation? what no. to be subtracted

Study solution A on the board above. Why was $8\frac{6}{12}$ renamed as $7\frac{18}{12}$ before the numbers were subtracted? In B you will see that 8 ft. 6 in. is expressed as 7 ft. 18 in. Then the numbers indicating inches and the numbers indicating feet are subtracted separately. In C you are shown the work in equation form. Is the answer the same in A, B, and C?

Suppose you were to subtract 1 gal. 2 qt. from 3 gal. You might proceed as shown.

$$3\frac{0}{4}\text{ gal.} = 2\frac{4}{4}\text{ gal.}$$
$$-1\frac{2}{4}\text{ gal.} = 1\frac{2}{4}\text{ gal.}$$
$$1\frac{2}{4}\text{ gal.} = 1\text{ gal. } 2\text{ qt.}$$

Why were the fractions expressed in fourths? How could you do the above subtraction using method B on the board?

3 gal. 0 qts
− 1 gal 2 qts.

2 g. 4 qts
−1 g. 2 qts
1 gal 2 qts

Oral Tell how to solve for n. How would you express each answer? numeral + a measure

See A, B, and C above and column 1.

1. $9\frac{2}{3}$ yd. − $3\frac{1}{3}$ yd. = n
   $9\frac{2}{3} - 3\frac{1}{3} = n$ $6\frac{1}{3}$ yd.
2. $18\frac{3}{4}$ pk. − $9\frac{2}{4}$ pk. = n
   $18\frac{3}{4} - 9\frac{2}{4} = n$ $9\frac{1}{4}$ pk.

Written Copy. Perform the indicated operation.

	a	*b*
1.	18 bu. 3 pk. −9 bu. 2 pk. 9bu. 1pk.	15 hr. 40 min. −3 hr. 19 min. 12hr. 21min.
2.	9 gal. 3 qt. −5 gal. 1 qt. 4gal. 2qt.	15 qt. 0 pt. −4 qt. 1 pt. 10qt. 1pt.
3.	8 wk. 5 da. −4 wk. 6 da. 3wk. 6da.	17 min. 30 sec. −15 min. 50 sec. 1min. 40sec.

4. $12\frac{1}{2}$ ft. − $6\frac{1}{3}$ ft. = n $6\frac{1}{6}$ ft.
5. $22\frac{1}{2}$ pt. − $14\frac{0}{2}$ pt. = n $8\frac{1}{2}$ pt.
6. 14 pt. − $6\frac{1}{2}$ pt. = n $7\frac{1}{2}$ pt.
7. $8\frac{3}{4}$ gal. − $2\frac{1}{2}$ gal. = n $6\frac{1}{4}$ gal.
8. 5 yd. − $2\frac{1}{3}$ yd. = n $2\frac{2}{3}$ yd.

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Suppose you wanted to know the total length of 5 pieces of material each $3\frac{1}{8}$ yd. in length. Would you expect to multiply $3\frac{1}{8}$ by 5? How would you express the answer?

Look at the computation below. What is the product of 6 and 3? Of 6 and 9?

9 gal. 3 qt.
×6
54 gal. 18 qt. = 58 gal. 2 qt.

How many quarts are equal in measure to 1 gallon? Does 18 qt. = 4 gal. 2 qt.? Do you see that 58 gal. 2 qt. is another name for 54 gal. 18 qt.?

You have solved many problems with measurements. They are really the same as other problems. However, before you try to express an equation for a problem about measurements, you may decide that the problem can be solved most easily in column form. In such cases the equation is unnecessary.

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Oral For the problems below decide whether to use an equation or to work in column form. Tell why you decided as you did. Equations are given.

1. Judy has 4 packages that weigh 1 lb. 3 oz. each. What is the total weight? 4x$1\frac{3}{16}$=n 4 lb. 12 oz.

2. Mary had 9 pieces of material each $3\frac{2}{3}$ yards long. How much material did she have altogether?
$3\frac{2}{3}$x9=n 33 yd.

3. John spends an average of 5 hr. 30 min. in school each day. How many hours does he spend in school each week?
5x$5\frac{1}{2}$=n 27 hr. 30 min.

4. Mr. Nelson can make a trip in 8 hr. 15 min. by car or in 1 hr. 25 min. by air. How much time will he save on the trip if he decides to fly?
$8\frac{15}{60}-1\frac{25}{60}$=n 6 hr. 50 min.

5. One container of orange juice holds 2 qt. A smaller container holds 1 qt. 1 pt. How much more orange juice is in the larger container than in the smaller one?
2-$1\frac{1}{2}$=n 1 pt.

Written Solve problems 1 through 5 above. Then find the answer for each multiplication below.

	a	*b*
6.	9 ft. 9 in. ×7 68ft.3in.	11 gal. 2 qt. ×25 287gal.2qt.
7.	7 ft. 3 in. ×4 29ft.	5 hr. 10 min. ×7 36hr.10min.
8.	10 lb. 5 oz. ×5 51 lb.9oz.	9 yd. 2 ft. ×6 58yd.
9.	8 qt. 1 pt. ×5 42qt.1pt.	10 min. 15 sec. ×5 51min.15sec.

17 ft. 6 in. ÷ 6 = ___ft. and ___in.

A

$$17\tfrac{1}{2} \div 6 = n$$
$$\tfrac{35}{2} \div 6 = n$$
$$\tfrac{35}{2} \times \tfrac{1}{6} = n$$
$$\tfrac{35}{12} = n$$
$$2\tfrac{11}{12} = n$$

B

$$\begin{array}{r} \text{2 ft. 11 in.} \\ 6\overline{)\text{17 ft. 6 in.}} \\ \text{12 ft.} \\ \hline \text{5 ft. 6 in.} \\ \text{66 in.} \\ \text{66 in} \\ \hline \end{array}$$

$2\frac{11}{12}$ ft. = 2 ft. 11 in.

Dividing Measures

Study solution A on the board. After writing an equation, you solve for n in the usual way. In B, how was the 2 of the quotient obtained? Why was 5 ft. 6 in. renamed as 66 in.? The 11 of the quotient was obtained by dividing 66 by 6.

In solving problems in which you divide measures, you will have to decide whether to work in equation or in column form. Make your decision after deciding which form will require the least written work.

Oral For the problems below, tell whether to work in equation or in column form and why.

1. Doris has a strip of crepe paper 1 foot 2 inches wide. She wants to cut it into 7 equal strips. How wide should she make each strip?
$1\frac{1}{6} \div 7 = n$ 2 in.

2. One week Nancy spent 4 hours 5 minutes practicing her violin. What was the average amount of time spent per day?
$4\frac{5}{60} \div 7 = n$ 35 min.

3. One box of soap flakes has contents of 2 pounds 15 ounces and another has contents of 3 pounds 4 ounces. What is the difference between the weights?
$3\frac{4}{16} - 2\frac{15}{16} = n$ 5 oz.

4. School hours are as follows: morning, 9 to 11:50; afternoon, 1:15 to 3:50. How many hours do sixth-grade pupils spend in school each week? $5 \times [(11\frac{50}{60} - 9) + (3\frac{50}{60} - 1\frac{15}{60})] = n$
27 hr. 5 min.

5. Roy made 11 feet 9 inches on his first try in a running broad jump and 12 feet 4 inches on his second try. How much longer was his second jump than his first?
$12\frac{4}{12} - 11\frac{9}{12} = n$ 7 in.

6. John and his father harvested 10 bushels 2 pecks of potatoes. They sold them at \$.85 a peck. How much did they get for the potatoes?
$[(10 \times 4) + 2] \times 85 = n$ \$35.70

7. One board is 8 ft. 6 in. long and another is 4 ft. 6 in. long. Find their total cost at \$.18 a foot.
$(8\frac{1}{2} + 4\frac{1}{2}) \times 18 = n$ \$2.34

Written Solve problems 1 through 7.

MORE PRACTICE PAGE 306

Review and Practice

You should be able to do the computations on this page without help.

Part 1 Solve for n.

	a		b	
1.	$8638 \div 43 = n$	200r38	$n = 2683 - 958$	1725
2.	$n = 79372 \div 37$	2145r7	$92 \times 62 = n$	5704
3.	$6954 - 3076 = n$	3878	$n = 3725 + 4691$	8416
4.	$51230 \div 84 = n$	609r74	$6001 - 4346 = n$	1655
5.	$n = 93 \times 276$	25668	$5876 \div 235 = n$	25r1

Part 2 Perform the operation indicated by the operational sign.

	a	b	c
1.	106576 852108 947683 +417457 2323824	579629 161761 913094 +228212 1882696	725938 390459 283867 +634345 2034609
2.	35187 92001 16679 64592 +62828 271287	20714 43225 53862 78431 +71943 268175	373661 840581 451472 509393 +892514 3067621
3.	84649 −18775 65874	325430 ×65 21152950	57234 −29498 27736
4.	476532 +7651 484183	884r275 535)473215	7056300 −43876 7012424
5.	37256 ×250 9314000	3407 125)425875	48750 ×485 23643750

Part 3 Solve for n.

	a		b	
1.	$9\frac{8}{9}+4\frac{5}{9}=n$	$14\frac{4}{9}$	$\frac{2}{5}+\frac{3}{10}=n$	$\frac{7}{10}$
2.	$n=\frac{2}{3}+\frac{5}{12}$	$1\frac{1}{12}$	$n=8\frac{3}{4}+6\frac{5}{8}$	$15\frac{3}{8}$
3.	$n=9\frac{3}{4}+3\frac{1}{3}$	$13\frac{1}{12}$	$9\frac{1}{5}+3\frac{3}{4}=n$	$12\frac{19}{20}$
4.	$\frac{7}{8}-\frac{3}{8}=n$	$\frac{1}{2}$	$9\frac{3}{7}-4\frac{2}{7}=n$	$5\frac{1}{7}$
5.	$5\frac{1}{8}-3\frac{7}{8}=n$	$1\frac{1}{4}$	$n=\frac{2}{3}-\frac{4}{9}$	$\frac{2}{9}$
6.	$11\frac{3}{5}-2\frac{1}{10}=n$	$9\frac{1}{2}$	$n=1\frac{1}{2}-\frac{5}{6}$	$\frac{2}{3}$
7.	$n=7\frac{1}{8}-4\frac{3}{4}$	$2\frac{3}{8}$	$n=\frac{3}{8}-\frac{1}{3}$	$\frac{1}{24}$
8.	$4\frac{3}{5}-1\frac{1}{2}=n$	$3\frac{1}{10}$	$9\frac{3}{4}-4\frac{1}{2}=n$	$5\frac{1}{4}$
9.	$n=32\frac{2}{3}-19\frac{3}{5}$	$13\frac{1}{15}$	$n=81\frac{4}{5}-39\frac{3}{4}$	$42\frac{1}{20}$

Part 4 Solve for n.

	a		b	
1.	$\frac{1}{8}\times 24=n$	3	$\frac{1}{8}\times\frac{2}{3}=n$	$\frac{1}{12}$
2.	$n=\frac{1}{4}\times 6\frac{2}{3}$	$1\frac{2}{3}$	$n=3\frac{2}{5}\times\frac{3}{7}$	$1\frac{16}{35}$
3.	$4\frac{1}{3}\times 5=n$	$21\frac{2}{3}$	$1\frac{2}{3}\times 3\frac{1}{4}=n$	$5\frac{5}{12}$
4.	$\frac{5}{6}\times\frac{3}{10}=n$	$\frac{1}{4}$	$25\frac{1}{2}\times 48=n$	1224
5.	$\frac{8}{15}\div\frac{7}{15}=n$	$1\frac{1}{7}$	$2\frac{1}{4}\div\frac{3}{4}=n$	3
6.	$12\frac{3}{8}\div 1\frac{3}{8}=n$	9	$9\div\frac{1}{4}=n$	36
7.	$n=6\div 1\frac{4}{7}$	$3\frac{9}{11}$	$n=\frac{3}{8}\div\frac{1}{4}$	$1\frac{1}{2}$
8.	$n=6\frac{2}{3}\div 2\frac{1}{2}$	$2\frac{2}{3}$	$n=\frac{7}{15}\div\frac{1}{6}$	$2\frac{4}{5}$
9.	$2\frac{1}{8}\div 1\frac{1}{4}=n$	$1\frac{7}{10}$	$n=\frac{3}{8}\div 1\frac{1}{2}$	$\frac{1}{4}$

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Measuring consists of finding how many times a certain unit of measure is contained in some given quantity. (113)

2. Measurements may be direct or indirect. (113-114)

3. The result of a measurement is expressed by using a numeral together with the unit of measure. (114)

4. Measurements are always approximate. (115)

5. The sign = used to show the relationship between measures means *equal in measure.* (116)

6. Formulas can be used in determining equivalent measures. (116)

Words to Know

1. Direct measurement and indirect measurement (113-114)

2. Graduated scale (115)

3. Error of measurement (115)

4. Formulas (116)

Questions to Discuss

See T129 for answers.

1. How may the result of a measurement be expressed? (113)

a numeral and a word

2. Why must every measurement be an approximation? (115)

3. How do you read and use the formulas $12 \times F = I$ and $F = \frac{I}{12}$? (116)

4. What units of measure are used in recording measurements of liquids? Of weight? Of time? (118-120)

5. How would 11:30 p.m. be shown on a 24-hour clock? (121)

Written Practice

Write a numeral in each blank that makes each statement true.

1. 35 yd. = 105 ft. (117)

35X3

2. 3850 yd. = $2\frac{3}{16}$ mi. (117)

$1760 \overline{)3850}$ 2; 3520; 330

3. 9 gal. = 36 qt. (118)

9X4=

4. 16 pt. = 8 qt. (118)

$2\overline{)16}$

5. 180 oz. = $11\frac{1}{4}$ lb. (119)

$16\overline{)180}$ 11, 4 oz.

6. $10\frac{1}{2}$ hr. = 630 min. (120)

$10\frac{1}{2} \times 60 = \frac{21}{2} \times 60$

7. 1:00 p.m. EST = 10 a.m. PST (122)

-3 hrs.

8. 6 yr. 3 mo. + 5 yr. 10 mo. = 12 yr. 1 mo. (124)

11 yrs 13 mos

9. 5 ft. 1 in. − 2 ft. 9 in. = 2 ft. 4 in. (125)

4-13; −2-9

10. 11 T. 1000 lb. ÷ 4 = 2 T. 1750 lb. (127)

$4\overline{)23000}$ 5750; 20; 3000; 2800; 200

11. 14 mi. 1700 ft. ÷ 5 = 2 mi. 4564 ft. (127)

15,124 ft.

$5\overline{)14}$ 2 miles; 10; 4

4X 5280 = 21120; 1700; $5\overline{)22820}$ 4564 ft.

Self-Evaluation

Part 1 Express each sentence below as a formula and each formula as a sentence.

1. 12 times the number of feet is equal in measure to the number of inches. 12xF=I

2. The number of inches divided by 36 is equal in measure to the number of yards. $\frac{I}{36}=y$

3. $5280 \times M = F$
See below.

4. $T = \frac{P}{2000}$
See below.

Part 2 Find a replacement for the letter which makes each sentence true.

	a	*b*
	1140	522
1.	19×60=S	(8×60)+42=S
	510	1052
2.	M=$8\frac{1}{2}$×60	M=(17×60)+32
	111	2r30
3.	$9\frac{1}{4}$×12=M	H=150÷60
	20	32r5
4.	C=2000÷100	D=325÷10
	288	244
5.	P=18×16	G=976÷4

Part 3 Copy. Write a numeral in each blank so that the statements below become true.

1. 39 in.=__3__ft.__3__in.

2. 99 ft.=__33__yd.__0__ft.

3. 7 gal. 1 qt.=__29__qt.

4. 28 hr.=__1__da.__4__hr.

Part 4 Perform the operation indicated by the operational sign. Express each answer in the largest possible units.

1. 5 lb. 6 oz.+9 lb. 3 oz.
14 lb. 9 oz.

2. $9\frac{1}{2}$ ft.÷2
$4\frac{3}{4}$ ft.

3. 16 yd. 27 in.+32 yd. 8 in.
48 yd. 35 in.

4. 35 ft. 9 in.−19 ft. 7 in.
16 ft. 2 in.

5. 12 hr. 15 min.−8 hr. 35 min.
3 hr. 40 min.

6. 4×9 ft. 2 in.
36 ft. 8 in.

7. 16 lb. 8 oz.÷3
5 lb. 8 oz.

8. $9\frac{1}{4}$ T.+$8\frac{9}{10}$ T.
$18\frac{3}{20}$ T.

Part 5 Solve these problems.

1. Betty bought 2 lb. 4 oz. of soap flakes for $.90. What was the cost per ounce?
[(2x16)+4]÷90=n $2\frac{1}{2}$¢

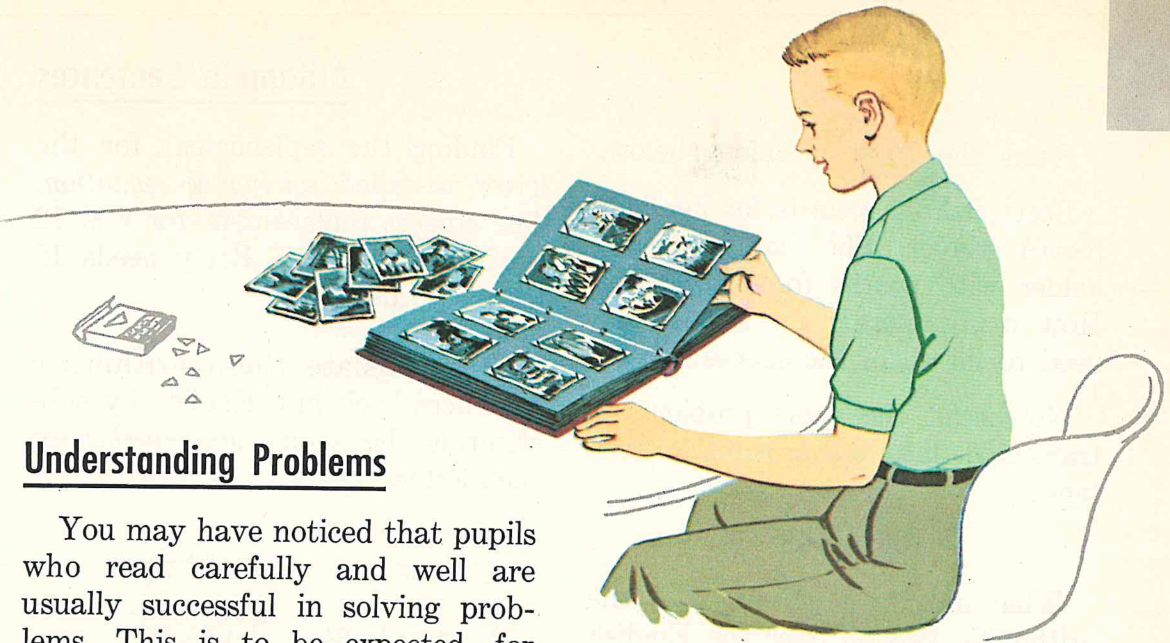
2. A television show ran for 90 minutes. There were 3 commercials each half hour. How many commercials were given during the program?
(90÷30)x3=n 9 commercials

3. John spends about $1\frac{1}{2}$ hours delivering papers each day of the week. He earns about 75¢ an hour. How much money does he earn each week?
(7x$1\frac{1}{2}$)x75=n $7.87

4. Mary bought 10 yards of material. How many pieces, each 1 yard 18 inches long, can she make from it? 10÷$1\frac{1}{2}$=n $6\frac{2}{3}$ pieces

3. 5280 times the number of miles is equal in measure to the number of feet.

4. The number of tons is equal in measure to the number of pounds divided by 2000.



Understanding Problems

You may have noticed that pupils who read carefully and well are usually successful in solving problems. This is to be expected, for one must understand the English language version of a problem before he can express it in the language of arithmetic.

Whenever you are to solve a problem, read it at least twice. The first reading should help you get a general idea of the problem situation. During this reading decide *how many things* and *what kind of things* are involved. You need to know the *number* of things because these numbers are needed in expressing the problem in the language of arithmetic. You need to know the *kind* of things to help you understand the life situation from which the problem arose.

The second reading should help you decide what you are to find. After you fully understand the problem situation and what you are to find, you are ready to translate the problem into an arithmetic sentence.

Oral Read each problem. Tell how many things and the kind of things that are told about. Tell what you are to find. See below for key.

1. Robert has a 96-page album with spaces for 4 pictures on each page. He has filled 60 pages. (How many more pictures will the album hold?)

2. Jack's older brother has a checkup for his scooter after each 1000 miles. He has driven 775 miles since his last checkup. If he averages 15 miles a day, (in how many days will he need another checkup?)

_____ indicates how many

===== indicates kinds of things

(   ) indicates what you are to find

Arithmetic Sentences

Read the story problem below.

Betty has 36 records for her new record player. She has a record holder with spaces for 48 records. How many more records does she need to fill all of the spaces?

Now read the same problem as translated into an arithmetic sentence.

$$36+r=48$$

What are the advantages of the arithmetic version over the English version?

The only symbols used in this arithmetic sentence are numerals, the letter r to represent a numeral, the operational sign +, and the relations sign =. Any letter besides r could have been selected to represent the numeral. Why was r a good letter to choose?

To translate this arithmetic sentence back into English you could say, *36 plus some number equals 48.*

Arithmetic sentences in which you use the sign = are called *equations.* To solve an equation you find the replacement for the letter that holds the place for a numeral. The replacement must make the sentence true. Which replacement would you select for r in 36+r=48?

Finding the replacement for the letter is called *solving an equation.* The correct replacement for r is 12 because 36+12=48. Betty needs 12 more records.

Oral Translate these arithmetic sentences back into English by substituting the words *some number* for each letter. See column 1.

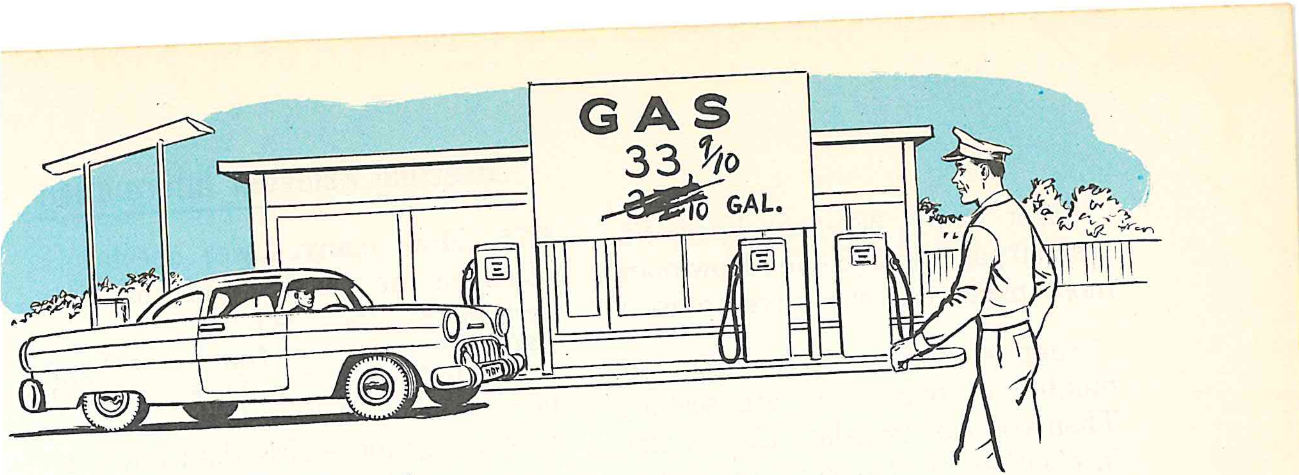
	a	*b*
1.	$17+6=\overset{23}{n}$	$5\times\overset{15}{c}=75$
2.	$\overset{13\frac{1}{2}}{a}-5\frac{1}{4}=8\frac{1}{4}$	$\overset{294}{b}\div7=42$
3.	$(8\times7)+6=\overset{62}{w}$	$(14-\overset{10}{x})+7=11$
4.	$9\times(5\times\overset{0}{n})=0$	$(6\times\overset{1}{w})+1=7$

Use the English language to make up problems which could be solved by using the equations below.

	a	*b*
5.	$\overset{15}{b}+35=50$	$\overset{36}{n}-12=24$
6.	$50-\overset{30}{w}=20$	$\overset{6}{p}\times5=30$

Written Solve each equation in rows 1 through 6 above. Then translate the following sentences into equations.

1. Some number minus 6 equals 2.
   $n-6=2$
2. Some number plus 9 equals 17.
   $n+9=17$
3. Some number times 0 equals 0.
   $nx0=0$



Determining the Operation

After you fully understand a problem, you need to decide how to operate on the numbers to obtain a solution. The statements below may help you reach a decision.

a. To find the sum or total of two different numbers, you *add*.

b. To find the total of two or more equal numbers, you may *add* or *multiply*.

c. To find the difference between two numbers, or by how much one number is more or less than the other, you *subtract*.

d. To find the number in each set or group of equal size, you *divide*.

e. To find the number of sets or groups of equal size, you *divide*.

Oral Tell which operation you would use in solving these problems and how you reached your decision.

See above.

1. After a price cut of $3\frac{2}{10}$ cents a gallon, gasoline was selling for $33\frac{9}{10}$ cents a gallon. What was its price before the cut was made?

$3\frac{2}{10}+33\frac{9}{10}=n$ $37\frac{1}{10}$ cents

2. A driver estimated that he averaged 16 miles per gallon of gasoline. How many gallons of gasoline would he use in driving 10,000 miles?

$10000\div16=n$ 625 gal.

3. Mr. Osburn's yearly gasoline bill was \$235. By careful driving he cut his bill by $\frac{1}{5}$. How much money did he save by driving carefully?

$235\div5=n$ or $\frac{1}{5}\times235=n$ \$47

4. An automobile club estimated the cost of driving a compact car 10,000 miles. The expenses included insurance at \$256.80; state license, \$10.50; local license, \$15; depreciation, \$637, and other expenses \$345. What was the total estimate?

$25680+1050+1500+63700+34500=n$ \$1264.30

5. Mr. Larsen averages $13\frac{1}{2}$ miles per gallon of gasoline. How far can he drive on a tank of gasoline if the tank holds 18 gallons?

$18\times13\frac{1}{2}=n$ 243 mi.

Written Express problems 1 through 5 as equations. Solve each equation. Write your answer.

Selecting Relevant Information

Janet had a 336-page book. She read 90 pages in $2\frac{1}{2}$ hours. How many more pages did she have to read?

Suppose you let p represent the number of pages you are seeking. Then you can translate the problem into either of the equations below. In what way do they differ?

$$336 - 90 = p \quad \text{or} \quad p = 336 - 90$$

Do you see that the $2\frac{1}{2}$ hours is not relevant, that is, it has no relation to the solution of the problem? See if you can make up a problem using only the numbers $2\frac{1}{2}$ and 90. Then make up a problem using 336, 90, and $2\frac{1}{2}$.

Before you can solve a problem, you must find a relationship between the data that is given and the question that is asked. If there is no relationship, the data is not relevant and you may disregard it.

In each problem below decide whether or not there is information that is not needed. If so, write the numerals you will *not* include in your equation for the problem.

1. The attendance at football games dropped from 1475 for the first game of the season to 989 for the next game when the price of tickets was increased from \$.50 to \$.75. How many fewer spectators attended the second game than the first game? (50; 75)

1475−989=n   486 fewer spectators

2. There are 541 girls and 527 boys enrolled in Forest Hills School. How many more girls than boys are enrolled in the school? (None)

541−527=n   14 more girls

3. The speed of sound is about 1100 feet a second or about 750 miles per hour. About how many seconds does it take sound to travel 1 mile (5280 ft.)? (750)

5280÷1100=n   4.8 sec.

4. The 32 sixth graders shared equally in the cost of a party. The total expenses were \$9.60. How much did each sixth grader pay as his share? (None)

960÷32=n   \$.30

5. Bob had a \$10 bill. He spent \$4.98 for a catcher's mask and \$1.75 for a bat. How much money did he spend altogether? (10)

498+175=n   \$6.73

Oral Follow these directions.

1. Tell which problems contained information that was not related to the question and which numerals you wrote. See above.

2. For each problem containing unnecessary information, supply a question which could be answered by using that information.

Answers will vary.

Written Write equations for problems 1 through 5. Solve them.

Supplying Missing Information

In many of your everyday problems, more than one answer may be correct. Occasionally, no solution is possible, because you have insufficient information. As you read each problem below, decide whether it has only one answer or no answer at all. Indicate your decision on your paper.

1. Bob traded 55 foreign stamps for 150 domestic stamps and had 225 foreign stamps left. How many foreign stamps did he have before making the trade?
55+225=n   280 foreign stamps

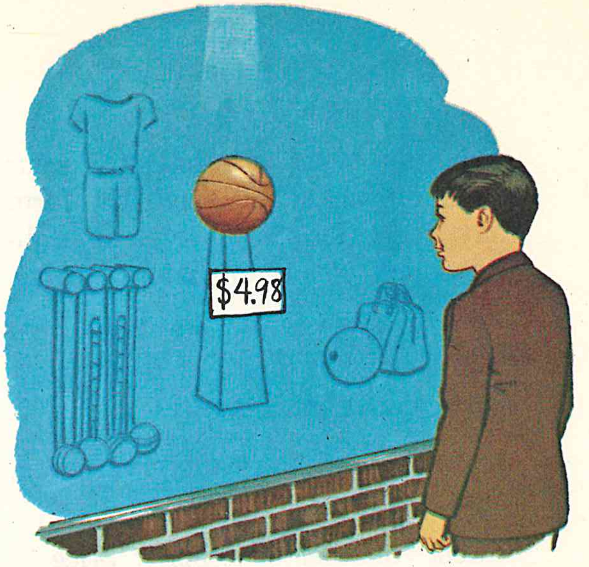
2. Doris paid a bill of $2.25. She still had $9.10. How much money did she have before she paid the bill?
225+910=n $11.35

3. Robert had $3.19 in his coin bank and used part of it to buy a baseball. How much of the money did he have left? No answer.
$3.19 — ? = n

4. Ann bought a package of 100 envelopes for 75¢ plus 3¢ tax. How much did she owe the clerk for her purchase? 75+3=n   78¢

5. Mary bought a sweater for $5.98, a pair of shoes for $9.79, and some rubbers for $2.98. How much change should she receive?
No answer.

6. A school cafeteria had enough tables to seat 72 pupils with 8 at a table. How many tables were needed to seat the pupils?
72÷8=n   9 tables



7. Al is saving money to buy a basketball that costs $4.98. He has saved $.75. How much more money must he have?
498−75=n $4.23

8. John made a 1060-mile airplane trip in 3 hrs. 20 min. What was the average speed of the plane? 1060÷200
$1060 \div 3\frac{1}{3} = n$ 318 mph

9. Ruth and her friends shared the cost of a picnic. How much did each girl pay as her share of the cost?
No answer.

Oral Tell your decision concerning each problem on this page and why you made it. See above.

Written Express each problem on this page in equation form. If necessary, supply the information that is missing. Solve each equation and write each answer. Then read each problem again. Is each answer sensible? See above. Equations for problems 3, 5, and 9 will vary.

Planning Solutions

Certain types of problems occur again and again, although the form may vary. Here is one type of problem which you often have to solve.

I can buy paperback books for \$.59. I want to buy 6 of them. How much money do I need?

A solution follows. Observe how a formula is used.

Cost = number of items × price

$C = n \times p$

$C = 6 \times 59$

$C = 354$

The cost is \$3.54.

In the formula $C = n \times p$, C represents the *number of cents in the cost.* What number is represented by n?

Here is another type of problem which occurs in many variations.

My aunt lives in a city that is 95 miles away. We average 40 miles per hour when we drive to her home. How many hours does the trip take?

A solution follows.

Time = distance ÷ speed

$t = d \div s$

$t = 95 \div 40$

$t = 2\frac{15}{40}$ or $2\frac{3}{8}$

The time is $2\frac{3}{8}$ hours.

The formula $t = \frac{d}{s}$ is used in solving the problem. The letter t represents the *number of hours.* What number is represented by d? By s?

One way to become a successful problem solver is to look for a similarity between a new problem and others you have solved. Use the same problem-solving technique for all problems of the same type.

Oral Tell and explain the formula to use in solving each problem below.

1. I spent \$14.32 for 8 books. What was the price of each book?

$p=C \div n$ $p=1432 \div 8$ \$1.79

2. A treasurer paid a bill for \$8.28 for rubber balls that cost \$.69 each. How many rubber balls had been purchased by his club?

$n=C \div p$ $n=828 \div 69$ 12 balls

3. Mr. Cox bought 18 gallons of gasoline that cost $36\frac{9}{10}$ cents a gallon. How much money did he spend?

$C=n \times p$ $C=18 \times 36\frac{9}{10}$ $\$6.64\frac{2}{10}$

4. Mr. Johnson averaged 43 miles per hour. One day he drove for $6\frac{1}{2}$ hours. How many miles did he drive that day?

$d=s \times t$ $d=43 \times 6\frac{1}{2}$ $279\frac{1}{2}$ mi.

5. A jet plane averaged 525 miles an hour on a trip of 1850 miles. How many hours did the trip take?

$t=d \div s$ $t=1850 \div 525$ $3\frac{11}{21}$ hr.

Written Write a plan, or formula, for solving problems 1 through 5. Solve each problem.

Estimating Answers

After you express a problem as an equation, you can often save the time of an unnecessary computation by estimating an answer. Perhaps the best way to make an estimate is to round the numbers to numbers you can use in mental computation.

Upon making an estimate, reread the problem and see whether your estimated answer seems reasonable. If not, think the entire problem through again. Perhaps you decided upon an equation that is not correct.

Write an equation to use in solving each problem below. Then write an estimated answer. Estimates will vary.

1. There were 24,798 persons at Friday's ball game and 27,010 at Saturday's game. How many more were at Saturday's game than at Friday's game? (Think: 27 thousand minus 25 thousand = 2 thousand.)
27010-24798=n 2212 people

2. The Smiths bought a load of coal that weighed 5850 lb. and another that weighed 5175 lb. Find their total weight.
5850+5175=n 11025 lb.

3. The population of a city grew from 8998 in 1950 to 10,005 in 1960. How much did it grow in ten years?
10005-8998=n 1007 people

4. Mr. Brown receives $79.50 a week for his work. How much does he receive in 52 weeks?
52x7950=n $4134.00

5. A gas station sold 6169 gallons of gasoline during the month of October. What was the average number of gallons sold per day?
6169÷31=n 199 gal.

6. One month Mr. Beck drove his car 756 miles. He estimated the cost at 11¢ per mile. What was the total cost for the month?
756x11=n $83.16

7. Mr. Jones has 1975 sq. ft. of lawn in front of his house, 990 sq. ft. at the side, and 4130 sq. ft. at the back. How many sq. ft. of lawn has he in all?
1975+990+4130=n 7095 sq. ft.

Oral Tell your estimated answer for each problem on this page and how you arrived at it.

Written Solve your equation for each problem. Read the problem again to see whether your solution meets the conditions of the problem. Compare your computed and estimated answers. Write your answer.

Can you do this? Use words to express the meaning of each number phrase below. To illustrate: $x+5$ is translated to *some number plus 5*.
See below.

	a	*b*
1.	$n+9$	$\frac{21}{t}$
2.	$x-7$	$12+r$
3.	$3\times d$	$25-x$

MORE PRACTICE PAGE 307

1. Some number plus 9; 21 divided by some number
2. Some number minus 7; 12 plus some number
3. 3 times some number; 25 minus some number

Using Addition and Another Operation

Many problems in arithmetic involve the use of more than one operation. As you read the following problem, try to determine which operations you would use in solving it. Then study the solution.

A milkman had 672 cartons of milk on his truck. He delivered 235 cartons of milk to Blaine School and 280 cartons to Adams School. How many cartons did he still have on his truck?

You can use the addition, 235+280, to find the number of cartons delivered. After finding this, you can use subtraction to complete the solution. You can express the problem as a single equation and solve it like this:

$$\begin{aligned} 672-(235+280)&=c\\ 672-515&=c\\ 157&=c \end{aligned}$$

The milkman still had 157 cartons of milk on his truck.

Other solutions are possible. To illustrate:

$$\begin{aligned} (672-235)-280&=c\\ 437-280&=c\\ 157&=c \end{aligned}$$

What does (672-235) represent? Try to think of still another solution. Write an equation for it.

Translate each problem below into a single equation. Use ( ) to indicate the operation to do first.

1. Mary spent $2\frac{3}{4}$ hrs. each morning and $2\frac{1}{2}$ hrs. each afternoon in school. She attended school for 176 days. How many hours altogether did she spend in school?
176x($2\frac{3}{4}$+$2\frac{1}{2}$)=n   924 hr.

2. Judy collected dues of $1.25 on Monday and $.70 on Tuesday. Dues were $.15. How many club members paid dues on the two days together?
(125+70)÷15=n   13 members

3. Mr. Daniels drove 295 miles on Monday, 330 miles on Tuesday, and 395 miles on Wednesday. On the average, how far did he drive each day?
(295+330+395)÷3=n   340 mi.

4. Betty had $3.19 in savings and earned $1.15 on a Saturday. After buying a blouse, she had $.35 left. What was the price of the blouse?
(319+115)-35=n $3.99

5. Five girls agreed to share the cost of a party. They spent $4.25 for food, $.90 for prizes, and $.75 for decorations. How much should each girl pay as her share?
(425+90+75)÷5=n $1.18

Oral Tell the equation you wrote for each problem. Also tell why.

Written Estimate an answer for each problem above. Solve your equations. Compare your computed and estimated answers.
Estimates will vary.

Using Subtraction and Another Operation

Mary has an allowance of \$1.50 a week. She plans to save at least \$.25 of her allowance each week. How much spending money will she have in a year? (1 yr. = 52 wk.)

You might think: I will find how much she has to spend each week. I will multiply the result by 52.

You could write the equation and solve it as shown below. Observe that the signs \$ and . are not part of the equation.

$$(150-25)\times 52=n$$
$$125\times 52=n$$
$$6500=n$$

Mary will have \$65 to spend.

Write the equations you will use in solving problems 1 through 8.

1. Mr. Nelson recorded his mileage at 9189 miles when he filled his gasoline tank. At his next stop for gasoline he needed $12\frac{8}{10}$ gallons. Then his mileage was 9435 miles. What was his average mileage per gallon?
$(9435-9189)\div 12\frac{8}{10}=n$ $19\frac{7}{32}$ mi.

2. At a special sale Jane can save \$.46 a yard on material that normally sold for \$1.25 a yard. How much will she have to pay for $3\frac{1}{4}$ yards of material? $3\frac{1}{4}\text{x}(125-46)=n$ \$2.57

3. Packages of books weighed $9\frac{3}{4}$ pounds. The packaging material weighed $1\frac{1}{8}$ pounds. What would be the combined weight of 24 packages of the books and packaging material?
$24\text{x}(9\frac{3}{4}+1\frac{1}{8})=n$ 261 lb.

4. John received \$4.25 in change when he gave a clerk a \$10 bill in payment for a box of 6 balls. How much did each ball cost? 96¢
$(1000-425)\div 6=n$ $95\frac{5}{6}$¢

5. Some girls earned \$7.02 selling candy at \$.39 a pound and had $6\frac{1}{2}$ pounds of candy left. How many pounds of candy did they have before selling any?
$(702\div 39)+6\frac{1}{2}=n$ $24\frac{1}{2}$ lb.

6. Bill saved \$38.25 to be used for buying a \$49.50 bicycle. He can save an average of \$.75 a week. In how many weeks can he buy the bicycle?
$(4950-3825)\div 75=n$ 15 wk.

7. Ted pays \$.17 each for magazines which he sells for \$.25. How much money will he make if he sells 35 magazines?
$35\text{x}(25-17)=n$ \$2.80

8. Mr. Ellis bought a car for \$2775. He made a payment of \$950 and agreed to pay the rest in 18 payments. If there were no additional charges, how much would each payment be?
$(2775-950)\div 18=n$ \$101.39

Oral Tell the class the equation you plan to use in solving each problem. Also make an estimate for each problem. Tell how you made it.
Estimates will vary.

Written Solve problems 1 through 8.

Using Multiplication and Another Operation

The clerk of a school store sold $16\frac{2}{3}$ dozen pencils in 20 days. On the average, how many pencils did he sell each day?

Think: There are 12 pencils in a dozen. In $16\frac{2}{3}$ dozen there are $(16\frac{2}{3}\times12)$ pencils. To find the average I must divide $(16\frac{2}{3}\times12)$ by 20. The equation and a solution are shown.

$$(16\tfrac{2}{3}\times12)\div20=n$$
$$200\div20=n$$
$$10=n$$

The clerk sold an average of 10 pencils a day.

Oral Make up problems which might be solved by using the equations below. Solutions only.

	a		b	
1.	$(5\times4)-3=n$	17	$(8\times\frac{3}{4})+\frac{1}{2}=n$	$6\frac{1}{2}$
2.	$(10\times12)\div6=n$	20	$(5\times2000)\div4=p$	2500

Written Solve problems 1 through 7 after writing an equation for each one.

1. Jane bought 2 games for presents. The games cost \$1.98 each. She gave the clerk a \$5 bill. How much change should she have received? 500-(2x198)=n $1.04

2. Mr. Kelley bought 2 gallons of paint at \$5.98 a gallon. By the quart, the same amount of paint would have cost \$14.00. How much did he save by buying the paint in gallons rather than in quarts?

1400-(2x598)=n $2.04

3. On the average the population of a city increased 15,000 a year. Its population was 1,110,098 in 1960. What would be a good estimate of its 1970 population? 1110098+(10x15000)=n

1,260,098 people

4. Bob walks $\frac{3}{10}$ of a mile to school and makes the trip 4 times a day. How much more than 5 miles does he walk each week?

$[5\mathrm{x}(4\mathrm{x}\frac{3}{10})]-5=n$ 1 mi.

5. Mary practiced her music lesson for $\frac{2}{3}$ of an hour each day for 5 days and $\frac{3}{4}$ of an hour on each of the other 2 days in a week. How much time did she spend in practice during an entire week?

$(5\mathrm{x}\frac{2}{3})+(2\mathrm{x}\frac{3}{4})=n$ $4\frac{5}{6}$ hr.

6. John bought 6 quarts of oil at \$.55 a quart. The tax was \$.13. How much did John owe?

(6x55)+13=n $3.43

7. Don bought a box of $2\frac{1}{2}$ dozen cookies. He ate 3 cookies. Then how many cookies were left in the box?

$(2\frac{1}{2}\mathrm{x}12)-3=n$ 27 cookies

A quick review Indicate whether each sentence below is true or false.

		a		b
T	1.	$7\frac{1}{2}\div3>9\frac{1}{3}\div4$	F	$3\frac{1}{2}\div1\frac{1}{3}<4\frac{1}{3}\div1\frac{3}{4}$
T	2.	$1\frac{1}{8}\times3\frac{1}{4}\neq1\frac{1}{4}\times3\frac{1}{8}$	T	$2\frac{3}{5}\times4\frac{1}{2}=4\frac{4}{8}\times2\frac{6}{10}$

Using Division and Another Operation

The equation on the board shows a solution for the problem below.

$$(12 \div \tfrac{3}{4}) - 4 = n$$
$$(12 \times \tfrac{4}{3}) - 4 = n$$
$$16 - 4 = n$$
$$12 = n$$

Fay bought 12 yards of ribbon and cut it into lengths of $\frac{3}{4}$ yards each. She used all but 4 of the pieces that she cut. How many pieces did she use?

Do you see that the numeral $12 \div \frac{3}{4}$ is another name for the number of pieces she can cut? Some number will equal 4 subtracted from $(12 \div \frac{3}{4})$. The equation is $(12 \div \frac{3}{4}) - 4 = n$. $16 - 4 = n$ and $12 = n$. Fay used 12 pieces of ribbon.

Read problems 1 through 5. Translate each one into an equation.

1. Larry spent \$5 for magazines that cost him \$.25 each. He made \$.10 on each magazine he sold. How much money did he make if he sold all of the magazines?

$(500 \div 25) \times 10 = n$ \$2.00

2. Don cut a 12-foot board into $1\frac{1}{3}$-foot lengths. He had paid \$1.35 for the board. What was the value of each piece he had cut?

$135 \div (12 \div 1\frac{1}{3}) = n$ \$.15

3. Phil earns \$1.30 each Saturday for 2 hours of mowing lawns. At this rate, how much should he charge for mowing a lawn that will take 3 hours to mow?

$3 \times (130 \div 2) = n$ \$1.95

4. Mr. Larsen used 396 pounds of apples to fill baskets with 44 pounds in each. He sold the baskets for \$2.95 each. How much did he receive for his apples?

$(396 \div 44) \times 295 = n$ \$26.55

5. John and his father drove 380 miles in 9 hours. They had hoped to average 45 miles an hour. By how many miles per hour did they miss their estimate?

$45 - (380 \div 9) = n$ $2\frac{7}{9}$ mi.

Oral Tell the class the equation you wrote for problems 1 through 5 and how you reached your decision.

Written Solve your equations for problems 1 through 5.

Can you do this? Tell why $x + (x + 100) = 110$ is a correct equation for the problem below. Solve the equation.

John paid \$1.10 for a theater ticket. The price included tax. The cost of the ticket was \$1.00 more than the tax. How much was the tax?

x is the tax. x=5¢

Problems with More Than Two Operations

Three girls shared the cost of a picnic. They bought 2 dozen cookies at $.49 a dozen, 3 pints of ice cream at $.35 a pint, and frozen lemonade for $.40. How much should each girl pay as her share?

More than two operations are required to solve this problem. The problem can be translated into a single equation as shown.

$$n = \frac{(2 \times 49) + (3 \times 35) + 40}{3}$$

$$n = \frac{98 + 105 + 40}{3}$$

$$n = \frac{243}{3}$$

$$n = 81$$

Each girl should pay $.81.

Oral Answer these questions about the equation above.

1. What item of cost is represented by 2×49? By 3×35? By 40?
cookies ice cream lemonade

2. How is division indicated?
By the fraction line

Explain how you think about the problem in order to arrive at the equation.

Written Solve problems 1 through 6 by writing an equation containing n. Solve for n. Then state what n represents in each equation.

MORE PRACTICE PAGE 307

1. Doris bought 5 stamps at $.25 each, a stamp album for $2.29, and mounting materials for $1.35. She gave the clerk $5.00. How much money should she receive in change?
500- [(5x25)+229+135]=n $.11

2. Committee members bought 3 jars of candy with 14 ounces in each jar, and 2 boxes of candy with $1\frac{1}{2}$ pounds in each box. They put the candy into bags that contained 4 ounces each. How many bags of candy did they fill?
[(3x14)+2x(16+8)] ÷4=n $22\frac{1}{2}$ bags

3. The Allisons averaged 35 miles per hour for 5 hours and 40 miles per hour for 3 hours. What was their speed per hour on their 8-hour trip?
[(5x35)+(3x40)] ÷8=n $36\frac{7}{8}$ mph

4. Bob had 1 gross, or 144, pencils to sell. He sold 5 dozen on Wednesday. On Thursday and Friday he sold all but 15 of the remaining pencils. How many pencils did he sell on Thursday and Friday together?
(144-60)-15=n 69 pencils

5. Nancy bought 2 records at $.79 each. The tax was $.06. She bought 3 other records at $1.75 each and paid $.21 tax on them. How much change should she get from $10? 1000-[(2x79)+6+(3x175)+21]=n $2.90

6. Jane practices violin for $\frac{1}{2}$ hour each morning and $\frac{3}{4}$ hour each afternoon on 5 days each week. How much time does she spend in practice in a year? $52\text{x}[(5\text{x}\frac{1}{2})+(5\text{x}\frac{3}{4})]=n$ 325 hr

7. Bob needed $10. He worked for $2\frac{1}{2}$ hours at $.50 an hour and $5\frac{1}{2}$ hours at $.75 an hour. How much money did he still need?
$1000-[(2\frac{1}{2}\text{x}50)+(5\frac{1}{2}\text{x}75)]=n$ $4.62

Practice in Solving Problems

You should be able to solve the problems on this page without any help. Read each problem at least twice before you translate it into an equation. Then write an estimated answer. Estimates will vary.

1. A plane flew 2075 miles between the hours of 4:00 a.m. and 9:00 a.m. What was its average speed per hour?

2075÷5=n   415 mph

2. Joe was building some shelves. He had a board 12 feet long and cut off one piece $3\frac{1}{2}$ feet long and another piece $2\frac{3}{4}$ feet long. How many feet of the board were left?

$12-(3\frac{1}{2}+2\frac{3}{4})=n$ $5\frac{3}{4}$ ft.

3. Lee picked 10 quarts of berries and sold them to the grocer for $18\frac{1}{2}$¢ a quart. How much money did Lee get? $10\times18\frac{1}{2}=n$ $1.85

4. Mary bought $\frac{1}{4}$ of a pound of butter at 92¢ a pound, $\frac{2}{3}$ of a dozen oranges at 63¢ a dozen, and a nickel candy bar. How much change should she get from $1?

$100-[(\frac{1}{4}\times92)+(\frac{2}{3}\times63)+5]=n$ 30¢

5. Mr. Adams sold 6 cans of milk that held 10 gallons each. 10 gallons of milk weighs 86 pounds. At 7¢ a pound, how much money did he get?

(6x86)x7=n $36.12

6. The amount of money received from the sale of tickets to the school play was $187.25. The expenses amounted to $21.95. What was the net amount of profit?

18725-2195=n $165.30

7. Four fathers shared the cost of 12 children's tickets at $.35 each, 4 adult tickets at $.50 each, and refreshments that cost $2.00. How much was each father's share?

[(12x35)+(4x50)+200]÷4=n $2.05

8. Ellen made $1\frac{1}{2}$ pounds of chocolate fudge and $2\frac{3}{4}$ pounds of maple fudge for the candy sale. How many pounds of candy did Ellen make?

$1\frac{1}{2}+2\frac{3}{4}=n$ $4\frac{1}{4}$ lb.

9. Mr. Roberts bought $14\frac{3}{10}$ gallons of gasoline at $36\frac{9}{10}$ cents a gallon and a quart of oil for $.65. How much did he pay?

$(14\frac{3}{10}\times36\frac{9}{10})+65=n$ $5.93

10. The distance between New York and London is 3450 miles. A jet plane made the flight in $8\frac{1}{2}$ hours. What was its average speed for each hour?

$3450\div8\frac{1}{2}=n$ $405\frac{15}{17}$ mph

11. At a major port 10,050,000 tons of freight were unloaded in 1962. Two years earlier 8,435,000 tons had been unloaded. How many more tons were unloaded in 1962 than in 1960?

10050000-8435000=n   1,615,000 T.

12. Three classes of 32 pupils each, 1 class of 34 pupils, 4 teachers, and 7 parents took a trip on 3 buses. Each bus took the same number of riders. How many riders were on each bus?

[(3x32)+34+4+7]÷3=n   47 riders

Written Solve your equation for each problem on this page. Read each problem again to see if your answer seems reasonable.

Review and Practice

You should be able to do all of the exercises on this page without referring to other pages in the book.

Part 1 Copy. Find the correct answer for each exercise by performing the indicated operations.

	a	*b*
1.	179r52 297)53215	980 605)592900
2.	\$4175.38 ×25 \$104384.50	\$31495.50 ×250 \$7873875.00
3.	\$91385.46 −32954.75 \$58430.71	\$800500.00 −395798.75 \$404701.25
4.	\$62536.50 19724.35 43895.82 10291.67 +54308.67 \$190757.01	\$937426.50 232054.28 914609.61 157048.24 +358697.73 \$2599836.36

Part 2 Copy. Write a numeral in each blank so that the statements below become true. Reminder: = means *equal in measure.*

1. 15 min. = 900 sec.
2. 2750 lb. = 1 T. 750 lb.
3. 5 lb. 6 oz. = 86 oz.
4. 245 in. = 20 ft. 5 in.
5. 6×35 min. = 210 min. = 3 hr. 30 min.
6. 9 lb. 8 oz. + 5 lb. 14 oz. = 15 lb. 6 oz.
7. 12 ft. 6 in. − 2 ft. 8 in. = 9 ft. 10 in.
8. 39 ft. 8 in. ÷ 4 = 9 ft. 11 in.

Part 3 Copy. Solve each equation in rows 1 through 9 below.

	a	*b*
1.	$5\frac{1}{4}$ $n=3\frac{3}{4}+1\frac{1}{2}$	$7\frac{5}{12}$ $x=2\frac{2}{3}+4\frac{3}{4}$
2.	$66\frac{1}{2}$ $d=7\times9\frac{1}{2}$	$27\frac{1}{2}$ $(5\times6)-2\frac{1}{2}=w$
3.	252 $(7\times12)\div\frac{1}{3}=n$	174 $t=(435\div5)\times2$
4.	3 $(36\div9)-n=1$	1 $n+(\frac{3}{4}\div\frac{1}{2})=2\frac{1}{2}$
5.	10 $x-(3\frac{1}{4}\times2)=3\frac{1}{2}$	$18\frac{1}{3}$ $(48\div3)+2\frac{1}{3}=n$
6.	30 $(12\div\frac{1}{2})+6=a$	$21\frac{1}{3}$ $p=2\times(4\frac{1}{3}+6\frac{1}{3})$
7.	25 $c=(2\times9\frac{1}{2})+6$	$18\frac{6}{7}$ $r=6\times(22\div7)$
8.	$37\frac{1}{2}$ $(3\frac{1}{8}\div\frac{1}{2})\times6=n$	$12\frac{3}{4}$ $x=(19\frac{1}{4}-12\frac{1}{2})+6$
9.	$15\frac{1}{2}$ $n=(6-2\frac{1}{8})\times4$	4 $7=n+(6-3)$

Part 4 Answer each question below.

1. Which operation is the inverse operation of addition? Of multiplication? Subtraction; division

2. Which 2 addends would you associate in 95+9+5=n? Why?
95+5=100

3. Which two operations of arithmetic have the commutative property? Addition; multiplication

4. Which number is the identity number of addition? Of multiplication? 0; 1

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Story problems must be translated into the language of arithmetic before they are solved. (131)

2. Arithmetic sentences usually consist of numerals, a letter or letters to hold the place for a numeral, and the = sign. (132)

3. Any letter may be selected to hold the place for a numeral. (132)

4. Arithmetic sentences in which you use the = sign are called equations. (132)

5. Solving an equation involves finding a replacement for the letter which makes the sentence true. (132)

Questions to Discuss

See T145 for answers.

1. Why should you read each problem at least twice? (131)

2. How would you translate the equation $(9\times6)+n=100$ into the English language? (132)

3. How do you determine which operation or operations to use when translating a problem into an equation? (133)

4. How can you determine whether the information in a problem is needed in its solution? (134)

5. What is one good method to use in estimating answers? (137)

6. Why may more than one equation be suitable for solving a problem? (138)

Written Practice

Solve each problem below by using an equation.

1275+500=n $17.75

1. A baseball club needed $25 for equipment. After earning $12.75, the club received a $5 gift. How much money did the club then have? (134)

6x79=n $4.74

2. Betty wanted to buy 6 records at $.79 each including tax. How much money did she need? (136)

3. At a tire sale, Mr. Cox bought 3 tires at $19.95 each and a fourth tire for $1. How much, on the average, did each tire cost? (138)

[(3x1995)+100]÷4=n $15.21

4. John's father bought $11\frac{3}{10}$ gallons of gasoline at $36\frac{9}{10}$ cents per gallon and $2\frac{1}{2}$ gallons of antifreeze at $2.98 a gallon. How much money did he owe? (140)

$(11\frac{3}{10}\text{x}36\frac{9}{10})+(2\frac{1}{2}\text{x}298)=n$ $11.62

5. Bob sells papers for 7¢ each. The papers cost him 4¢. Last week he delivered 125 papers. At the same rate, how much money can he expect to earn in 4 weeks? (142)

4x[125x(7-4)]=n $15.00

Self-Evaluation

Part 1 Express as equations.

1. 15 plus 9 equals some number.
$15+9=n$

2. Some number minus $4\frac{1}{4}$ equals 6.
$x-4\frac{1}{4}=6$

3. 3 divided by a number.
$\frac{3}{n}$ or $3\div n$

4. 5 times some number multiplied by 8 equals 0. $5\times(n\times 8)=0$ or $(5\times n)\times 8=n$

5. Some number plus 50 equals 75.
$n+50=75$

Part 2 Each of the problems below is followed by four equations. Write the equation or equations you might use in solving each problem.

1. Ann has 17 records. This is 5 more records than Betty has. How many records does Betty have?

a. $17+5=r$ **c.** $17=r+5$
b. $5+r=17$ **d.** $r=17-5$

2. John had enough books to pack 6 boxes with 12 books in each box. He had 5 books left over. How many books did he have in all?

a. $(6+12)+5=b$ **c.** $(6\times 12)+b=5$
b. $b=(6\times 12)+5$ **d.** $(5+6)\times 12=b$

3. Barbara had $5.00. She spent 98¢ for a book and 79¢ for lunch. How much of the money was left?

a. $(500-98)-79=n$
b. $n=(500-79)=98$
c. $500-(98+79)=n$
d. $(98+79)+n=500$

Part 3 Write an equation for each of the following problems. Estimate an answer and record it. Solve the equation, and compare the result with your estimated answer.

Estimates will vary.

1. Each classroom had seats for 35 pupils. There were 18 classrooms in the school. The average enrollment per classroom was 32. How many pupils were enrolled in the school?
$18\times 32=n$ 576 pupils

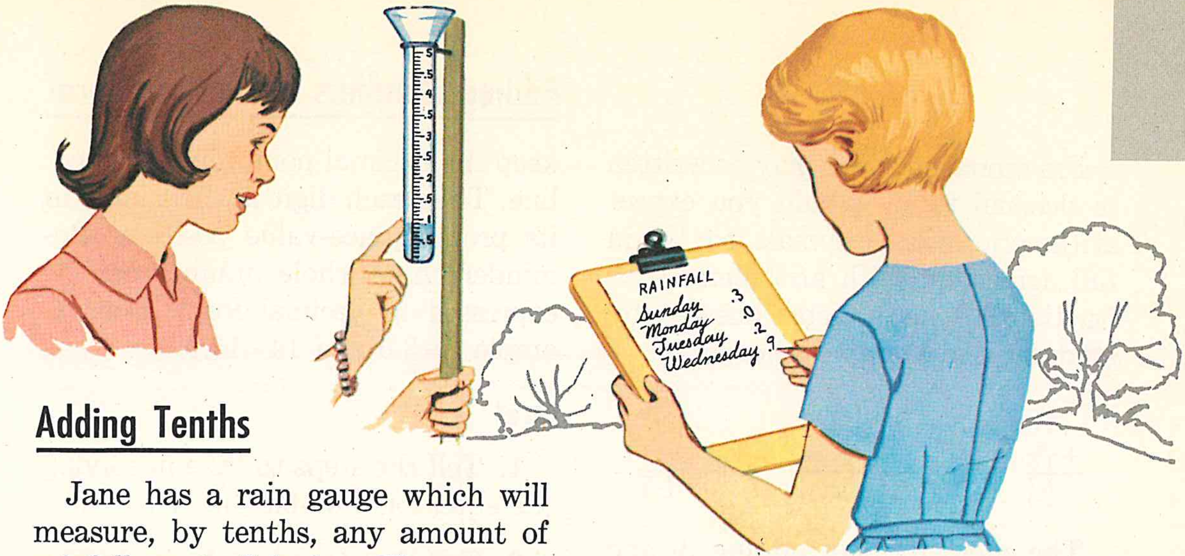
2. Mary read a 324-page book in one week and a 198-page book in the next week. On the average, how many pages did she read each day during the 2-week period?
$(324+198)\div(2\times 7)=n$ $37\frac{2}{7}$ pages

3. In its first 3 games a football team scored 14 points, 35 points, and 20 points. Last year the school team averaged 20 points per game. This year's average score is how much above last year's average?
$[(14+35+20)\div 3]-20=n$ 3 points

4. Don charged 50¢ an hour for washing windows. He worked for Mrs. Green for $2\frac{1}{2}$ hours. She paid him and gave him 25¢ for bus fare. How much money did he get from Mrs. Green?
$(2\frac{1}{2}\times 50)+25=n$ $1.50

5. Mr. Brown drove for 8 hours at an average speed of 42 miles per hour. His company allows 9¢ a mile for car expenses. How much should his company pay him for his car expenses that day?
$(8\times 42)\times 9=n$ $30.24

9



Adding Tenths

Jane has a rain gauge which will measure, by tenths, any amount of rainfall up to 5 inches. The present reading of the gauge is .9, or nine tenths. How much rain fell on Sunday? On Monday? On Tuesday?

Suppose you wished to know how much rain fell on Tuesday and Wednesday together. You would add .2 and .9. Observe how this addition is shown on the number line on this page. Look at the fraction scale first. Do you see that

$$\frac{2}{10}+\frac{9}{10}=\frac{2+9}{10}=\frac{11}{10}=1\frac{1}{10}?$$

Now look at the decimal scale. Do you see that

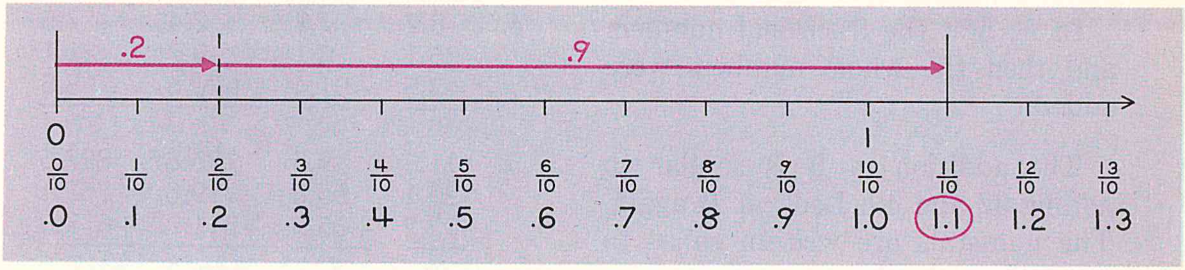
$$.2+.9=1.1?$$

Look again at the addition in decimal form. The decimal point in .2 and .9 reminds you that the denominator is tenths. Since the denominator is common to both fractions, simply add 2 and 9. Add .2 and .9 as you add 2 and 9. Use the decimal point in the sum to show that you were adding tenths.

Oral

1. How would you add .4 and .3 in fractional form? In decimal form?
See below.

2. How would you add .8 and .9 in fractional form? In decimal form?
See below.



1. $\frac{4}{10}+\frac{3}{10}=\frac{7}{10}$; $.4+.3=.7$

2. $\frac{8}{10}+\frac{9}{10}=1\frac{7}{10}$; $.8+.9=1.7$

Adding Numbers in Decimal Form

Fractional numbers may be written in decimal form. Would you expect arithmetic with decimals to be in full agreement with arithmetic with fractions? Check your answer by studying the example below.

$$\begin{array}{r} \frac{9}{10} \\ +\frac{8}{10} \\ \hline \frac{17}{10} \end{array} \qquad \begin{array}{r} 9\text{ tenths} \\ +8\text{ tenths} \\ \hline 17\text{ tenths} \end{array} \qquad \begin{array}{r} .9 \\ +.8 \\ \hline 1.7 \end{array}$$

The short form of adding .9 and .8 is to add 9 and 8 and then to write the sum in tenths.

Suppose you were to solve the equation 3.2+4.9=n. The steps are shown in fractional and in decimal form in A and B.

The whole numbers and the fractional numbers are added separately.

A

$$\begin{array}{l} \;\;3\frac{2}{10} \\ +4\frac{9}{10} \\ \hline \;\;7\frac{11}{10}=(7+1)+\frac{1}{10}=8\frac{1}{10} \end{array}$$

B

$$\begin{array}{r} 3.2 \\ +4.9 \\ \hline 8.1 \end{array}$$

In A, first the fractional numbers and then the whole numbers were added.

The addition in B is similar to adding 32 and 49. Look at B again. The numerals are written so as to keep the decimal points in a vertical line. Then each digit in the sum is in its proper place-value position. Reminder: Any whole number can be expressed in decimal form. For example 3=3.0 and 14=14.0.

Oral

1. Tell the steps to take in solving the equation 4.6+6.5=n. See column 1.

2. Tell the steps to take in finding each sum.

$$\begin{array}{rrrr} a & b & c & d \\ .9 & 3.6 & 32.7 & 175.8 \\ +.7 & +4.5 & +14.6 & +69 \\ \hline 1.6 & 8.1 & 47.3 & 244.8 \end{array}$$

Written Solve for n.

	a	b
1.	$.4+6.5=\overset{6.9}{n}$	$\overset{9.6}{n}=8.7+.9$
2.	$8.4+.7=\overset{9.1}{n}$	$\overset{20.1}{n}=10.5+9.6$

Copy. Write each sum.

$$\begin{array}{lrrrr} & a & b & c & d \\ \textbf{3.} & 7.6 & 8.5 & 27 & 147.5 \\ & +3.9 & +7 & +35.8 & +258.5 \\ \cline{2-5} & 11.5 & 15.5 & 62.8 & 406.0 \\ \textbf{4.} & 8.2 & 7.2 & 2.4 & 7.5 \\ & 35.1 & 8.1 & 3.7 & 21.1 \\ & +7.5 & +91.4 & +71.5 & +8.5 \\ \cline{2-5} & 50.8 & 106.7 & 77.6 & 37.1 \\ \textbf{5.} & 11.3 & 19.5 & 175.8 & 897.6 \\ & 23.1 & 85.8 & 719.6 & 762.1 \\ & +2.2 & +65.9 & +541.2 & +657.8 \\ \cline{2-5} & 36.6 & 171.2 & 1436.6 & 2317.5 \end{array}$$

Adding Hundredths

Suppose you were to solve the equation .54+.60=x. Since .54 is a little more than .50, which is $\frac{1}{2}$, and .60 is also somewhat more than $\frac{1}{2}$, and $\frac{1}{2}+\frac{1}{2}=1$, the value of x will be greater than 1. Could it possibly be as much as 2? Why not?

Study A, B, and C.

A	B	C
$\frac{54}{100}+\frac{60}{100}=x$	$.54+.60=x$	$.54$
$\frac{114}{100}=x$	$1.14=x$	$+.60$
		1.14

Which numeral can you use as a replacement for x in A? In B? Then $x=1\frac{14}{100}$ or 1.14.

You are most likely to use the column form shown in C above. In this form, you write the numerals in a way that keeps the decimal points in a vertical line. Why?

You had experience in adding decimal numbers when you solved problems involving dollars and cents. Thus you should be able to solve such equations as 1.67+3.25=n. Study the solution below.

$$\begin{array}{rr} 1\frac{67}{100} & 1.67 \\ +3\frac{25}{100} & +3.25 \\ \hline 4\frac{92}{100} & 4.92 \end{array}$$

Oral Tell the reason for each step you would take in solving for n.
See column 1.

	a	b
1.	.34+.52=n .86	.49+.85=n 1.34
2.	.97+.6=n 1.57	.59+.8=n 1.39
3.	4.31+.58=n 4.89	3.96+7.32=n 11.28

Written Copy. Solve for n.

	a	b
1.	.12+.66=n .78	.25+.33=n .58
2.	.75+.80=n 1.55	.94+.44=n 1.38
3.	.47 n=.19+.28	.81 n=.36+.45
4.	5.79 n=5.67+.12	3.67 n=.43+3.24
5.	3.96+4.75=n 8.71	6.32+1.78=n 8.10

Copy. Write each sum. Check your work for accuracy.

	a	b	c
6.	67.57 +9.65 77.22	3.33 +26.77 30.10	55.64 +4.76 60.40
7.	95.67 +8.98 104.65	4.58 +97.63 102.21	98.88 +3.4 102.28
8.	17.87 +29.23 47.10	46.7 +47.94 94.64	54.45 +48.67 103.12
9.	60.17 32.0 +83.75 175.92	72.3 30.47 +54.05 156.82	90.4 37.08 +42.70 170.18

The Associative Property of Addition

$7 + 3 + 2 = X$	$\frac{7}{10} + \frac{3}{10} + \frac{2}{10} = X$	$.7 + .3 + .2 = X$
$(7+3) + 2 = X$	$\left(\frac{7}{10} + \frac{3}{10}\right) + \frac{2}{10} = X$	$(.7 + .3) + 2 = X$
$10 + 2 = X$	$\frac{10}{10} + \frac{2}{10} = X$	$1.0 + .2 = X$
$12 = X$	$\frac{12}{10} = X$	$1.2 = X$

Study the computation as shown above. Then take any three decimal numbers. Group them in different ways before adding them. The grouping of addends will not change the sum. Thus the associative property of addition applies also to numbers in decimal form.

By grouping, or associating, the addends, you can sometimes do additions without using pencil and paper. To illustrate, solve the equation below without using pencil and paper.

$$.82+7+.18=n$$

You probably know that $.82+.18=1.00$. Then $(.82+.18)+7=1+7=8$.

See what happens when you take any two decimals, for example .7 and .5, and add them in either order, like this: .7+.5 and then .5+.7. In each case the sum is the same. Thus the *commutative property* of addition applies also to decimal numbers.

Practice in Adding Decimal Numbers

Oral Solve for n. Tell how you associated the addends.

	a	*b*
1.	$\underline{.8}+.5+\underline{.2}=n$ 1.5	$\underline{.6}+.7+\underline{.4}=n$ 1.7
2.	$3+\underline{.7}+4+\underline{.3}=n$ 8.0	$\underline{.37}+.46+\underline{.63}=n$ 1.46
3.	$\underline{.85}+\underline{.15}+3=n$ 4.0	$\underline{.06}+.19+\underline{.94}=n$ 1.19

Written Copy. Write each sum. Look for groupings by 10's.

	a	*b*	*c*	*d*
1.	8.15	9.27	7.22	6.81
	3.47	4.98	1.46	4.79
	8.99	1.53	3.64	7.29
	+2.05	+6.87	+3.58	+3.31
	22.66	22.65	15.90	22.20
2.	4.10	7.92	1.32	9.74
	1.54	3.43	8.07	8.89
	2.96	1.17	2.33	6.20
	+6.34	+5.76	+5.74	+1.16
	14.94	18.28	17.46	25.99

Can you do this? Express $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{4}{5}$ first as tenths and then as hundredths in decimal form. .5, .2, .8; .50, .20, .80

Adding Thousandths

Suppose you were to add .009 and .007. The common denominator is one thousand. The addition may be done in any of the ways shown below.

A $\frac{9}{1000}+\frac{7}{1000}=\frac{16}{1000}$

B $.009+.007=.016$

C $\begin{array}{r} .009 \\ +.007 \\ \hline .016 \end{array}$

Do you see that you add thousandths in the same way as you add tenths and hundredths?

Now study the addition in D. Ts represents tenths, Hs hundredths, and Ths thousandths.

	T	O	Ts	Hs	Ths
		8.	9	6	5
D	3	4.	7	2	0
	+	1.	9	0	8
	4	5.	5	9	3

Sometimes you add decimal numbers expressed with different denominators, for example .375 and .28. Study the two solutions below.

$$\frac{375}{1000}+\frac{28}{100}=\frac{375}{1000}+\frac{280}{1000}=\frac{655}{1000}$$

$$.375+.28=.375+.280=.655$$

The fractional form shows that $\frac{28}{100}$ was written as $\frac{280}{1000}$. Then the fractions had a common denominator and addition was possible. In decimal form .28 was written as .280. Does $\frac{28}{100}=\frac{280}{1000}$?

If the addition of .375 and .28 is written in column form, you need not write .28 as .280. However, be sure to write the numerals so that the decimal points are in a vertical line.

Oral

1. How would you express .5 as hundredths? As thousandths?
.50; .500

2. In column form how would you write the solution for the equation .3+2.50+.395=n? See below.

Written Copy. Write each sum.

	a	*b*	*c*
1.	.009 +.037 .046	.038 +.196 .234	.785 +.693 1.478
2.	3.721 +2.986 6.707	14.721 +39.985 54.706	39.921 +145.899 185.820
3.	.514 7.000 .22 +.107 7.841	.251 .03 4.152 +.204 4.637	.217 .14 4.402 +.829 5.588
4.	3.234 61.24 2.00 +1.429 67.903	4.205 1.247 31.328 +3.026 39.806	4.029 7.213 .322 +54.03 65.594

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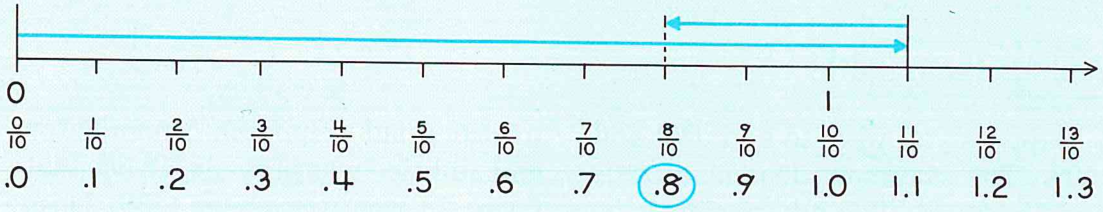
2. $\begin{array}{r} .3 \\ 2.50 \\ .395 \\ \hline 3.195 \end{array}$


0
1
$\frac{0}{10}$ $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ $\frac{10}{10}$ $\frac{11}{10}$ $\frac{12}{10}$ $\frac{13}{10}$
.0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3

Subtracting Decimal Numbers

Ann lives 1.1 miles from school. Betty's house is .3 of a mile from school. How much closer to school is Betty's home than Ann's?

You are to find by how much one number is greater than the other; so you subtract. The equation is $1.1-.3=n$.

Study the solution on the number line above. Does $\frac{11}{10}-\frac{3}{10}=\frac{8}{10}$? Does $1.1-.3=.8$?

Betty's home is .8 of a mile closer to school than Ann's.

Study the example below. You will see again that the arithmetic of numbers in decimal form is in full agreement with the arithmetic of numbers in fractional form.

$\frac{8}{10}$	8 tenths	.8
$-\frac{3}{10}$	−3 tenths	−.3
$\frac{5}{10}$	5 tenths	.5

The short form of subtracting .3 from .8 is to subtract 3 from 8 and then to write the answer in tenths.

If you remember to write the numerals so that the decimal points are in a vertical line, your answer will be expressed correctly.

Now study A, B, and C.

A	B	C
7.8	3.06	73.803
−3.9	−1.87	−2.65
3.9	1.19	71.153

In A, B, and C, the computation is the same as with whole numbers. Do you see that the numerals are written so as to keep the decimal points in a vertical line?

Oral Answer these questions.

1. In example A, how was the minuend renamed to make subtraction possible? 6 and 18 tenths

2. What changes in form were made in B before subtraction was possible? See below.

3. In C, why is 3 written in thousandths place of the answer? See below.

2. As 2 and 9 tenths and 16 hundredths.
3. Because there are no thousandths in the subtrahend to subtract from the 3 thousandths in the minuend.

4. In B and C on page 152, how would you check the computation?
Answer+subtrahend=minuend

Written Copy. Replace each letter with the numeral that makes each statement true.

	a		b	
1.	.7 − .4 = n	.3	x = 1.4 − .4	1.0
2.	9.1 − .9 = t	8.2	p = .72 − .60	.12
3.	.42 − .35 = d	.07	w = 4.46 − .24	4.22
4.	.092 − .075 = m	.017	n = 3.150 − 1.335	1.815

Perform the indicated operations. Check for accuracy.

	a	b	c
5.	14.7 −8.5 6.2	54.8 −15.4 39.4	58.7 −4.9 53.8
6.	68.2 −14.9 53.3	36.4 −18.6 17.8	99.2 −27.5 71.7
7.	54.78 −23.05 31.73	64.30 −21.26 43.04	72.40 −13.08 59.32
8.	81.70 −19.07 62.63	60.12 −28.07 32.05	51.90 −37.83 14.07
9.	6.008 −2.004 4.004	9.998 −3.427 6.571	8.964 −3.547 5.417
10.	8. −3.006 4.994	25.850 −7.006 18.844	72.050 −5.209 66.841
11.	540.025 −12.675 527.350	225.350 −114.275 111.075	962.875 −387.998 574.877

Solve these problems.

12. One metal rod had a length of 1.17 ft. A similar rod had a length of 1.29 ft. What was the difference between the two lengths?
1.29−1.17=n .12 ft.

13. One piece of metal measured .075 in. in thickness. A second piece measured .080 in. What was their combined thickness?
.075+.080=n .155 in.

14. Janet had a temperature of 101.0°. This was 2.5° above her normal temperature. What was her normal temperature?
101.0−2.5=n 98.5°

15. Jack wants to spend $5.98 for a sweater. He had $2.75 and earned $.85. How much more money does he need?
5.98−(2.75+.85)=n $2.38

16. After buying a book for $1.29 and a ball for $.79, Arthur had $2.00. How much money did he have before making his purchases?
1.29+.79+2.00=n $4.08

Another way Study the solution of the equation n = 7.8 − 3.9 below.

n = (7 + .8) − (3 + .9)
n = (6 + 1.8) − (3 + .9)
n = (6 − 3) + (1.8 − .9)
n = 3 + .9 = 3.9

7.8
−3.9
3.9

Use the way presented above to show all of the steps taken in subtracting 8.2 from 12.1. See below.

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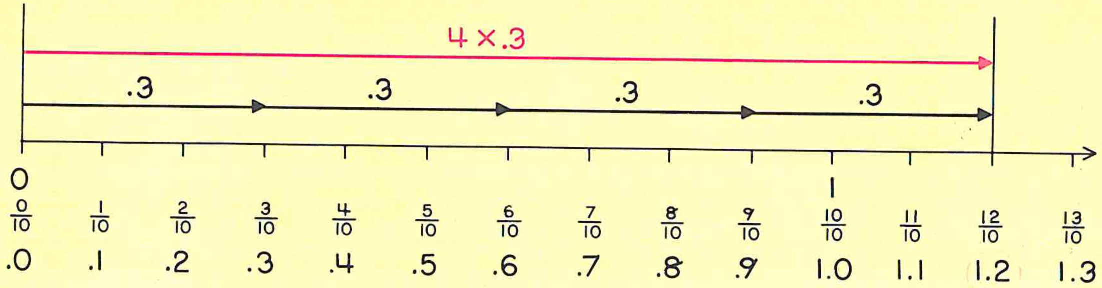
n=(12+.1)−(8+.2)
n=(11+1.1)−(8+.2)
n=(11−8)+(1.1−.2)
n= 3 + .9=3.9

12.1
−8.2
3.9



Multiplying Tenths

Each trip across the lake was .3 mile. How far did Dick row if he made the trip 4 times?

You are to find the sum of 4 equal numbers; so you multiply. Let m represent miles. Then the equation is $4 \times .3 = m$.

Observe how the multiplication is shown on the number line above. Look at the fraction scale first. Do you see that

$$4\times\frac{3}{10}=\frac{3}{10}+\frac{3}{10}+\frac{3}{10}+\frac{3}{10}=\frac{12}{10} \text{ or } 1\frac{2}{10}?$$

Now look at the decimal scale. Do you see that

$$4\times .3 = .3+.3+.3+.3 = 1.2?$$

The multiplication can be done in column form as shown below. The abbreviation Ts means tenths.

$$\begin{array}{r} 3 \text{ tenths} \\ \times 4 \\ \hline 12 \text{ tenths} \end{array} \qquad \begin{array}{r|l} 0 & \text{Ts} \\ & 3 \\ \times 4 & \\ \hline 1 & 2 \end{array} \qquad \begin{array}{r} .3 \\ \times 4. \\ \hline 1.2 \end{array}$$

Except for the decimal point, the multiplication itself is identical to multiplying 3 by 4. The decimal point shows that tenths were being multiplied.

Try multiplying .3 by 9 both in fractional and in decimal form. Do you obtain 2.7 as your answer? Is the number of decimal places in the product the same as the number of decimal places in the decimal factor?

Oral How would you multiply .2 by 4 and then .5 by 8 in fractional form? In decimal form? See column 1.

Written Solve for n.

	a	*b*
1.	$3\times .3 = n$ .9	.4 $n = 2\times .2$
2.	$2\times .4 = n$ .8	2.8 $n = 4\times .7$
3.	$6\times .8 = n$ 4.8	2.4 $n = 8\times .3$
4.	$3\times .7 = n$ 2.1	6.3 $n = 7\times .9$
5.	$6\times .4 = n$ 2.4	4.5 $n = 9\times .5$

Multiplying Tenths, Hundredths, and Thousandths

Roger bought 6 stamps at \$.04 each. How much did he spend?

Since \$.04 is 4 hundredths of a dollar, you can think of \$.04 as .04. Let c represent the cost of the stamps. Then the problem may be expressed as $6\times .04=c$. Study the solutions below. Do you see again that arithmetic with decimal numbers is the same as arithmetic with fractional numbers?

$$6\times\frac{4}{100}=\frac{6\times 4}{100}=\frac{24}{100}$$
$$6\times .04=.24$$

Roger spent .24 of a dollar or \$.24.

Multiplying .04 by 6 is similar to multiplying 4 by 6. However, the decimal point *must* be used in the answer. Is the number of decimal places in the product and in the decimal factor the same?

How would you multiply .004 by 6? Compare the multiplication of numbers in fractional and decimal form.

$$6\times\frac{4}{1000}=\frac{6\times 4}{1000}=\frac{24}{1000}$$
$$6\times .004=.024$$

Since you are multiplying thousandths, the answer must also be in thousandths.

Suppose you were to multiply .735 and 15. Two solutions are shown.

$$\frac{15}{1}\times\frac{735}{1000}=\frac{15\times 735}{1000}=\frac{11025}{1000}=11\frac{25}{1000}$$

	T	O	Ts	Hs	Ths
			.7	3	5
	×1	5			
5×.735 =		3	.6	7	5
10×.735 =		7	.3	5	
	1	1	.0	2	5

Do you multiply .735 by 15 as you multiply 735 by 15? Where is the decimal point in the answer? Why?

Oral Answer each question below. Explain your answer. See column 1.

1. How would you multiply .412 by 15 in fractional form?

2. What is the advantage of working in decimal rather than in fractional form?

Written Solve for n.

	a		*b*	
1.	2×.24 = n	.48	4×.42 = n	1.68
2.	3×.43 = n	1.29	5×.16 = n	.80
3.	6×.14 = n	.84	3×.84 = n	2.52
4.	17×.85 = n	14.45	n = 45×.96	43.20
5.	3×.032 = n	.096	n = 6×.035	.210
6.	8×.463 = n	3.704	n = 9×.199	1.791
7.	35×.975 = n	34.125	n = 87×.567	49.329
8.	97×.32 = n	31.04	n = 54×.750	40.500

Multiplying a Decimal Number by a Whole Number

Bob's father averages 15.6 miles on a gallon of gasoline. How far can he drive on a full tank of gasoline if the tank holds 18 gallons?

The equation is $18\times15.6=n$. Since you multiply decimal numbers just as you do whole numbers, the multiplication can be done as shown at the right. Multiply 15.6 by 8 as you multiply 156 by 8. Then multiply 15.6 by 10 as you multiply 156 by 10. Then add the partial products. Are the decimal points in a vertical line?

$$\begin{array}{r} 15.6 \\ \times 18. \\ \hline 124.8 \\ 156. \\ \hline 280.8 \end{array}$$

Bob's father can drive 280.8 miles on a full tank of gasoline.

Study A and B. Notice the position of the decimal point in 3.26 and 42.357 and in each partial product and product.

A

$$\begin{array}{r} 3.26 \\ \times 12. \\ \hline 6.52 \\ 32.6 \\ \hline 39.12 \end{array}$$

B

$$\begin{array}{r} 42.357 \\ \times 25. \\ \hline 211.785 \\ 847.14 \\ \hline 1058.925 \end{array}$$

On your paper write the factors given in A and B above. Do the multiplications yourself and compare your answers with those given.

Oral Tell where to place the decimal point in the replacement for n. Reminder: The number of decimal places in the product must be the same as the number of decimal places in the decimal factor.

	a	*b*
1.	6.3 $3\times2.1=n$	63.0 $n=30\times2.1$
2.	9.0 $2\times4.5=n$	90.0 $n=20\times4.5$
3.	8.48 $4\times2.12=n$	84.80 $n=40\times2.12$
4.	8.048 $4\times2.012=n$	6.025 $5\times1.205=n$

Written Solve for n in rows 1 through 4 above. Then copy and find each product in rows 5 through 9.

	a	*b*	*c*
5.	$\begin{array}{r} 3.4 \\ \times 12. \\ \hline 40.8 \end{array}$	$\begin{array}{r} 2.3 \\ \times 31. \\ \hline 71.3 \end{array}$	$\begin{array}{r} 3.2 \\ \times 22. \\ \hline 70.4 \end{array}$
6.	$\begin{array}{r} 6.3 \\ \times 15. \\ \hline 94.5 \end{array}$	$\begin{array}{r} 7.5 \\ \times 34. \\ \hline 255.0 \end{array}$	$\begin{array}{r} 6.8 \\ \times 36. \\ \hline 244.8 \end{array}$
7.	$\begin{array}{r} .96 \\ \times 8. \\ \hline 7.68 \end{array}$	$\begin{array}{r} 4.44 \\ \times 2. \\ \hline 8.88 \end{array}$	$\begin{array}{r} 5.22 \\ \times 3. \\ \hline 15.66 \end{array}$
8.	$\begin{array}{r} 8.94 \\ \times 2. \\ \hline 17.88 \end{array}$	$\begin{array}{r} 1.25 \\ \times 2. \\ \hline 2.50 \end{array}$	$\begin{array}{r} 9.27 \\ \times 2. \\ \hline 18.54 \end{array}$
9.	$\begin{array}{r} .356 \\ \times 95. \\ \hline 33.820 \end{array}$	$\begin{array}{r} 6.008 \\ \times 94. \\ \hline 564.752 \end{array}$	$\begin{array}{r} 9.037 \\ \times 25. \\ \hline 225.925 \end{array}$

Solving Problems

Read each problem carefully at least twice. Then translate each problem into an equation. Make an estimate and record it. Estimates may vary.

1. An airplane traveled at an average speed of 475.645 miles per hour for 5 hours. How far did it travel in the 5 hours?
5x475.645=n 2378.225 mi.

2. The average rainfall per year in Bob's state is 15.24 inches. Bob is 12 years old. About how many inches of rain have fallen in Bob's state during his lifetime?
15.24x12=n 182.88 in.

3. Carolyn spends $.20 a day for bus fare and $.25 a day for lunch. In 5 days how much will she spend for both?
5x(.20+.25)=n $2.25

4. Sally's father earns $480 per month. He pays $\frac{1}{5}$ of it for rent. How much rent does he pay each month?
$\frac{1}{5}$x480=n $96

5. A three-pound box of candy costs $1.98. A one-pound box of the same candy costs $.75. How much per pound would you save by getting the three-pound box rather than 3 one-pound boxes?
(3x.75)-1.98=n $.27

6. Sally walks to a school .7 of a mile from her house. She comes home for lunch each day. How many miles will she walk to and from school in 5 days?
5x(4x.7)=n 14 mi.

7. A gallon of water weighs 8.33 lb. A gallon of milk weighs 8.6 lb. How much more will 25 gallons of milk weigh than 25 gallons of water?
(25x8.6)-(25x8.33)=n 6.75 lb.

8. Tom ran 100 yd. in 12.9 seconds. How much slower was this than the Jefferson school record of 11.8 seconds?
12.9-11.8=n 1.1 sec.

9. A sheet of thin paper is .002 in. thick. How thick would a stack of 75 sheets of the paper be?
75x.002=n .150 in.

10. If the average monthly rainfall in a city is 2.19 in., how much rain can be expected in a year?
12x2.19=n 26.28 in.

Oral Tell your estimated answers for problems 1 through 10.

Written Solve your equation for each problem on this page. Compare your estimated and computed answers. Read each problem again to determine if your answer is reasonable. Write your answer.

A quick review

	a	*b*
1.	$8+(9\times\frac{9}{10})=r$ $16\frac{1}{10}$	$3\frac{13}{16}$ $p-(\frac{9}{16}\div\frac{3}{16})=\frac{13}{16}$
2.	135 $t=\frac{1}{2}\times4\frac{1}{2}\times60$	5 $n=17-(6\div\frac{1}{2})$
3.	12.54 $m=(9\times.06)+12$	49 $w=(236\div4)-10$
4.	24 $(n+12)\div6=6$	20 $r=(7\frac{1}{8}\times0)+20$

Multiplying a Whole Number by a Decimal Number

A repair crew expected to use 3 tons of asphalt to repair a mile of highway. How many tons of asphalt should the crew use to repair .1 of a mile of highway?

If you let t represent tons, you could translate the problem into the equation $t=.1\times 3$. Study the two solutions of the equation. Which solution seems easier to you?

$$t=\frac{1}{10}\times 3 \qquad\qquad t=.1\times 3$$
$$=\frac{1\times 3}{10} \qquad\qquad =3\times .1$$
$$=\frac{3}{10}\text{ or }.3 \qquad\qquad =.3$$

The men should use .3 of a ton of asphalt.

Do you suppose it is possible to develop a method of multiplying a whole number by a decimal number without changing to fractional form each time? Should it be possible to multiply by a decimal number without using the commutative property of multiplication? Before trying to answer these questions, look for a pattern in the multiplications below. Hint: Notice especially the number of zeros in the factor in fractional form and in the product.

$$\frac{1}{10}\times 3=\frac{3}{10}=.3 \qquad \frac{1}{10}\times 30=\frac{30}{10}=3.0$$
$$\frac{1}{100}\times 7=\frac{7}{100}=.07 \qquad \frac{1}{100}\times 70=\frac{70}{100}=.70$$
$$\frac{1}{1000}\times 9=\frac{9}{1000}=.009 \qquad \frac{1}{1000}\times 90=\frac{90}{1000}=.090$$
$$\frac{1}{10000}\times 5=\frac{5}{10000}=.0005 \qquad \frac{1}{10000}\times 50=\frac{50}{10000}=.0050$$

Did you discover the pattern summarized below?

Multiplier	Number of decimal places in the product
$\frac{1}{10}$	1
$\frac{1}{100}$	2
$\frac{1}{1000}$	3
$\frac{1}{10000}$	4

Do you see that the number of decimal places in the product is the same as the number of zeros in the multiplier?

See if you can find a pattern to help you determine the number of decimal places in the product as you study the multiplication below.

$$.2\times3=\frac{2}{10}\times3=\frac{6}{10}=\frac{6\times1}{10}=6\times\frac{1}{10}=.6$$

$$.03\times7=\frac{3}{100}\times7=\frac{21}{100}=\frac{21\times1}{100}$$
$$=21\times\frac{1}{100}=.21$$

$$.007\times9=\frac{7}{1000}\times9=\frac{63}{1000}=\frac{63\times1}{1000}$$
$$=63\times\frac{1}{1000}=.063$$

Is the number of decimal places in the product and in the decimal factor the same?

Oral Solve the following equations without using pencil and paper. Tell how you decided upon the number of decimal places in the product.

	a			b
1.	.3×2=n	.6	2.0	n=.5×4
2.	.5×6=n	3.0	4.8	n=.8×6
3.	.2×15=n	3.0	10.8	n=.6×18
4.	.03×3=n	.09	.12	n=.02×6
5.	.03×25=n	.75	45.00	n=.15×300
6.	.002×6=n	.012	.125	n=.005×25
7.	.004×50=n	.200	.180	n=.009×20
8.	.003×63=n	.189	.370	n=.010×37

Written Copy. Find each product. In partial products write the numerals in a way that keeps the decimal points in a vertical line.

	a	b	c
1.	4. ×.2 .8	6. ×.5 3.0	8. ×.9 7.2
2.	7. ×.16 1.12	9. ×.23 2.07	2. ×.008 .016
3.	8. ×.036 .288	32. ×.24 7.68	18. ×.6 10.8
4.	69. ×.08 5.52	125. ×.06 7.50	48. ×.015 .720
5.	112. ×.009 1.008	356. ×.015 5.340	527. ×.362 190.774

Translate each problem into an equation. Then solve each problem.

6. Mr. Kremer sold .75 of his 15 acres of land. How many acres of land did he sell?
.75x15=n   11.25 acres

7. Bob lives .6 of a mile from school. He makes the trip 4 times each day. At least how far must he walk in a school week of 5 days?
5x(4x.6)=n   12.0 mi.

8. A new highway cost about $875,000 a mile. How much would .8 of a mile of the highway cost?
.8x875000=n $700,000

9. Jack was taking a 3.7-mile hike. After hiking 1.8 mile, how much farther did he still have to hike?
3.7-1.8=n   1.9 mi.

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A $.3 \times .2 = \frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = \frac{6 \times 1}{100} = 6 \times \frac{1}{100} = .06$

B $.15 \times .3 = \frac{15}{100} \times \frac{3}{10} = \frac{45}{1000} = \frac{45 \times 1}{1000} = 45 \times \frac{1}{1000} = .045$

C $.24 \times .03 = \frac{24}{100} \times \frac{3}{100} = \frac{72}{10000} = \frac{72 \times 1}{10000} = 72 \times \frac{1}{10000} = .0072$

Using Two Decimal Factors

In the computations on the board both factors are decimal numbers. Study the computations. Notice especially the digits printed in blue. Then count the digits in both of the fractional factors. In A there are 2 decimal places in the product. How many decimal places are in both factors together? Look at B. Are there 3 decimal places in the product and also in both factors together? How many decimal places are in the product and in both factors together in C?

Knowing that the number of decimal places in the product is the same as the number of decimal places in both factors together is very helpful. You multiply the decimal numbers as you multiply whole numbers, and provide as many decimal places in the product as in both of the factors.

Suppose you were to multiply 3.125 and .25. How many decimal places are in both factors together? There are 5. Then there should be 5 decimal places in the product.

Study the multiplication below. Observe the position of the numerals and decimal points in the partial products and in the product.

$$\begin{array}{r} 3.125 \\ \times .25 \\ \hline .15625 \\ .6250 \\ \hline .78125 \end{array}$$

Oral Tell the steps you would take in multiplying 4.622 and .37 in decimal form. See column 1.

Written Copy. Write each product.

	a	*b*	*c*	*d*
1.	.7 ×.06 .042	1.6 ×.08 .128	2.9 ×.07 .203	.04 ×.02 .0008
2.	.05 ×.08 .0040	.12 ×.03 .0036	.17 ×.06 .0102	1.12 ×.04 .0448
3.	2.37 ×.02 .0474	3.91 ×.08 .3128	4.78 ×.65 3.1070	.003 ×.2 .0006
4.	.031 ×.03 .00093	.125 ×.05 .00625	.375 ×.87 .32625	4.224 ×.02 .08448

Rounding Off Decimals

John is attempting to round off \$1.17 to the nearer 10 cents. He must decide whether the answer is \$1.20 or \$1.10. Read his sentences. Is \$1.17 closer in value to \$1.20 or to \$1.10? To the nearer 10 cents, \$1.17 is rounded to \$1.20.

Try rounding off \$1.17 to the nearer dollar as shown.

\$1.17 < \$2.00 by \$.83

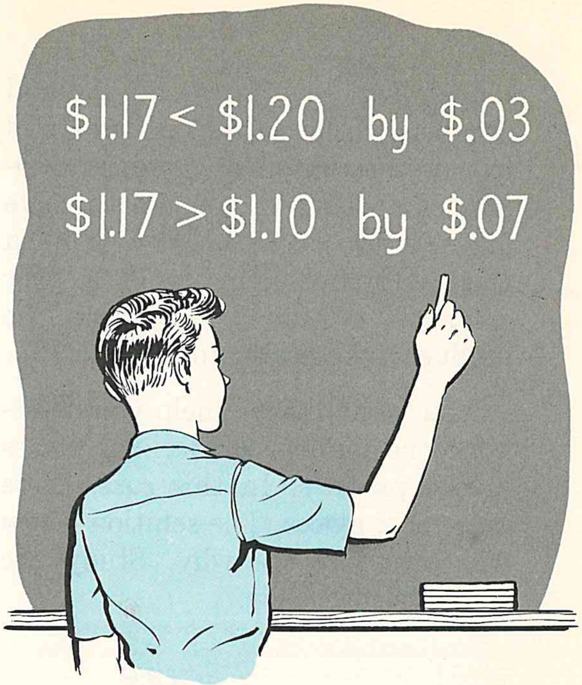
\$1.17 > \$1.00 by \$.17

Since \$1.17 is closer in value to 1 dollar than to 2 dollars, \$1.17 may be rounded off to 1 dollar.

Suppose you were to round off \$3.50 to the nearer dollar. Of course \$3.50 is no closer in value to \$4.00 than to \$3.00. In cases like this you round off "to make even." This means that you make the digit preceding the digit being rounded off an even digit, as 2, 4, 6, 8, ... Since the 3 in \$3.50 is odd, round off \$3.50 to \$4.00 rather than to \$3.00.

Suppose you were rounding \$4.50 to the nearer dollar. You could choose \$4 or \$5. Since 4 is already even, round \$4.50 to \$4.00.

If you can round off such numbers as \$1.17, and \$3.50, you can round off 1.17 and 3.50. Why?



Write sentences like those on the board to help you round off 3.6 to the nearer one, and 3.783 to the nearer hundredths, tenths, and one.

Oral Show the class the sentences you wrote. Explain each sentence.

Written

1. Round off 3.2, 9.6, 8.5, and 3.5 to the nearer one. 3.0; 10.0; 8.0; 4.0

2. Round off 6.53, 9.35, 10.50, and 11.50 to the nearer tenth and to the nearer one. See below.

3. Round off 9.007, 5.899, 7.353, and 8.639 to the nearer one.
9, 6, 7, 9

2. 6.5, 9.4, 10.5, 11.5,
7, 9, 10, 12

Multiplying Decimal Numbers

Knowing how to round decimal numbers to the nearer one is helpful to you in estimating a product. Suppose, for example, you are to solve for n in n = 9.3×8.6. Why is 81 a good estimate? Why is 72 a poor estimate? Would you expect the product to be nearer to 81 or to 72?

Your estimate will help you determine the number of decimal places in the product. In this case, there must be 2 places. The solution below will help you see why. Study the computation below.

$$\begin{array}{lr} & 9.3 \\ & \times 8.6 \\ \hline \frac{6}{10}\times\frac{3}{10}=\frac{18}{100}= & .1\,8 \\ \frac{6}{10}\times 9=\frac{54}{10}= & 5.4 \\ 8\times\frac{3}{10}=\frac{24}{10}= & 2.4 \\ 8\times 9=72= & 72. \\ \hline & 79.98 \end{array}$$

Observe that the decimal points in the partial products are on a straight vertical line. Where is the decimal point in the product?

Now study the solution of the equation n = 3.85×42.9 shown below.

$$\begin{array}{lr} & 42.9 \\ & \times 3.85 \\ \hline \frac{1}{100}\times\frac{1}{10}=\frac{1}{1000}= & 2.145 \\ \frac{1}{10}\times\frac{1}{10}=\frac{1}{100}= & 34.32 \\ 1\times\frac{1}{10}=\frac{1}{10}= & 128.7 \\ \hline & 165.165 \end{array}$$

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Is the number of decimal places in the product the same as the number of decimal places in both of the factors together?

Oral Make two whole-number estimates for n in rows 1 through 6. One estimate should be a number larger and the other a number smaller than the exact value of n.

	a	*b*
1.	4.00 1.6×2.5 = n	35.52 n = 4.8×7.4
2.	10.44 3.6×2.9 = n	10.488 n = 8.74×1.2
3.	41.552 7.42×5.6 = n	3.3642 n = 2.67×1.26
4.	18.5935 9.07×2.05 = n	77.7750 n = 1.22×63.75
5.	187.4495 76.51×2.45 = n	5.2500 n = 1.25×4.20
6.	5.3808 3.54×1.52 = n	67.9112 n = 7.24×9.38

Written Record an estimate before solving for n in rows 1 through 6 above. Compare your estimated and computed answers. Write your answers. Then solve problems 7 and 8.

7. At 16.7 miles per gallon how far can a car be driven on 12.5 gallons of gasoline?

12.5x16.7=n   208.75 mi.

8. Jane's mother bought a $5\frac{1}{2}$-pound roast that cost $.84 a pound. How much change should she get from $10.00?

10.00-($5\frac{1}{2}$x.84)=n $5.38

Problem Solving

Read each problem carefully at least twice. Then translate each problem into an equation. Make an estimate and record it. Keep your paper for reference. Estimates may vary.

1. Find the cost of 14.9 gallons of gasoline at 36.2¢ per gallon. (Write 36.2¢ as .362.)

14.9x.362=n $5.39

2. When John's father filled his gas tank, the speedometer read 17832.9. He filled the tank again when the speedometer read 18072.0. How far had he driven?

18072.0-17832.9=n 239.1 mi.

3. A 10-ride bus ticket costs $4. Find the cost per ride.

4.00÷10=n $.40

4. Jim had a board 4 feet $3\frac{1}{2}$ inches long. He needed a piece 4 feet $6\frac{3}{8}$ inches long. How much too short was the board which he had?

See below.

5. Helen's father bought 15 gal. of gas at 35.9¢ per gal. and 1 qt. of oil at 65¢. What was the total of these purchases?

(15x.359)+.65=n $6.04

6. Sally's mother paid 85¢ for a gallon of milk. What was the price per quart?

.85÷4=n $.21$\frac{1}{4}$

7. At the beginning of a trip the speedometer on Mr. Johnson's car read 10976.3 miles. At the end of the trip it read 12531.8 miles. How far did he drive on the trip?

12531.8-10976.3=n 1555.5 mi.

8. At one stop Mr. Johnson bought 9.5 gal. of gas. At 37.9¢ a gal. how much did the gas cost?

9.5x.379=n $3.60

9. Mr. Johnson's car gets an average of 16.9 miles per gallon of gasoline. At this rate how many miles can he drive on 9.5 gallons?

9.5x16.9=n 160.55 mi.

10. Dick's father can get a 5-gallon can of oil for $4.84. What is the price per quart?

4.84÷(5x4)=n $.242

11. Bob's uncle sold 75 young chicks for $48.75. What was the average price per chick?

48.75÷75=n $.65

Oral Tell your estimated answer for problems 1 through 11.

Written Solve your equation for each problem on this page. Compare your estimated and computed answers. Read each problem again to see if your answer is reasonable. Write your answer.

Can you do this? Reconstruct the multiplications below by replacing each dot with a numeral. There is only one possible solution for each.

a

$$\begin{array}{r} 37 \\ \times 2\bullet \\ \hline 2\bullet 6 \\ 7\bullet \\ \hline \bullet\bullet\bullet 6 \end{array} \qquad \begin{array}{r} 37 \\ \text{x}28 \\ \hline 296 \\ 74 \\ \hline 1036 \end{array}$$

b

$$\begin{array}{r} \bullet 56 \\ \times 3\bullet \\ \hline 17\bullet 0 \\ \bullet\bullet\bullet\bullet \\ \hline \bullet\bullet\bullet\bullet\bullet \end{array} \qquad \begin{array}{r} 356 \\ \text{x}35 \\ \hline 1780 \\ 1068 \\ \hline 12460 \end{array}$$

4. $[(4\text{x}12)+6\frac{3}{8}]-[(4\text{x}12)+3\frac{1}{2}]=n$ $2\frac{7}{8}$ in.

Multiplying by 10, 100, or 1000

Study each multiplication below. Which factor is the same in each one? Do both digits of this factor also appear in the product?

7.5	7.5	7.5
×10	×100	×1000
75.0	750.0	7500.0

How is the place-value position of the 7 and 5 changed when you multiply 7.5 by 10? By 100? By 1000?

Take other numbers expressed in tenths. Multiply them by 10, 100, and by 1000. As you look at each product, do you always see the digits of the number you chose? How has the place-value position of the digits been changed?

Now study these multiplications.

.75	.75	.75
×10	×100	×1000
7.50	75.00	750.00

How is the place-value position of the 7 and 5 changed when you multiply .75 by 10? By 100? By 1000?

Try multiplying other decimals by 10, 100, and 1000. Convince yourself that the same digits appear in the product as in the number you chose as the other factor.

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By now you should have discovered a pattern to help you decide upon the place-value position of each digit in the product when you multiply by 10, 100, or 1000. The number of zeros in 10, in 100, and in 1000 helps you make your decision. Why must you sometimes affix zeros in the product?

Oral Read. Supply each product. Tell how you decided upon it.

	a	*b*
1.	10×.8 = n 8.0	100×.8 = n 80.0
2.	1000×.8 = n 800.0	10×.065 = n .650
3.	100×.065 = n 6.500	1000×.065 = n 65.000
4.	10×9.5 = n 95.0	100×.369 = n 36.900
5.	1000×1.2 = n 1200.0	1000×2.5 = n 2500.0

Written Solve for n in rows 1 through 5. Perform the indicated operation in rows 6 through 9.

	a	*b*	*c*	*d*
6.	.08 ×10 .80	.08 ×100 8.00	.08 ×1000 80.00	.008 ×100 .800
7.	2.4 ×10 24.0	.24 ×100 24.00	.024 ×1000 24.000	3.68 ×100 368.00
8.	.368 ×10 3.680	5 ×100 500	1.7 ×100 170.0	7.641 ×1000 7641.000
9.	2.55 ×10 25.50	8.2 ×100 820.0	.27 ×100 27.00	37.5 ×1000 37500.0

Review and Practice

You should be able to do the work on this page without any help.

Part 1 Copy. Perform the indicated operation. Check your work.

	a	*b*	*c*
1.	16r33 54)897	20r376 435)9076	12r102 761)9234
2.	579.4 286.3 529.2 +347.7 1742.6	\$24.36 86.94 57.28 +49.17 \$217.75	\$3.47 2.99 8.63 +5.52 \$20.61
3.	896.54 −328.75 567.79	\$9836.42 −7947.58 \$1888.84	\$500.00 −437.29 \$62.71
4.	$36\frac{0}{6}$ $-21\frac{5}{6}$ $14\frac{1}{6}$	$39\frac{2}{5}$ $+27\frac{5}{6}$ $67\frac{7}{30}$	$87\frac{1}{2}$ $-27\frac{7}{10}$ $59\frac{4}{5}$
5.	$32\frac{3}{4}$ $\times 12\frac{1}{2}$ $409\frac{3}{8}$	$87\frac{5}{6}$ $\times 29\frac{2}{3}$ $2605\frac{13}{18}$	$34\frac{1}{4}$ $\times 38\frac{4}{5}$ $1328\frac{9}{10}$
6.	473.21 ×.25 118.3025	947.075 ×1.25 1183.84375	600.96 ×1000 600960.00

Part 2 Solve for n.

	a		*b*	
1.	$4\frac{1}{2}\times 5\frac{2}{3}=n$	$25\frac{1}{2}$	$n=9\frac{5}{8}\times 10\frac{2}{5}$	$100\frac{1}{10}$
2.	$6\frac{3}{4}\times 7\frac{5}{6}=n$	$52\frac{7}{8}$	$n=17\frac{3}{8}\times 15\frac{1}{5}$	$264\frac{1}{10}$
3.	$384\times 7.46=n$	2864.64	$n=47.3\times 29.8$	1409.54
4.	$9.58\times 9.32=n$	89.2856	$n=36.7\times 53.1$	1948.77
5.	$24\frac{1}{2}\div 6\frac{3}{4}=n$	$3\frac{17}{27}$	$n=72\frac{3}{4}\div 13\frac{1}{8}$	$5\frac{19}{35}$

Part 3 Solve for n.

	a		*b*	
1.	$(27+3)\div 10=n$	3	$(81\div 9)\times n=27$	3
2.	$(86+14)\div 25=n$	4	$(56-29)\div 3=n$	9
3.	$(144\div 12)\times n=24$	2	$(56\div 8)\times n=49$	7
4.	$72\div(9\times 12)=n$	$\frac{2}{3}$	$8\times(8\div n)=8$	8
5.	$(4\times 4)\div n=8$	2	$(10\times 9)\div 180=n$	$\frac{1}{2}$

Part 4 Solve each problem.

1. Ted's father, while on a business trip, had to rent a car for a day. He had to agree to pay \$10.00 plus 10¢ for each mile that he drove the car. If he drove 322.9 miles, how much rent did he have to pay when he turned in the car?

10.00+(322.9x.10)=n \$42.29

2. During the first week in August, Jerry earned \$5.65. He earned \$6.75 during the second week, \$9.35 during the third week, and \$7.95 in the fourth week. On the average, how much money did Jerry earn each week during that month?

(5.65+6.75+9.35+7.95)÷4=n $\$7.42\frac{1}{2}$

3. Steve rode 3.2 miles to Dan's house. From there he rode 2.4 miles to Larry's house. Then both boys rode 1.8 miles to the beach. Steve rode directly home from the beach, a distance of 4.9 miles. How far had he ridden altogether that day?

3.2+2.4+1.8+4.9=n 12.3 mi.

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. When adding and subtracting decimal numbers, write the numerals so as to keep the decimal points in a vertical line. (148)

2. You add and subtract tenths, hundredths, and thousandths in the same way as ones, tens, and hundreds. (148-149, 151, 152)

3. Arithmetic with decimal numbers is in full agreement with arithmetic with fractional numbers. (148, 152, 154)

4. There must be as many digits *after* the decimal point in the product as are after the decimal point in the factors together. (162)

Questions to Discuss

See T166 for answers.

1. How would you use fractions to prove that .2+.9=1.1? (147)

2. Which of the properties of addition of whole numbers apply to addition of decimal numbers? (150)

3. How would you use fractions to prove that .8−.3=.5? (152)

4. How would you use fractions to prove that 4×.3=1.2? (154)

5. What is the denominator of the answer when you multiply tenths by tenths? Hundredths by tenths? Thousandths by tenths? (158)

6. Between which two whole numbers will the exact product lie when you multiply 6.7 and 9.2? (162)

7. What short method can you use to help you place the decimal point correctly in a product? (162)

Written Practice

Perform the indicated operations.

	a	*b*	
1.	145.9 +32.5 178.4	460.87 +935.79 1396.66	(148, 149)
2.	7.932 +8.675 16.607	14.237 +35.929 50.166	(151)
3.	12.17 −6.39 5.78	18.357 −6.295 12.062	(152)
4.	.96 ×12 11.52	4.653 ×35 162.855	(155, 156)
5.	19. ×.08 1.52	3.17 ×.34 1.0778	(158, 159)
6.	5.75 ×.43 2.4725	6.12 ×.95 5.8140	(160)
7.	6.398 ×4.85 31.03030	9.435 ×7.95 75.00825	(162)

Self-Evaluation

Part 1 Copy. Perform the indicated operations.

	a	*b*	*c*
1.	60r21 31)1881	6 237)1422	100r13 326)32613
2.	88.5 −63.8 24.7	324.6 −13.3 311.3	2.142 −.970 1.172
3.	9.008 −.644 8.364	18.765 −12.777 5.988	54.976 −19.893 35.083
4.	.4 .8 +.6 1.8	2.0 .2 +.4 2.6	.8 2.6 +9.0 12.4
5.	2.012 8.42 7.21 +5.9 23.542	5.082 9.821 6.401 +9.765 31.069	17.890 9.861 8.943 +5.805 42.499
6.	983 ×.6 589.8	31.3 ×.03 .939	989 ×2.34 2314.26
7.	3.402 ×5.34 18.16668	195.6 ×3.79 741.324	84.560 ×3.33 281.58480

Part 2 In place of each dot use the sign =, >, or <, whichever makes the statement true.

1. $\frac{9}{10}+\frac{8}{10}+\frac{2}{10} \overset{=}{\bullet} .9+(.8+.2)$
2. $6.17-3.98 \overset{<}{\bullet} 6-3$
3. $3\times .5 \overset{=}{\bullet} .5+.5+.5$
4. $.07\times 35 \overset{=}{\bullet} 35\times .07$
5. $9.017\times 8.039 \overset{>}{\bullet} 72$
6. $3\times(1.096+2.975) \overset{>}{\bullet} 3$
7. $(2\times 5)\times .0096 \overset{=}{\bullet} .096$
8. $1000\times .12 \overset{=}{\bullet} 1000\times .120$
9. $(3\times .2)\times 6 \overset{=}{\bullet} 3\times(.2\times 6)$

Part 3 Solve these problems.

1. The average annual snowfall in one city is 71.10 in. In another city the average snowfall is .02 in. Find the difference.

71.10−.02=n 71.08 in.

2. A building lot is 55.7 ft. wide. The lot next to it is 69.9 ft. wide. What is the total width of the two lots together?

55.7+69.9=n 125.6 ft.

3. In three months Joe's father bought gasoline as follows: 35.9 gal., 41.8 gal., and 49.9 gal. Find the total number of gallons he bought.

35.9+41.8+49.9=n 127.6 gal.

4. Mr. Johnson owns 5 city lots. Each has a width of 51.5 feet. What is the total width of the 5 lots that Mr. Johnson owns?

5x51.5=n 257.5 ft.

5. The sixth graders sold 264 student tickets at $.25 each and 72 adult tickets at $.75 each. How much money did the sixth graders receive for all of the tickets?

(264x.25)+(72x.75)=n $120.00

Midyear Review

On your paper, write the answer to each exercise on these two pages. Then you will have answers to check during the class discussion. The numerals in ( ) refer to pages where you can turn for help. See T168 and T169 for answers.

Number Meanings

1. What is the place value of each digit in 9075.346? (6, 9)

2. How would you show the meaning of $\frac{3}{4}$ by using a number line? (12)

3. How do you rename $\frac{1}{2}$ as sixths and $\frac{6}{8}$ as fourths? (15, 16) $1 \times \frac{1}{2} =$ $\frac{3}{3} \times \frac{1}{2} = \frac{3}{6}$

NO **4.** How do you round a decimal number to the nearer one? (161)

Addition and Subtraction

1. Why is addition called a binary operation? (21)

2. Using whole numbers and then fractional numbers, give examples to show that addition has the commutative property and the associative property. (21, 22)

3. What steps would you take in solving a story problem? (26, 31)

4. Using whole numbers and then fractional numbers, give examples to show that subtraction is the inverse operation of addition. (27)

5. Explain a good way to check subtraction. (29)

6. Explain how to solve the equation $145 - 18 = n$ in two steps. (30)

Multiplication

1. Which of the properties of addition also apply to multiplication? (41, 45)

2. Give an example to show how you use the associative property of multiplication. (45)

3. Give an example to show how the commutative property of multiplication may save time when you are multiplying a whole number by a fractional number. (75)

4. What is the advantage of reducing fractions *before* multiplying fractional numbers? (86)

NO **5.** Give examples to show that the arithmetic used in multiplying fractional numbers is the same as the arithmetic used in multiplying decimal numbers. (158-160)

Division

1. Give examples to show that division is the inverse operation of multiplication. (58)

2. Give examples to show that any multiple of a number is divisible by that number. (60)

3. How do you find the average of five different numbers? (65)

4. What is a good way of determining a trial quotient and also its place-value position? (61, 66)

5. What steps do you take in dividing numbers in fractional form when the denominators are equal? When they are unequal? (94, 103)

Measures

1. What steps do you take in measuring something? (113)

2. How do direct measurement and indirect measurement differ? (113-114)

3. Give several examples of formulas used in converting from one measure to another. Be ready to explain how to use the formulas. (116)

4. How does the scale of a 24-hour clock differ from the scale of a clock you usually use? (121)

Problem Solving

1. What are some advantages of reading a problem several times? (131, 134, 135)

2. What are some symbols you might use in writing arithmetic sentences? (132)

3. What steps do you take in determining the operation to use in solving a problem? (133)

Written Practice

Copy. Find each answer as indicated by the operational sign.

	a		*b*	
1.	72975 +89386 162361	(25)	97026 −69385 27641	(29)
2.	$3\frac{2}{3}$ $+4\frac{2}{5}$ $8\frac{1}{15}$	(36)	$9\frac{5}{6}$ $-4\frac{9}{10}$ $4\frac{14}{15}$	(38)
3.	38750 ×675 26156250	(52)	$198\frac{176}{223}$ $892\overline{)177320}$	(69)
4.	95.438 +37.954 133.392	(151)	17.215 −9.638 7.577	(152)
5.	6.735 ×35 235.725	(156)	9.720 ×8.95 86.994	(162)

yes

NO

Solve the following equations. Replace each letter with a numeral to make each statement true.

6. $x = 3\frac{1}{2} \times 4\frac{2}{5}$ ~~$16\frac{2}{5}$~~ $15\frac{2}{5}$ (85)

7. $\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4} = n$ $\frac{1}{4}$ (88)

8. $\frac{7}{4} \div \frac{3}{4} = p$ $2\frac{1}{3}$ (93)

9. $12\frac{3}{4} \div 2\frac{1}{8} = m$ 6 (105)

10. $975 - (212 + 419) = c$ 344 (138)

11. $(290 - 87) \times 35 = n$ 7105 (139)

12. $(18\frac{1}{2} \times 20) \div 5 = d$ 74 (140)

yes

Midyear Tests

Test 1 Copy. Replace each letter with the numeral that makes each statement true.

	a	*b*
1.	$93200-9758=\overset{83442}{n}$	$\overset{15}{d}=(0\div39)+15$
2.	$\overset{44}{n}=9+18+12+5$	$\overset{48}{w}=2\times(9+15)$
3.	$\overset{.945}{b}=.9\times(.75+.3)$	$8\frac{9}{16}-2\frac{5}{8}=\overset{5\frac{15}{16}}{n}$
4.	$17530+987=\overset{18517}{s}$	$(95+0)\times1=\overset{95}{r}$
5.	$(.45+.12)\times3=\overset{1.71}{n}$	$(8\frac{1}{2}\div2\frac{1}{4})+9=\overset{12\frac{7}{9}}{r}$
6.	$8+(1\frac{3}{5}+2\frac{2}{3})=\overset{12\frac{4}{15}}{n}$	$\overset{87}{t}=96-(12\times\frac{3}{4})$
7.	$\overset{.049}{x}=.7\times(.97-.9)$	$3\frac{1}{2}+9\frac{3}{4}=\overset{13\frac{1}{4}}{n}$
8.	$(.7-.3)\times.8=\overset{.32}{f}$	$(.9-.3)+9=\overset{9.6}{a}$

Test 2 Copy. Replace each dot with =, <, or >, whichever makes the statement true.

1. $.0815\times8 \overset{=}{\bullet} 8\times.0815$

2. $18+19+82 \overset{=}{\bullet} (18+82)+19$

3. $.076\times.435 \overset{<}{\bullet} .76\times.435$

4. 315 in. $\overset{=}{\bullet}$ 26 ft. 3 in.

5. 12.7 mi. $\overset{>}{\bullet}$ 6700 ft.

6. 575 sec. $\overset{=}{\bullet}$ 9 min. 35 sec.

7. 25 lb. 8 oz. $\overset{>}{\bullet}$ 400 oz.

8. 15.75 mi. $\overset{>}{\bullet}$ 80,000 ft.

Test 3 Copy. Find each answer as indicated by the operational sign.

	a	*b*
1.	435 47)20445	3146r57 298)937565
2.	319.747 917.536 +389.724 1627.007	4910.57 8310.47 +4763.29 17984.33
3.	14.097 −7.326 6.771	354.213 −193.860 160.353
4.	4.325 ×75 324.375	19.706 ×.83 16.35598
5.	9.112 ×4.98 45.37776	16.235 ×8.05 130.69175

Test 4 Solve each problem below.

1. Mr. Taylor was making a 700-mile trip. Before lunch he drove for $5\frac{1}{2}$ hours at an average speed of 42 miles an hour. How many more miles did he have to drive after lunch?

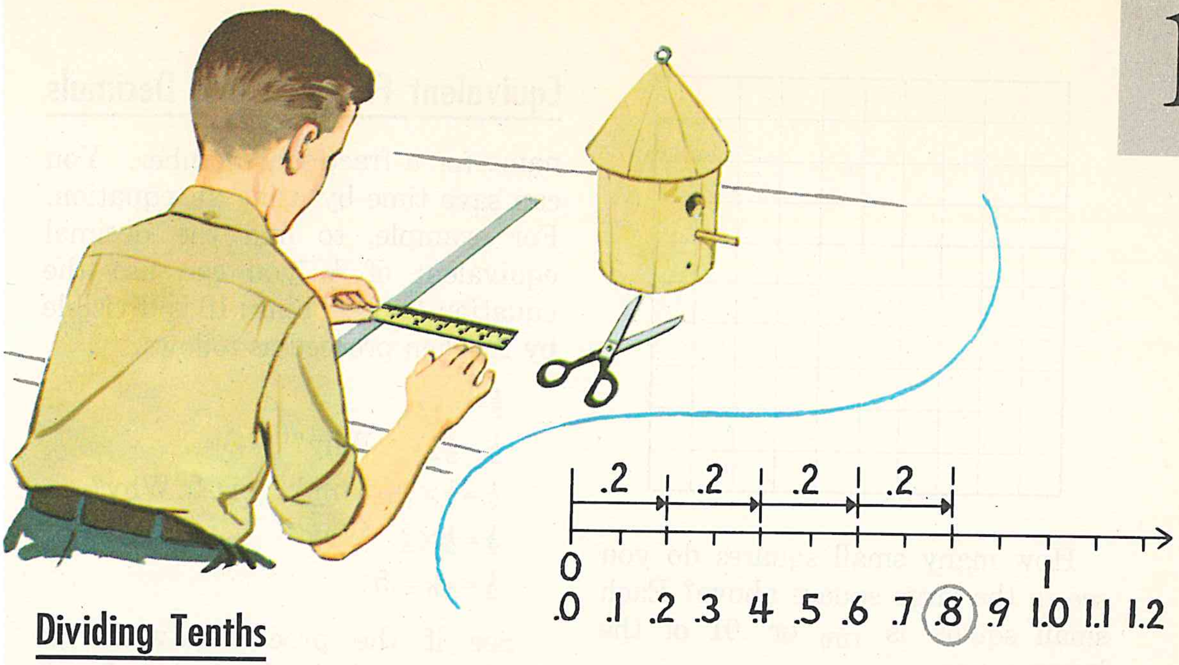
$700-(5\frac{1}{2}x42)=n$ 469 mi.

2. John bought one record for $1.29 and another for $2.98. The tax was $.17. How much money must he pay for his purchase?

$1.29+2.98+.17=n$ $4.44

3. Janet had a new spool of ribbon. From it she cut 8 pieces each $4\frac{1}{3}$ yards long and had $1\frac{1}{3}$ yards of ribbon left. How many yards of ribbon did the spool hold?

$(8x4\frac{1}{3})+1\frac{1}{3}=n$ 36 yd.



Dividing Tenths

How many strips, each .2 of an inch wide, can the boy cut from a strip .8 of an inch wide? You can find the answer by setting up the equation $.8 \div .2 = n$. Since division is the inverse of multiplication, the equation can be solved as $.8 = n \times .2$. One way to solve for n is to use a number line. On the number line above, how many segments .2 of an inch in length are contained in a segment .8 of an inch in length?

Does $.8 \div .2 = 4$? Does $.8 = 4 \times .2$?

The boy can cut 4 strips.

Suppose now that you translated the equation $.8 \div .2 = n$ into the fractional form $\frac{8}{10} \div \frac{2}{10} = n$. You could solve for n as shown below.

$$\frac{8}{10} \div \frac{2}{10} = \frac{8 \div 2}{10 \div 10} = \frac{4}{1} = 4$$

Why is 1 the denominator of the quotient? Could you find a replacement for n in $\frac{8}{10} \div \frac{2}{10} = n$ and in $.8 \div .2 = n$ by dividing 8 and 2 as though they were whole numbers?

Oral Tell the reasons for the steps you would take in solving for n in the equations below. Solve for n. Check by using a number line.

See column 1.

	a	*b*
1.	$\frac{9}{10} \div \frac{3}{10} = n$ 3	$.9 \div .3 = n$ 3
2.	$\frac{6}{10} \div \frac{2}{10} = n$ 3	$.6 \div .2 = n$ 3
3.	$\frac{16}{10} \div \frac{8}{10} = n$ 2	$1.6 \div .8 = n$ 2
4.	2 $n \times .2 = .4$	$.4 \div .2 = n$ 2
5.	3 $n \times .3 = .9$	$.9 \div .3 = n$ 3
6.	2 $n \times .4 = .8$	$.8 \div .4 = n$ 2

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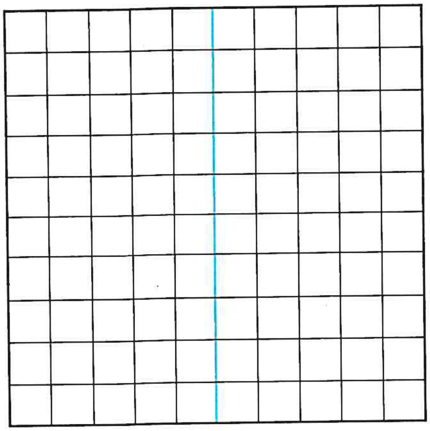
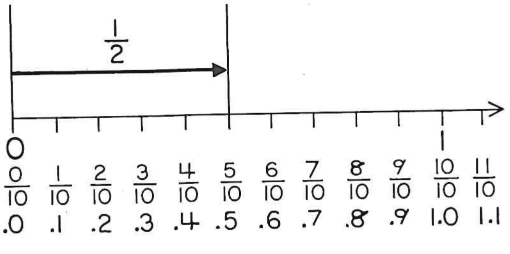


How many small squares do you see in the large square above? Each small square is $\frac{1}{100}$ or .01 of the larger square.

The blue line separates the large square into halves. How many hundredths do you see in each $\frac{1}{2}$?

$\frac{1}{2}=\frac{50}{100}=.50$ $\qquad$ $\frac{2}{2}=\frac{100}{100}=1.0$

Suppose you are to express $\frac{1}{2}$ as a decimal and do not wish to use graph paper. You could use a number line to help you. On the number line below do you see that $\frac{1}{2}=\frac{5}{10}=.5$?



$\frac{1}{2}$
0
$\frac{0}{10}$ $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ $\frac{10}{10}$ $\frac{11}{10}$
.0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1

Graph paper and number lines are awkward to use in finding a decimal name for a fractional number. You can save time by using an equation. For example, to find the decimal equivalent of $\frac{1}{2}$, you can use the equation $\frac{1}{2}=\frac{n}{10}$. Think: 10 is divisible by 2. Then proceed as follows.

$\frac{1}{2}=\frac{n}{2\times5}$

$\frac{1}{2}=\frac{1\times n}{2\times5}$ Why?

$\frac{1}{2}=\frac{1}{2}\times\frac{n}{5}$ Think: $\frac{n}{5}$ is 1. Why?

$\frac{1}{2}=\frac{1}{2}\times\frac{5}{5}$ Why?

$\frac{1}{2}=\frac{5}{10}=.5$

See if the procedure works in finding the decimal number for $\frac{1}{4}$. Study the equations below. Which would you use? Why?

$\frac{1}{4}=\frac{n}{10}$ $\qquad$ $\frac{1}{4}=\frac{n}{100}$

Is 10 divisible by 4? Is 100 divisible by 4? Since 100 is divisible by 4, work with the equation $\frac{1}{4}=\frac{n}{100}$. You rename $\frac{n}{100}$ as $\frac{n}{4\times25}$ and proceed as follows.

$\frac{1}{4}=\frac{n}{4\times25}$

$\frac{1}{4}=\frac{1\times n}{4\times25}$ Why?

$\frac{1}{4}=\frac{1}{4}\times\frac{n}{25}$ Why?

$\frac{1}{4}=\frac{1}{4}\times\frac{25}{25}$ Why?

$\frac{1}{4}=\frac{25}{100}=.25$

Suppose you wish to find the decimal number for $\frac{3}{4}$. What might you do? Study the solution below.

$\frac{3}{4}=3\times\frac{1}{4}=3\times.25=.75$

Now begin with $\frac{1}{8}$. Since 1000 is divisible by 8, use the equation $\frac{1}{8}=\frac{n}{1000}$. You can rename $\frac{n}{1000}$ as $\frac{n}{8\times 125}$ and proceed as follows.

$\frac{1}{8}=\frac{n}{8\times 125}$

$\frac{1}{8}=\frac{1\times n}{8\times 125}$ Why?

$\frac{1}{8}=\frac{1}{8}\times\frac{n}{125}$ Why?

$\frac{1}{8}=\frac{1}{8}\times\frac{125}{125}$ Why?

$\frac{1}{8}=\frac{125}{1000}=.125$

Suppose that you wanted to find the decimal equivalent of $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, and so on. You could use the procedure shown below.

$\frac{2}{8}=2\times\frac{1}{8}=2\times.125=.250$

$\frac{3}{8}=3\times\frac{1}{8}=3\times.125=.375$

$\frac{4}{8}=4\times\frac{1}{8}=4\times.125=.500$

Oral Answer each question below.
See page 173 and above.

1. What steps would you take in finding the decimal equivalent of $\frac{5}{8}$? Of $\frac{6}{8}$? Of $\frac{7}{8}$?

2. What is the reason for the following steps taken in finding the decimal equivalent of $\frac{1}{20}$? Of $\frac{7}{20}$?

$\frac{1}{20}=\frac{n}{100}$

$\frac{1}{20}=\frac{1\times n}{20\times 5}$ or $\frac{1}{20}\times\frac{n}{5}$ or $\frac{1}{20}\times\frac{5}{5}$

$\frac{1}{20}=\frac{5}{100}=.05$

$\frac{7}{20}=7\times\frac{1}{20}=7\times.05=.35$

3. What steps would you take in finding the decimal equivalent of $\frac{8}{20}$? Of $\frac{9}{20}$?

Written Use the equation form to find the decimal equivalent for each fraction below.

	a	*b*	*c*	*d*	*e*
1.	$\frac{10}{20}$ .5	$\frac{2}{25}$ .08	$\frac{3}{4}$ .75	$\frac{2}{10}$ .2	$\frac{5}{10}$ .5
2.	$\frac{3}{10}$ .3	$\frac{1}{4}$ .25	$\frac{6}{10}$ .6	$\frac{5}{25}$ .20	$\frac{3}{5}$ .6
3.	$\frac{4}{50}$ .08	$\frac{2}{5}$ .4	$\frac{4}{10}$ .4	$\frac{15}{20}$ .75	$\frac{6}{50}$ .12
4.	$\frac{5}{50}$ .10	$\frac{1}{20}$ .05	$\frac{8}{10}$ .8	$\frac{7}{25}$ .28	$\frac{7}{10}$ .7
5.	$\frac{9}{10}$ .9	$\frac{1}{5}$ .2	$\frac{9}{25}$ .36	$\frac{1}{125}$ .008	$\frac{11}{50}$ .22
6.	$\frac{3}{25}$ .12	$\frac{4}{20}$ .20	$\frac{7}{50}$ .14	$\frac{200}{500}$ .400	$\frac{19}{200}$ .095
7.	$\frac{3}{125}$ .024	$\frac{2}{4}$ .50	$\frac{5}{125}$ .040	$\frac{3}{20}$ .15	$\frac{2}{125}$ .016
8.	$\frac{8}{25}$ .32	$\frac{12}{50}$ .24	$\frac{35}{500}$ .070	$\frac{350}{500}$ .700	$\frac{4}{25}$ .16
9.	$\frac{6}{20}$ .30	$\frac{4}{125}$ .032	$\frac{5}{20}$ .25	$\frac{3}{200}$ .015	$\frac{4}{125}$ .032
10.	$\frac{9}{50}$ .18	$\frac{2}{20}$ .10	$\frac{6}{25}$ .24	$\frac{4}{5}$ .8	$\frac{8}{50}$ .16

A quick review

59.22
1. $.9\times(34.5+12.6+18.7)=n$

648.02350
2. $(435.176-298.75)\times 4.75=b$

196.71605170
3. $w=.98\times(4.007\times 50.095)$

4.54506250
4. $x=.25\times(3.005\times 6.050)$

4.885
5. $r=50.015-(35.060+10.07)$

4655
6. $5280=n+625$

594
7. $w+1956=2550$

$160\frac{2}{3}$
8. $(6\frac{1}{2}\times 5\frac{1}{3})+126=a$

12.99
9. $c=.47+3.58+6.19+2.75$

The equation method is a good one to use in finding the decimal equivalent for $\frac{7}{20}$. However, there is a still faster way of doing this. In this faster way you skip the fractional form entirely.

Recall that $\frac{7}{20}$ means $20\overline{)7}$. In this form, division is not possible, but if you rename 7 as 700 hundredths, you can divide as in A.

A

$$\begin{array}{rl} & 35 \text{ hundredths} \\ 20 & \overline{)700 \text{ hundredths}} \\ & \underline{60} \\ & 100 \\ & \underline{100} \end{array}$$

You will save time if you use the decimal point in place of the word hundredths. Recall that

$$7=\frac{700}{100}=7.00 \quad \text{and} \quad \frac{35}{100}=.35$$

Study B and C. Observe how the decimal point is used.

B

$$\begin{array}{rl} & . \\ 20 & \overline{)7.00} \end{array}$$

C

$$\begin{array}{rl} & .35 \\ 20 & \overline{)7.00} \\ & \underline{6.0} \\ & 1.00 \\ & \underline{1.00} \end{array}$$

In B the decimal point in the place for the quotient is in a vertical line with the decimal point in the dividend. In C the quotient is to the right of the decimal point.

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Suppose you wish to find the decimal equivalent of $\frac{7}{8}$. Since 1000, but not 10 or 100, is divisible by 8, you may work in thousandths as shown: $8\overline{)7.000}$. Then divide as you divide whole numbers. Does $7000 \div 8 = 875$? Does $7.000 \div 8 = .875$?

There is a way to predict whether or not a given fraction has an exact decimal equivalent. The steps are given below.

a. Make sure the given fraction is expressed in lowest terms. If it is not, reduce it.

b. Determine if the denominator of the reduced fraction divides 10, 100, 1000, and so on. If so, you can be certain that the fraction has an exact decimal equivalent.

Oral Tell if division can be used to find an exact decimal equivalent for each fraction below. Yes. Also see below.

	a	*b*	*c*	*d*
1.	$\frac{7}{20}$ .35	$\frac{3}{8}$ .375	$\frac{11}{32}$	$\frac{5}{16}$ .3125
2.	$\frac{9}{64}$	$\frac{38}{128}$	$\frac{12}{25}$ .48	$\frac{12}{40}$ .3
3.	$\frac{60}{80}$ .75	$\frac{17}{50}$ .34	$\frac{18}{125}$ .144	$\frac{60}{160}$ .375

Written Use division to find a decimal equivalent for each fraction above. Consider the division complete when the remainder is 0.

1c. .34375
2a. .140625
2b. .296875

Approximate Decimal Equivalents

Study the work which Jack is doing at the board. Which digit would you expect him to write next? And next? And next? No matter how long Jack works, he will never obtain a 0 remainder. He may either repeat the writing of 3's over and over again; or after the final 3, place three dots, like this: 3=.333· · · . The dots after the final 3 are read *and so on indefinitely*. Now try dividing 2 by 3, 8 by 33, and 27 by 37. The results are shown below. Find the numerals that are repeated.

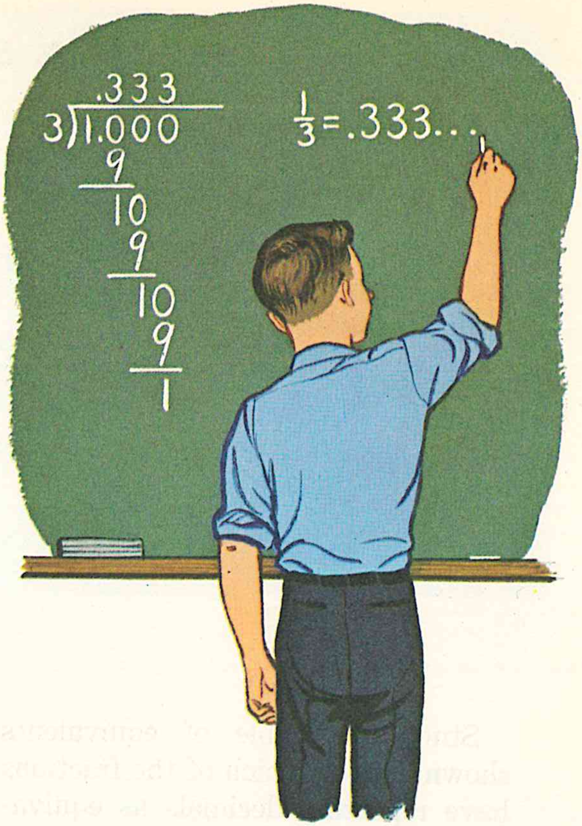
$\frac{2}{3}=.66666\cdots$ repeats 1 numeral
$\frac{8}{33}=.242424\cdots$ repeats 2 numerals
$\frac{27}{37}=.729729\cdots$ repeats 3 numerals

The decimals shown above are called **repeating decimals.** In a repeating decimal the same numeral or the same set of numerals is repeated indefinitely.

There is a way to predict whether or not a given fraction has a repeating decimal equivalent. The steps are given below.

a. Make sure the given fraction is expressed in lowest terms.

b. Determine if the denominator of the fraction divides 10, 100, 1000, and so on. If not, division will produce a repeating decimal.



Oral Predict which decimal equivalents will be repeating decimals. Tell why.

See column 1. Also see below.

	a	*b*	*c*	*d*
1.	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{7}{9}$	$\frac{5}{9}$
2.	$\frac{3}{8}$	$\frac{11}{40}$	$\frac{15}{90}$	$\frac{19}{20}$
3.	$\frac{5}{32}$	$\frac{8}{11}$	$\frac{13}{18}$	$\frac{23}{25}$
4.	$\frac{6}{7}$	$\frac{11}{12}$	$\frac{9}{13}$	$\frac{12}{95}$

Written Write the decimal equivalent of each fraction above. Express repeating decimals in thousandths. Place three dots · · · after such equivalents.

1. .666...; .166...; .777...; .555...
2. .375...; .275; .166...; .95
3. .15625; .727...; .722...; .92
4. .857...; .916...; .692...; .126...

FRACTION EQUIVALENTS

Common Fraction	Decimal Equivalent	Common Fraction	Decimal Equivalent
$\frac{1}{2}$	.5	$\frac{2}{3}$	.666···
$\frac{1}{4}$	.25	$\frac{1}{6}$	.166···
$\frac{3}{4}$	.75	$\frac{5}{6}$	.833···
$\frac{1}{5}$	.2	$\frac{1}{8}$	.125
$\frac{2}{5}$	.4	$\frac{3}{8}$	.375
$\frac{3}{5}$	.6	$\frac{5}{8}$	.625
$\frac{4}{5}$	.8	$\frac{7}{8}$	.875
$\frac{1}{3}$	.333···	$\frac{1}{10}$	.1

Practice with Decimal Equivalents

Study the table of equivalents shown above. Which of the fractions have repeating decimals as equivalents? These fractions, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{5}{6}$, are often rounded to the nearer thousandth. Why is $\frac{1}{3}$ rounded to .333 while $\frac{2}{3}$ is rounded to .667? Which digit is in ten-thousandths place when you divide 1 by 6? 5 by 6?

How would you round the decimal equivalent of $\frac{1}{3}$ to the nearer ten-thousandth? To the nearer hundred-thousandth? How would you round the decimal equivalent of $\frac{2}{3}$ to the nearer ten-thousandth? To the nearer hundred-thousandth?

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For most purposes, you will find it unnecessary to round equivalents beyond thousandths.

Oral Why is it important to learn the decimal equivalent for each fraction in the above chart?

Written Solve for n. Round repeating decimals to thousandths.

See column 1.

	a	*b*
1.	14 $(15+27)\div 3 = n$	7.125 $n = (3\times 19)\div 8$
2.	105 $(35\times 12)\div 4 = n$	13.666... $n = (98-16)\div 6$
3.	137.2 $(49\times 14)\div 5 = n$	487.5 $n = (75\times 65)\div 10$
4.	40.375 $(17\times 19)\div 8 = n$	112 $n = (42\times 16)\div 6$
5.	11 $(82-16)\div 6 = n$	48 $n = (79+65)\div 3$

Can you do this? Find a pattern in which digits are repeated when you find decimal equivalents for $\frac{1}{7}$, $\frac{1}{11}$, and $\frac{1}{12}$.

Dividing a Decimal Number by a Whole Number

John ran 4 laps around the indoor track in 61.2 seconds. What was his average time for each lap?

Let n represent the number you are seeking. Then the equation is $61.2 \div 4 = n$. Why?

Before solving for n, you may wish to make an estimate as shown.

Since $4 \times 15 = 60$, $n > 15$.

Since $4 \times 16 = 64$, $n < 16$.

The answer must be a number between 15 and 16.

You divided tenths and hundredths when you solved problems involving dollars and cents. Recall also that tenths and hundredths were divided in the same way as ones and tens, and that the decimal points were on the same vertical line. Then you should have no trouble dividing 61.2 by 4. Study the solution and its check in A.

A

$$\begin{array}{r|r} & 15.3 \\ 4 & \overline{61.2} \\ & 4 \\ \hline & 21 \\ & 20 \\ \hline & 1\,2 \\ & 1\,2 \\ \hline \end{array}$$

Check

$$\begin{array}{r} 15.3 \\ \times 4 \\ \hline 61.2 \end{array}$$

John made a lap in 15.3 seconds.

Estimating a quotient is always worthwhile, because your estimate will guide you in placing the decimal point.

Suppose you are to solve for n in $176.40 \div 14 = n$ and are to make an estimate. You may disregard .40. Why? Then proceed as in B.

B

$10 \times 14 = 140$ and $140 < 176$

$20 \times 14 = 280$ and $280 > 176$

Do you see that n must be a number between 10 and 20?

Now divide 176.40 by 14 as you would divide $176.40 by 14. You should obtain 12.60 as an answer. Check the computation. Is 12.60 a number between 10 and 20?

Oral Make an estimate of a replacement for n. Tell the steps you took in making your estimate.

Estimates may vary.

	a	*b*
1.	$75.6 \div 3 = n$ 25.2	$27.86 \div 7 = n$ 3.98
2.	$47.94 \div 3 = n$ 15.98	$105.0 \div 15 = n$ 7
3.	$133.2 \div 18 = n$ 7.4	$121.6 \div 38 = n$ 3.2
4.	$109.2 \div 14 = n$ 7.8	$193.5 \div 45 = n$ 4.3

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Written Solve for n in each equation in rows 1 through 4 above.

Writing Remainders as Decimals

Some ways to rename 2 are shown below. Read the numerals in each row. Do you see any pattern?

$$2=\frac{20}{10}=\frac{200}{100}=\frac{2000}{1000}\ \cdots$$

$$2=2.0=2.00=2.000\ \cdots$$

The dots at the end of each row show that you could go on and on indefinitely. How do you know that $2=\frac{20}{10}$? Does $1=\frac{10}{10}$, or 1.0? Does $4=4.00$? Does $.2=.20$? Why?

Now suppose that you are to solve for n in $25\div4=n$ and that your answer is to be expressed in decimal form.

Study the solution below.

A

$$\begin{array}{r} 6\\ 4\overline{)25}\\ \underline{24}\\ 1 \end{array}$$

B

$$\begin{array}{r} 6.2\\ 4\overline{)25.0}\\ \underline{24}\\ 1.0\\ \underline{.8}\\ .2 \end{array}$$

C

$$\begin{array}{r} 6.25\\ 4\overline{)25.00}\\ \underline{24}\\ 1.0\\ \underline{.8}\\ .20\\ \underline{.20}\\ 0 \end{array}$$

What is the remainder of ones in A? Does 1 equal ten tenths? What is the remainder of tenths in B? Do two tenths equal twenty hundredths?

In B, the remainder of 1 was renamed as 1.0. How was the remainder of .2 renamed in C?

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You do not need to write the decimal points in the partial dividends, providing you keep each digit in its proper place-value position. The division is considered complete when the final remainder is 0.

Oral Tell the steps to take in solving for n if n is to be a number in decimal form. See column 1.

	a	*b*
1.	8.5 $17\div2=n$	3.2 $16\div5=n$
2.	2.5 $15\div6=n$	4.2 $63\div15=n$

Written Perform the indicated operation. Express the quotient as a number in decimal form.

	a	*b*	*c*
1.	4.8 $5\overline{)24}$	4.5 $8\overline{)36}$	16.5 $4\overline{)66}$
2.	19.2 $5\overline{)96}$	1.25 $8\overline{)10}$	6.75 $4\overline{)27}$
3.	2.75 $28\overline{)77}$	3.75 $24\overline{)90}$	2.75 $12\overline{)33}$
4.	30.5 $26\overline{)793}$	50.5 $12\overline{)606}$	40.5 $18\overline{)729}$
5.	14.9 $50\overline{)745}$	21.5 $40\overline{)860}$	54.25 $12\overline{)651}$
6.	20.02 $50\overline{)1001}$	90.5 $12\overline{)1086}$	30.5 $46\overline{)1403}$
7.	72.5 $16\overline{)1160}$	83.75 $12\overline{)1005}$	94.5 $14\overline{)1323}$
8.	85.5 $14\overline{)1197}$	345.5 $18\overline{)6219}$	51.2 $50\overline{)2560}$

Dividing by Hundredths

Suppose you are to solve the equation $.96 \div .12 = n$.

Since you know how to divide with fractions, solve the equation in fractional form.

$$\frac{96}{100} \div \frac{12}{100} = \frac{96 \div 12}{100 \div 100} = \frac{8}{1} = 8$$

This shows that $.96 \div .12 = 8$.

This division can also be done using the $\overline{)}$ sign as shown.

$$\begin{array}{r} 8 \\ 12 \text{ hundredths } \overline{)96 \text{ hundredths}} \\ \underline{96} \end{array} \qquad \begin{array}{r} 8 \\ .12\overline{).96} \\ \underline{96} \end{array}$$

Think "How many sets of 12 hundredths are contained in 96 hundredths?" There are 8. Does $8 \times .12 = .96$?

The quotient of $.96 \div .12$ is 8, and the quotient of $96 \div 12$ is also 8. Therefore it should be possible to solve for n in $.96 \div .12 = n$ by dividing 96 by 12. Study the example below to find why this is true.

$$1 \times \frac{.96}{.12} = \frac{100}{100} \times \frac{.96}{.12}$$
$$= \frac{100 \times .96}{100 \times .12} = \frac{96}{12} = 8$$

Suppose you are to divide 1.215 by .15. You know how to divide by a whole number. Study the solution below. Why was 1 renamed as $\frac{100}{100}$?

$$1 \times \frac{1.215}{.15} = \frac{100}{100} \times \frac{1.215}{.15}$$
$$= \frac{100 \times 1.215}{100 \times .15} = \frac{121.5}{15} = 8.1$$

Oral Answer these questions.

1. What is the main advantage of changing a divisor to a whole number before you divide? To permit using arithmetic of whole numbers

2. In solving such equations as $9.6 \div .12 = n$, why should you multiply both 9.6 and .12 by 100 rather than by 10? Because the divisor is expressed in hundredths

Written Write two solutions for each equation below. In the first solution work with numbers in fractional form. In the second solution, rename the divisor as a whole number.

Answers only

	a	*b*
1.	$.72 \div .06 = \overset{12}{n}$	$.96 \div .24 = \overset{4}{n}$
2.	$.84 \div .21 = \overset{4}{n}$	$.95 \div .05 = \overset{19}{n}$
3.	$.80 \div .16 = \overset{5}{n}$	$.72 \div .36 = \overset{2}{n}$

Copy. Find each quotient.

	a	*b*
4.	$\begin{array}{r} 21. \\ .42\overline{)8.82} \end{array}$	$\begin{array}{r} 33. \\ .24\overline{)7.92} \end{array}$
5.	$\begin{array}{r} 60. \\ .74\overline{)44.40} \end{array}$	$\begin{array}{r} 90. \\ .85\overline{)76.50} \end{array}$
6.	$\begin{array}{r} 8.1 \\ .15\overline{)1.215} \end{array}$	$\begin{array}{r} 7.3 \\ .18\overline{)1.314} \end{array}$
7.	$\begin{array}{r} 24. \\ .73\overline{)17.52} \end{array}$	$\begin{array}{r} 105. \\ .50\overline{)52.50} \end{array}$
8.	$\begin{array}{r} 9.0 \\ .53\overline{)4.770} \end{array}$	$\begin{array}{r} 70. \\ .18\overline{)12.60} \end{array}$
9.	$\begin{array}{r} .57 \\ .45\overline{).2565} \end{array}$	$\begin{array}{r} 200. \\ .31\overline{)62.00} \end{array}$

Division with Decimals

Suppose you are to solve the equation $5.92 \div 3.2 = n$. You know how to divide when the divisor is a whole number. You also know that you can change a decimal divisor to the whole number form by multiplying by 1 in the most convenient form. By which form of 1 would you multiply $\frac{5.92}{3.2}$ to change 3.2 to a whole number? Would it be $\frac{10}{10}$?

See what happens when you choose $\frac{10}{10}$ as the name for 1, and solve $5.92 \div 3.2 = n$. Study this solution.

$$\frac{5.92}{3.2} = \frac{5.92}{3.2} \times 1 = \frac{5.92}{3.2} \times \frac{10}{10}$$
$$= \frac{5.92 \times 10}{3.2 \times 10} = \frac{59.2}{32}$$

Then you perform the division as with any whole number divisor. The quotient is 1.85.

Oral Tell the reason for each step taken in solving for n.

See T180, Using the text

1. $n = .5 \div 2.5$

$$= \frac{.5}{2.5} = \frac{.5}{2.5} \times 1 = \frac{.5}{2.5} \times \frac{10}{10}$$
$$= \frac{.5 \times 10}{2.5 \times 10} = \frac{5}{25}$$
$$= 5 \div 25 = .20$$

2. $n = 6 \div 1.5$

$$= \frac{6.}{1.5} = \frac{6.}{1.5} \times 1 = \frac{6.}{1.5} \times \frac{10}{10}$$
$$= \frac{6. \times 10}{1.5 \times 10} = \frac{60}{15}$$
$$= 60 \div 15 = 4$$

Tell the steps to take in solving for n in $n = .68 \div 8.5$.

Written Copy. Write each quotient.

	a	*b*	*c*
1.	$7.5\overline{)4}$ .533...	$37.5\overline{)6}$ .16	$22.5\overline{)18}$ .8
2.	$5.5\overline{)4}$ .727...	$3.6\overline{)9}$ 2.5	$7.5\overline{)10}$ 1.33
3.	$1.25\overline{)8.4}$ 6.72	$.75\overline{).045}$ .06	$.35\overline{)74.9}$ 214
4.	$9.5\overline{)513}$ 54	$.85\overline{)15.3}$ 18	$2.12\overline{)954}$ 450
5.	$2.8\overline{)8.4}$ 3	$8.5\overline{)5.1}$ .6	$1.8\overline{)9}$ 5
6.	$1.6\overline{)48}$ 30	$2.95\overline{)236}$ 80	$4.7\overline{)23.5}$ 5

Solve for n.

	a		*b*
7.	$98.1 \div 32.7 = n$	3 4	$n = 9.92 \div 2.48$
8.	$9.30 \div 12.4 = n$	.75 2.4	$n = 8.40 \div 3.50$
9.	$.92 \div 3.68 = n$	.25 20	$n = 69 \div 3.45$
10.	$52.75 \div 1.25 = n$	42.2 31.25	$n = 80 \div 2.56$

Solve these problems.

11. A contractor finished 3.2 miles of new highway in 5.5 days. On the average, how many miles of new highway were finished each day?
$3.2 \div 5.5 = n$ .581... mi.

12. The driver of a racing car drove 119 miles in 1.4 hours. What was his average speed expressed in miles per hour? $119 \div 1.4 = n$ 85 mph

Dividing by Thousandths

Suppose you are to solve the equation 15÷.008=n, and want to do the division by using whole numbers. Would you choose $\frac{10}{10}$, $\frac{100}{100}$, or $\frac{1000}{1000}$ as a name for 1 in order to change .008 from the decimal form to the whole number form?

See what happens when you choose $\frac{1000}{1000}$ as the name for 1 and solve 15÷.008=n. Study the solution below.

$$\frac{15}{.008}=\frac{15}{.008}\times 1=\frac{15}{.008}\times\frac{1000}{1000}$$
$$=\frac{15\times 1000}{.008\times 1000}=\frac{15000}{8}=1875$$

When the divisor is expressed in thousandths, you can multiply it by 1000 to produce a whole number divisor. Then you will also multiply the dividend by 1000. Multiplying by $\frac{1000}{1000}$ is the same as multiplying by 1, because $\frac{1000}{1000}=1$.

Do you see that 15÷.008=n may be solved as 15000÷8=n? Then n is 1875. Check the answer.

Oral Tell how you know that each statement below is true.
See above.

1. .002).008 becomes 2)8
2. .019).095 becomes 19)95
3. .015).9 becomes 15)900
4. .320)96 becomes 320)96000

Written Solve for n. Round each quotient to the nearer thousandth.

	a	*b*
1.	.819÷.009=n 91.000	n=36.9÷.006 6150.000
2.	4.57÷.007=n 652.857	n=.013÷.052 .250
3.	.081÷.027=n 3.000	n=.012÷.026 .462
4.	1.064÷.019=n 56.000	n=2.58÷.049 52.653
5.	1.58÷.035=n 45.143	n=14.50÷.018 805.556
6.	1.37÷.017=n 80.588	n=.7210÷.036 20.028
7.	22.65÷.453=n 50.000	n=1789÷.198 9035.354
8.	7.360÷.320=n 23.000	n=.5852÷.209 2.800

Estimate the answer for each problem below. Then solve each problem.
Estimates may vary.

9. Mr. Carlson bought 9.8 gallons of gasoline at 37.9¢ a gallon and a quart of oil for 65¢. How much money did he spend for both?
(9.8x37.9)+65=n $4.36

10. Mr. Carlson made a trip of 123 miles. His car usually averages 16.4 miles per gallon. How many gallons of gasoline did he probably use on the trip?
123÷16.4=n 7.5 gal.

11. In one city the rainfall for April, May, and June was as follows: 5.19 in., 3.85 in., and 6.21 in. On the average, how much rain fell in each month? (5.19+3.85+6.21)÷3=n
5.083... in.

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75× 10 = 750	75÷ 10 = 7.5	7.5× 10 = 75	7.5÷ 10 = .75
75× 100 = 7500	75÷ 100 = .75	7.5× 100 = 750	7.5÷ 100 = .075
75×1000 = 75000	75÷1000 = .075	7.5×1000 = 7500	7.5÷1000 = .0075

Dividing by 10, 100, and 1000

Study the work in the chart above. Look for the related patterns when multiplying and dividing by 10, by 100, and by 1000. How is the place-value position of the 7 and 5 changed when you multiply and then divide 75 by 10, by 100, and by 1000? When you multiply and then divide 7.5 by 10, by 100, and by 1000?

Choose any other number, for example 150. Multiply it and then divide it by 10, then by 100, and then by 1000. As you look at each product and at each quotient, you will see the digits of the numeral you chose. You will also see that the place-value position of each digit is changed as the decimal point is moved.

Perhaps you have discovered that the number of zeros in a divisor of 10, 100, or 1000 helps you decide where to place the decimal point in the quotient. If there is 1 zero in the divisor, the digits of the quotient are in the place-value position 1 place to the right of their original position.

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What happens when the divisor has 2 zeros? 3 zeros?

Oral Read. Supply each quotient. Tell how you decided upon it. See column 1.

	a	*b*
1.	1250÷10 = n 125	1250÷100 = n 12.50
2.	125÷100 = n 1.25	125÷1000 = n .125
3.	12.5÷10 = n 1.25	12.5÷100 = n .125
4.	12.5÷1000 = n .0125	7÷1000 = n .007

Written Solve for n in rows 1 through 8.

	a	*b*
5.	1.25÷10 = n .125	1.25÷100 = n .0125
6.	1.25÷1000 = n .00125	467÷10 = n 46.7
7.	467÷100 = n 4.67	360÷1000 = n .360
8.	4.5÷1000 = n .0045	4.5÷100 = n .045

A quick review To the nearer thousandth give the decimal equivalent of each fraction that follows: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{1}{6}$. .500; .333; .667; .750; .200; .167

To find this	You do this
The sum of two or more different numbers	add
The sum of two or more equal numbers	multiply
The difference between two numbers	subtract
How many times one number is contained in another	divide

Solving Problems

Write the equation for each problem below. Be ready to tell the class why you decided upon the equation you wrote.

1. A distance swimmer averages 3.5 miles per hour. At this rate, how far will he swim in $2\frac{1}{4}$ hours?
2.25x3.5=n 7.875 mi.

2. A hiker walked 11 miles in 3 hours. To the nearest tenth of a mile, how many miles did he average per hour? 11÷3=n 3.7 mi.

3. Ronnie's father bought gasoline as follows: 14.7 gallons, 9.8 gallons, 10.5 gallons. He averaged 12.8 miles per gallon. How many miles did he drive before using all of the gasoline?
(14.7+9.8+10.5)x12.8=n 448 mi.

4. Virgil gained 5.25 pounds in 6 months. To the nearer hundredth pound, what was his average gain in each of the months?
5.25÷6=n .88 lb.

5. A milk can that weighed 18.5 pounds when empty was filled with 8 gallons of water. Water weighs 8.33 pounds per gallon. How much did the can full of water weigh?
18.5+(8x8.33)=n 85.14 lb.

6. In a road test, car A averaged 27.452 miles per gallon of gasoline. Car B averaged 25.979 miles per gallon of gasoline. At this same rate how far would each car go on 10 gallons of gasoline? See below.

7. Mr. Larsen used 65.9 gallons of gasoline in December and 51.6 gallons in January. His gasoline bill for the two months was $42.18. On the average, how much did he pay for each gallon of gasoline?
42.18÷(65.9+51.6)=n 35.9¢

Oral Tell the class how you decided upon each equation for problems 1 through 7.

Written Write an estimated answer for problems 1 through 7. Solve each equation. Compare your estimated and computed answers. Write the answer for each problem.
Estimates may vary.

6. A: 10x27.452=n 274.52 mi.
B: 10x25.979=n 259.79 mi.

Review and Practice

Part 1 Copy. Find each sum, difference, or product by performing the indicated operation.

	a	*b*	*c*
1.	1987 2362 5950 +6478 16777	\$13.25 1.60 94.80 +9.58 \$119.23	26.8 43.0 69.7 +73.5 213.0
2.	47.88 94.32 65.40 +18.76 226.36	97.495 6.281 5.426 +23.657 132.859	94.872 15.941 37.555 +84.766 233.134
3.	$8\frac{9}{10}$ $+7\frac{1}{4}$ $16\frac{3}{20}$	$19\frac{5}{16}$ $+24\frac{7}{8}$ $44\frac{3}{16}$	$27\frac{5}{6}$ $+43\frac{8}{9}$ $71\frac{13}{18}$
4.	6284 −3517 2767	76.324 −5.867 70.457	98.475 −39.686 58.789
5.	$8\frac{3}{10}$ $-4\frac{1}{4}$ $4\frac{1}{20}$	$20\frac{8}{9}$ $-16\frac{1}{6}$ $4\frac{13}{18}$	$37\frac{1}{10}$ $-14\frac{3}{4}$ $22\frac{7}{20}$
6.	45.62 ×16 729.92	8.2 ×4.3 35.26	6.14 ×.3 1.842

Part 2 Solve for n.

	a	*b*
1.	$\frac{3}{8}$ $\frac{3}{4}\times\frac{1}{2}=n$	$\frac{1}{2}$ $n=\frac{4}{5}\times\frac{5}{8}$
2.	14 $4\frac{3}{8}\times3\frac{1}{5}=n$	775 $n=62\times12\frac{1}{2}$
3.	$\frac{5}{6}\div\frac{5}{8}=n$ $1\frac{1}{3}$	$2\frac{4}{5}$ $n=8\frac{4}{5}\div3\frac{1}{7}$
4.	$\frac{4}{9}$ $2\div4\frac{1}{2}=n$	$\frac{20}{39}$ $n=5\div9\frac{3}{4}$

Part 3 Solve for n.

	a	*b*
1.	32 800÷25 = n	121 n = 9075÷75
2.	50 3000÷60 = n	89.428... n = 8764÷98
3.	5.1 35.7÷7 = n	22.7 n = 6.81÷.3
4.	9.6 6.24÷.65 = n	2.25 n = 7.2÷3.2
5.	5 .545÷.109 = n	30 n = 7.20÷.240

Part 4 Copy. Write a numeral in each blank so that the statements below become true. Reminder: = means equal in measure.

1. 35 min. = 2100 sec.

2. 4500 lb. = 2 T. 500 lb.

3. 9 lb. 5 oz. = 149 oz.

4. 115 in. = 3 yd. 7 in.

5. 3 hr. 12 min. = 192 min.

Part 5 Indicate the operation or operations you think of as you read each word below.

	a	*b*
1.	addend addition	product multiplication
2.	associative add., mult.	sum addition
3.	commutative add., mult.	factors multiplication
4.	minuend subtraction	quotient division
5.	divisor division	subtrahend subtraction

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Whole numbers may be renamed as fractions and as decimals. (172-173)

2. The digits in any place-value position are divided in the same way as ones are divided. (174)

3. Decimal equivalents of fractional numbers may always be determined by division. For example, $\frac{8}{12}$ means $8 \div 12$. (174)

4. Multiplying both the divisor and the dividend either by 10, by 100, or by 1000, and so on does not change the quotient. (179-181)

5. After renaming the dividend and divisor to make the divisor a whole number, you place the decimal point in the quotient. (179)

Words to Know

Repeating decimal (175)

Questions to Discuss

See T185 for answers.

1. How do you find a decimal number equivalent for a fractional number by using division? (172, 174)

2. How can you tell whether or not the decimal equivalent of a fractional number will be a repeating decimal? (174-175)

3. In what way is dividing decimal numbers similar to dividing numbers expressed as money? (177)

4. How would you estimate the number that n stands for in the equation $68.2 \div 3 = n$? (177)

5. Why is it unnecessary to write the decimal points in each partial dividend? (178)

6. How can you change a divisor that is in decimal form to a whole number form? (179-181)

7. What is a short way to divide 15.1 by 1000? (182)

Written Practice

Copy. Perform the indicated operation. Round answers to thousandths.

	a	*b*	
1.	$\overset{3.000}{.9 \div .3 = n}$	$\overset{4.000}{n \times .2 = .8}$	(171)
2.	$\overset{.175}{40\overline{)7}}$	$\overset{.750}{20\overline{)15}}$	(174)
3.	$\overset{.833}{6\overline{)5}}$	$\overset{.684}{19\overline{)13}}$	(175)
4.	$\overset{42.167}{12\overline{)506}}$	$\overset{26.200}{35\overline{)917}}$	(178)
5.	$\overset{2.771}{3.5\overline{)9.7}}$	$\overset{1.526}{21.6\overline{)32.97}}$	(180)
6.	$\overset{3.000}{.003\overline{).009}}$	$\overset{201.900}{.421\overline{)85}}$	(181)

Self-Evaluation

Part 1 Write the numeral which is the best estimate of a replacement for n. Then solve for n. Answers given to the nearer thousandth

1. 127.86÷7=n 18.267

a. 10 [✓ c. 20]
b. 100 d. 1000

2. 14.52÷.79=n 18.380

a. 10 c. 1000
b. 100 [✓ d. 20]

3. 435.9÷10.2=n 42.735

[✓ a. 40] c. 10
b. 50 d. 100

4. 989.35÷28.7=n 34.472

a. 300 c. 30
b. 350 [✓ d. 35]

5. 750÷.017=n 44117.647

a. 100 [✓ c. 10,000]
b. 1000 d. 100,000

Part 2 Solve for n. Round answers to thousandths.

	a	*b*
1.	7.000 5.6÷.8=n	11.700 n=46.8÷4
2.	23.562 754÷32=n	5.790 n=92.64÷16
3.	2.150 41.5÷19.3=n	106.857 n=112.2÷1.05
4.	2.464 32.4÷13.15=n	4800.000 n=72÷.015
5.	.271 .312÷1.15=n	298.571 n=2.09÷.007

Part 3 Copy. Find each quotient.

	a	*b*	*c*
1.	4.6 5)23	6.75 4)27	7.5 4)30
2.	4 5)2	1.76 3)5.28	3.575 4)14.3
3.	.583 12)7	3 .3).9	12 .07).84
4.	4 .05).2	200 .02)4	2000 .003)6
5.	.06 6).36	.6 .3).18	1.2 .15).18
6.	.2 2.5).50	12.5 .4)5	2 3.5)7
7.	.769 5.2)4	38.4 3.75)144	.9 7)6.3
8.	2.725 12)32.7	15.5 .6)9.3	8 4.3)34.40
9.	4 8.9)3.56	2.107 2.8)5.9	204.44.. .045)9.2

Part 4 Solve these problems.

1. Henry earned $4.95 in $5\frac{1}{2}$ hours. What was his pay per hour?

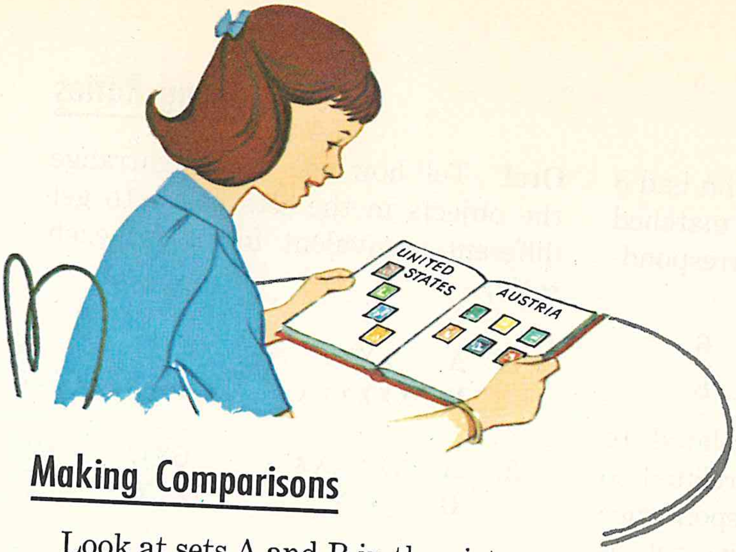
4.95÷5.5=n $.90

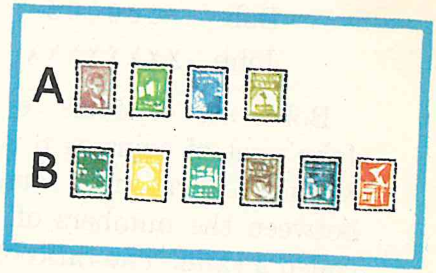
2. Mary wanted to earn money to buy a $24.50 bicycle. At $.75 an hour, how many hours would she have to work to earn the money?

24.50÷.75=n $32\frac{2}{3}$ hr.

3. A can filled with milk weighed 70 pounds. The empty can weighed 12 pounds. If milk weighs 8.6 pounds per gallon, how many gallons of milk were in the can?

(70-12)÷8.6=n 6.744 gal.





Making Comparisons

Look at sets A and B in the picture above. How many less stamps are in set A than in set B? How many more stamps are in set B than in set A? In either case the answer may be obtained by matching the stamps, one-to-one. This type of matching establishes a **one-to-one correspondence** between the objects in one set and the objects in another set.

A ☐ ☐ ☐ ☐ ? ?

B ☐ ☐ ☐ ☐ ☐ ☐

The two sets cannot be matched one-to-one, for there are 2 more objects in set B than in set A.

Suppose you match the stamps in sets A and B in another way as shown below.

A ☐ ☐ ☐ ☐

B ☐ ☐ ☐ ☐ ☐ ☐

The sets are now matched by making a **two-to-three correspondence.** This means that for every 2 objects in set A there are 3 objects in set B. Take two sets of objects. In one set have 4 objects and in the other have 8 objects. See how many different correspondences you can establish by matching the objects in the sets.

Oral What two correspondences are suggested by each pair of set drawings below?

1. **A** XXX XXX 3 to 4
   B XXXX XXXX 6 to 8

2. **A** XXXX XXXX 4 to 5
   B XXXXX XXXXX 8 to 10

3. **A** XXXXX XXXXX 5 to 2
   B XX XX 10 to 4

Using Ratios

Bob had 6 pennies and John had 8 dimes. The two sets may be matched by making a six-to-eight correspondence as follows.

Bob	000000	6
John	XXXXXXXX	8

Bob's set of coins is related to John's set of coins as 6 is related to 8. Such a relation or correspondence between the numbers of two sets is called a **ratio.** The ratio of 6 to 8 can be written as the fraction $\frac{6}{8}$.

By rearranging the objects in the sets, you get a different form for the ratio as shown below.

Bob	000	000	3
John	XXXX	XXXX	4

Since the same two sets are used in both arrangements above, you can conclude that the ratios 6 to 8, or $\frac{6}{8}$, and 3 to 4, or $\frac{3}{4}$, are equivalent. You might have expected this, for you already know from your work with fractions that $\frac{6}{8}=\frac{3}{4}$.

You can use drawings of sets to show that the ratio of 2 to 3 is equivalent to the ratio of 6 to 9. You might proceed as shown.

XXXXXX			6	$\frac{6}{9}\times 9=6$
XXXXXXXXX			9	
XX	XX	XX	2	$\frac{2}{3}\times 9=6$
XXX	XXX	XXX	3	

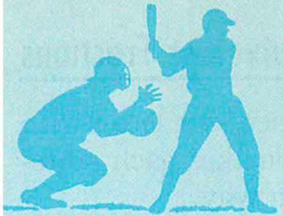
Oral Tell how you would rearrange the objects in the sets below to get different equivalent forms for each ratio.

1.	**A**	XXXX	4	XX		XX
	B	XXXXXX	6	XXX		XXX
2.	**A**	XXXXXX	6	XXX		XXX
	B	XX	2	X		X
3.	**A**	XX	2	X		X
	B	XXXX	4	XX		XX
4.	**A**	XXX	3	X	X	X
	B	XXXXXX	6	XX	XX	XX
5.	**A**	XXXXXXXX	8	XXXX		XXXX
	B	XXXXXX	6	XXX		XXX

Written Make set patterns to illustrate each ratio below. Then, if possible, rearrange the objects in the sets to get a different but equivalent form for your ratio.

See column 1.

	a	*b*
1.	8 to 10	10 to 6
2.	4 to 12	12 to 8
3.	3 to 9	9 to 6
4.	2 to 3	3 to 5
5.	4 to 5	3 to 1
6.	10 to 4	6 to 2



	Games Played	*Games Won*	*Games Lost*
Yankees	12	8	4
White Sox	10	6	4
Angels	14	8	6

Writing Ratios

Team standings are often expressed as the ratio of games won to games played. The Yankees won 8 games out of 12. The ratio is expressed below.

$$\begin{array}{lcl} \text{Games won} & 8 & \text{(Numerator)} \\ \hline \text{Games played} & 12 & \text{(Denominator)} \end{array}$$

Does $\frac{8}{12}=\frac{2}{3}$? Then the ratio of games won to games played may be expressed as $\frac{2}{3}$, or 2 to 3.

Suppose you are to give the ratio of the number of games won to the number of games lost. You must find out how many games were lost. Then you can write the ratio as follows.

$$\frac{\text{Games won}}{\text{Games lost}} \quad \frac{8}{4} \quad \text{or} \quad \frac{2}{1}$$

This means that for any 8 games won, 4 were lost, or that for any 2 games won, 1 was lost.

Oral Answer these questions.

1. What is the ratio of games won to games played for the Yankees? For the White Sox? For the Angels? $\frac{8}{12}$; $\frac{6}{10}$; $\frac{8}{14}$

2. What is the ratio of wins to losses for the Yankees? For the White Sox? For the Angels? $\frac{8}{4}$; $\frac{6}{4}$; $\frac{8}{6}$

3. What is the ratio of losses to wins for the Yankees? For the White Sox? For the Angels? $\frac{4}{8}$; $\frac{4}{6}$; $\frac{6}{8}$

Written Express as ratios in fractional form.

1. 4 hits for 9 times at bat $\frac{4}{9}$

2. 2 runs in 3 innings $\frac{2}{3}$

3. 9 riders for 2 cars $\frac{9}{2}$

4. 15 cents for 2 apples $\frac{15}{2}$

5. 1 table for 4 pupils $\frac{1}{4}$

6. 8 pages for 3 envelopes $\frac{8}{3}$

7. 5 cookies for 1 boy $\frac{5}{1}$

8. 10 apples for 5 boxes $\frac{2}{1}$

9. 3 cups for 4 cakes $\frac{3}{4}$

10. 12 glasses for 2 jars $\frac{6}{1}$

11. 90 feet to 5 minutes $\frac{18}{1}$

12. 75 miles to 5 gallons $\frac{15}{1}$

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Reviewing Equivalent Fractions

You may recall that the same fractional number may have many names as shown below.

$$\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}\ldots$$
$$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}=\frac{10}{15}\ldots$$
$$\frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{4}{16}=\frac{5}{20}\ldots$$

In each row, called an **equivalence row,** all fractions are different names for the same number. The 3 dots indicate that each row goes on indefinitely.

Suppose you wished to find a replacement for n in each of the following.

A	**B**	**C**	**D**
$\frac{2}{3}=\frac{n}{6}$	$\frac{2}{3}=\frac{4}{n}$	$\frac{n}{3}=\frac{4}{6}$	$\frac{2}{n}=\frac{4}{6}$

By using the equivalence rows, you can quickly find the correct replacement for n. Another way to find the replacement for n is to determine a name for 1, for example, $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}\ldots$, and to multiply or divide both terms of the fraction by the name you chose. For example, in A, 6 is divisible by 3. $6 \div 3 = 2$. Therefore choose $\frac{2}{2}$ as a name for 1.

$$\frac{2}{2}\times\frac{2}{3}=\frac{4}{6} \qquad \frac{2}{3}=\frac{4}{6}$$

In B you would also choose $\frac{2}{2}$ as a name for 1. Why? How would you decide upon $\frac{2}{2}$ as the name for 1 in C and also in D?

Oral Read. Replace each letter with the numeral that makes each statement true. Explain why.

`See column 1.`

	a	*b*	*c*
1.	$\frac{1}{4}=\frac{n}{12}$ 3	$\frac{2}{5}=\frac{n}{30}$ 12	$\frac{9}{12}=\frac{3}{r}$ 4
2.	$\frac{r}{12}=\frac{3}{4}$ 9	$\frac{4}{5}=\frac{m}{60}$ 48	$\frac{n}{6}=\frac{12}{36}$ 2
3.	$\frac{9}{12}=\frac{x}{4}$ 3	$\frac{1}{4}=\frac{a}{16}$ 4	$\frac{12}{16}=\frac{a}{4}$ 3
4.	$\frac{n}{7}=\frac{9}{21}$ 3	$\frac{c}{4}=\frac{5}{20}$ 1	$\frac{3}{n}=\frac{12}{20}$ 5

Written Write each pair of ratios in rows 1 through 4. Replace each letter with the numeral that makes the statement true. Solve each of the following equations.

	a	*b*	*c*
5.	$\frac{10}{5}=\frac{n}{10}$ 20	$\frac{48}{16}=\frac{a}{4}$ 12	$\frac{14}{30}=\frac{x}{90}$ 42
6.	$\frac{34}{24}=\frac{t}{72}$ 102	$\frac{7}{8}=\frac{21}{w}$ 24	$\frac{1}{12}=\frac{r}{72}$ 6
7.	$\frac{28}{4}=\frac{a}{1}$ 7	$\frac{20}{60}=\frac{c}{6}$ 2	$\frac{5}{1}=\frac{m}{7}$ 35
8.	$\frac{14}{7}=\frac{28}{n}$ 14	$\frac{32}{4}=\frac{b}{1}$ 8	$\frac{10}{25}=\frac{2}{n}$ 5
9.	$\frac{6}{10}=\frac{a}{50}$ 30	$\frac{12}{48}=\frac{2}{c}$ 8	$\frac{27}{18}=\frac{e}{2}$ 3
10.	$\frac{3}{15}=\frac{9}{n}$ 45	$\frac{14}{18}=\frac{7}{n}$ 9	$\frac{24}{14}=\frac{12}{a}$ 7
11.	$\frac{18}{10}=\frac{54}{t}$ 30	$\frac{14}{35}=\frac{a}{105}$ 42	$\frac{35}{5}=\frac{x}{25}$ 17
12.	$\frac{9}{10}=\frac{81}{a}$ 90	$\frac{4}{32}=\frac{d}{128}$ 16	$\frac{24}{16}=\frac{144}{m}$ 9
13.	$\frac{20}{10}=\frac{n}{20}$ 40	$\frac{24}{8}=\frac{a}{2}$ 6	$\frac{17}{12}=\frac{t}{36}$ 5
14.	$\frac{25}{10}=\frac{n}{2}$ 5	$\frac{48}{12}=\frac{c}{2}$ 8	$\frac{18}{14}=\frac{d}{7}$ 9

X X

X X X

Proportion

Mary was making little bouquets of flowers for table decorations. She used 2 zinnias and 3 marigolds in every bouquet. She had 24 zinnias. How many marigolds did she need so that she could use all of the zinnias?

In A you see the ratio of zinnias to marigolds in every bouquet.

A XX / XXX $\frac{2}{3}$

Now suppose you were to show the ratio of all of the zinnias to the number of all of the marigolds. Let m represent the marigolds. Then you can show this ratio as in B.

B $\frac{24}{m}$

The ratio of all zinnias to all marigolds is the same as that in any one bouquet. Thus you can write = between the two ratios as in C.

C $\frac{2}{3}=\frac{24}{m}$

Such a statement is called a **proportion.** A proportion is a statement of equality between two ratios.

Look at C again. To find the number represented by m, you can use the arithmetic of fractional numbers. The thinking steps are shown below.

$\frac{2}{3}=1\times\frac{2}{3}$ Why?
$\frac{2}{3}=\frac{12}{12}\times\frac{2}{3}$ Choose $\frac{12}{12}$ for 1.
$\frac{2}{3}=\frac{12\times2}{12\times3}=\frac{24}{36}$

Now compare the proportions.

$$\frac{2}{3}=\frac{24}{m} \text{ and } \frac{2}{3}=\frac{24}{36}$$

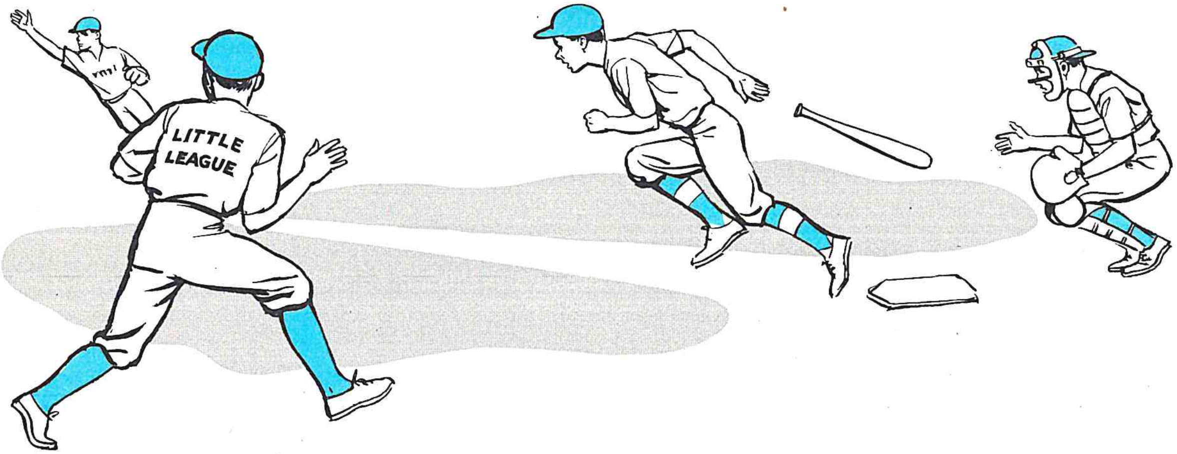
What number does m represent? Mary needs 36 marigolds.

For a shorter method, find the factor m so that $m\times2=24$. This factor is 12. Then multiply 3 by 12. What factor would you use in finding n in $\frac{2}{3}=\frac{18}{n}$? Think: $n\times2=18$. Then $n=9$. I will use 9 as a factor and multiply 3 by it. Then $\frac{2}{3}=\frac{18}{27}$.

Oral Read each proportion below. Tell the reason for the steps to take in finding the number represented by n. See column 1 and above.

	a	*b*	*c*
1.	$\frac{2}{3}=\frac{n}{6}$ 4	$\frac{6}{9}=\frac{n}{3}$ 2	$\frac{3}{2}=\frac{n}{4}$ 6
2.	$\frac{4}{6}=\frac{n}{3}$ 2	$\frac{3}{2}=\frac{n}{8}$ 12	$\frac{6}{4}=\frac{n}{8}$ 12
3.	$\frac{3}{2}=\frac{n}{6}$ 9	$\frac{2}{3}=\frac{n}{9}$ 6	$\frac{9}{6}=\frac{n}{2}$ 3
4.	$\frac{6}{9}=\frac{n}{18}$ 12	$\frac{6}{4}=\frac{n}{2}$ 3	$\frac{9}{6}=\frac{n}{18}$ 27
5.	$\frac{8}{12}=\frac{n}{3}$ 2	$\frac{12}{8}=\frac{n}{2}$ 3	$\frac{4}{3}=\frac{n}{9}$ 12
6.	$\frac{8}{12}=\frac{n}{24}$ 16	$\frac{27}{18}=\frac{9}{n}$ 6	$\frac{10}{15}=\frac{20}{n}$ 30
7.	$\frac{30}{20}=\frac{15}{n}$ 10	$\frac{18}{12}=\frac{n}{48}$ 72	$\frac{12}{18}=\frac{4}{n}$ 6
8.	$\frac{32}{24}=\frac{n}{72}$ 96	$\frac{27}{18}=\frac{n}{54}$ 81	$\frac{21}{14}=\frac{n}{28}$ 42

Written Copy the proportions in rows 1 through 8. Solve for n.



Using Proportions in Problem Solving

Roger made 8 hits in the first 3 games his team played. The team has 9 more games to play. If Roger can keep the same batting average, how many more hits might he expect to make before the season ends?

The ratio of hits to games for the first 3 games is $\frac{8}{3}$. Let h represent the number of hits he must make in the next 9 games. To show the ratio of hits to the number of games you use the ratio $\frac{h}{9}$.

The ratio of 8 to 3 is the same as the ratio of h to 9. Therefore you can translate the problem into the proportion below.

$$\frac{8}{3}=\frac{h}{9}$$

Since 3 is contained in 9 exactly 3 times, h is a number in which 8 must be contained 3 times. Does $3\times8=24$? Then $h=24$.

$$\frac{8}{3}=\frac{24}{9}$$

Roger might expect to make 24 hits in the remaining games.

Translate each of the following problems into a proportion or into an equation. Be ready to tell why you decided upon the proportion or equation you wrote.

See column 1.

1. Betty plans to mount 8 pictures on each 2 facing pages of a scrapbook. She has 56 pictures to mount. How many pages will she need for her pictures? $\frac{8}{2}=\frac{56}{p}$ 14 pages

2. In his motor, Mr. Cox uses 2 quarts of oil in each 5 gallons of gasoline. How many quarts of oil will he need if he uses 20 gallons of gasoline in a week? $\frac{2}{5}=\frac{n}{20}$ 8 qt.

3. The girls can serve 6 people from each quart of ice cream. They need enough ice cream to serve 48 people at a picnic. How many quarts of ice cream should they buy?

$\frac{6}{1}=\frac{48}{n}$ 8 qt.

4. Eight boys planned a cookout. They decided they would buy enough hamburgers so that each boy could have 3. They can make 4 hamburgers from a pound of meat. How much meat should the boys buy? $\frac{8}{3}$ $\frac{4}{x}$ 6

$\frac{4}{1}=\frac{24}{n}$ 6 lb.

5. Mr. Rogers used 3 quarts of paint to cover 225 square feet of surface. He has another 6 quarts of paint of the same kind. How many square feet of surface might he expect to cover with it? $\frac{3}{225}=\frac{6}{x}$ 450

$\frac{3}{225}=\frac{6}{n}$ 450 sq. ft.

6. Mr. Dixon bought $1\frac{1}{2}$ pounds of sweet-corn seed at 45¢ a pound. How much change should he receive from the $2.00 he gave to the clerk?

$200-(45\times1\frac{1}{2})=n$ $1.32

7. Mr. Johnson drove 1125.9 miles in March, 987.5 miles in April, and 1417.8 miles in May. On the average, how far did he drive each month?

$(1125.9+987.5+1417.8)\div3=n$ 1177 mi.

8. The Whitmers drove 85 miles in 2 hours. At this rate, how far could they expect to drive in 6 hours?

$\frac{85}{2}=\frac{n}{6}$ 255 mi.

9. Mr. Anderson bought 12.5 gallons of gasoline and had an oil change that required 5 quarts of oil. The gasoline cost 37.9¢ a gallon and the oil cost 65¢ a quart. How much money did he owe?

$(12.5\times.379)+(5\times.65)=n$ $7.99

10. Mr. Thompson averages 14.7 miles per gallon of gasoline. How many miles can he expect to drive on 12.5 gallons?

$\frac{14.7}{1}=\frac{n}{12.5}$ 183.75 mi.

11. Sally weighed 85 pounds. Her little sister weighed only 15 pounds. Sally's weight was how many times that of her sister? $\frac{85}{15}=\frac{n}{1}$ $5\frac{2}{3}$

12. John worked for 17 hours one week. Bill worked 5 hours. John worked how many times as many hours as Bill? $\frac{17}{5}=\frac{n}{1}$ $3\frac{2}{5}$

13. A librarian bought 10 fiction books for each 7 nonfiction books. If she is placing an order for 150 fiction books, how many nonfiction books should she order?

$\frac{10}{7}=\frac{150}{n}$ 105 books

Oral Tell the class the plan which you wrote to use in solving problems 1 through 13 and why you decided as you did.

Written Record an estimated answer for each of the problems. Solve each problem and compare your computed answer with your estimated answer. Reread the problem to see whether your computed answer seems reasonable. Write your answer.

Estimates will vary.

A quick review

	a	*b*	*c*
1.	$\frac{3}{124}=\frac{9}{n}$ 372	$\frac{9}{175}=\frac{36}{r}$ 700	$\frac{12}{48}=\frac{60}{s}$ 240
2.	$\frac{24}{144}=\frac{a}{12}$ 2	$\frac{35}{175}=\frac{x}{15}$ 3	$\frac{7}{224}=\frac{w}{16}$ .5
3.	$\frac{14}{16}=\frac{t}{48}$ 42	$\frac{52}{68}=\frac{r}{4}$ $3\frac{1}{17}$	$\frac{112}{48}=\frac{n}{96}$ 224
4.	$\frac{12}{5}=\frac{n}{25}$ 60	$\frac{8}{16}=\frac{32}{n}$ 64	$\frac{150}{10}=\frac{n}{100}$ 1500
5.	$\frac{8}{168}=\frac{a}{42}$ 2	$\frac{19}{57}=\frac{c}{3}$ 1	$\frac{75}{225}=\frac{25}{k}$ 75

Using Decimals in Ratios

John saved \$.25 a week. His average earnings were \$.75 a week. What was the ratio of his savings to his earnings?

You are to use a ratio in making a comparison. The \$.75 is the base of the comparison; so the ratio is as follows.

Savings $\frac{.25}{.75}$ (Numerator)
Earnings (Denominator)

Note that the \$ sign is *not* part of the ratio.

You would not change the comparison if you multiplied both of the numbers by 100, as shown in the following equation.

Then $\frac{.25}{.75}=\frac{100\times.25}{100\times.75}=\frac{25}{75}=\frac{1}{3}$

The ratio of savings to earnings is 1 to 3.

The advantage of multiplying both .25 and .75 by 100 is that you are then working with whole numbers. Is it easier to work with a ratio of $\frac{1}{3}$ than with a ratio of $\frac{.25}{.75}$?

Suppose that a man earned \$6.75 in 3 hours and wanted to use a ratio to compare his earnings to the number of hours worked. As a fraction the ratio is expressed as $\frac{6.75}{3}$. The ratio can be reduced to $\frac{2.25}{1}$ which is equivalent to $\frac{225}{100}$ or $\frac{9}{4}$.

Oral Express each of the following ratios in simplest fractional form.

1. \$.50 to \$.75 $\frac{2}{3}$
2. \$.30 to \$1.00 $\frac{3}{10}$
3. \$1.50 to \$3.00 $\frac{1}{2}$
4. \$2.75 to \$5.00 $\frac{11}{20}$
5. \$.90 for 1 hour $\frac{90}{1}$

Written Each fraction below is a ratio. Write each ratio as a common fraction in lowest terms.

	a	*b*	*c*	*d*
1.	$\frac{.16}{.24}$ $\frac{2}{3}$	$\frac{.12}{.08}$ $\frac{3}{2}$	$\frac{.21}{.28}$ $\frac{3}{4}$	$\frac{.10}{.30}$ $\frac{1}{3}$
2.	$\frac{.14}{.20}$ $\frac{7}{10}$	$\frac{.12}{.14}$ $\frac{6}{7}$	$\frac{.18}{.30}$ $\frac{3}{5}$	$\frac{.16}{.48}$ $\frac{1}{3}$
3.	$\frac{.12}{.24}$ $\frac{1}{2}$	$\frac{.40}{.48}$ $\frac{5}{6}$	$\frac{.30}{.60}$ $\frac{1}{2}$	$\frac{.24}{.36}$ $\frac{2}{3}$
4.	$\frac{.21}{28}$ $\frac{3}{400}$	$\frac{.30}{40}$ $\frac{3}{400}$	$\frac{.32}{60}$ $\frac{2}{375}$	$\frac{.21}{24}$ $\frac{7}{80}$
5.	$\frac{3}{.30}$ $\frac{10}{1}$	$\frac{4}{.20}$ $\frac{20}{1}$	$\frac{5}{.25}$ $\frac{20}{1}$	$\frac{9}{.10}$ $\frac{90}{1}$
6.	$\frac{1.70}{3.40}$ $\frac{1}{2}$	$\frac{2.80}{1.20}$ $\frac{7}{3}$	$\frac{10.00}{5.00}$ $\frac{2}{1}$	$\frac{9.50}{2.25}$ $\frac{3}{9}$

A quick review Solve for n. Check your work for accuracy.

1. $(3\times1.12)+4.69=n$ 8.05
2. $n=(4.05\div5)\times3.25$ 2.6325
3. $25=n\times(2.5\div.5)$ 5
4. $37.5+137.9+34.52+864=n$ 1073.92
5. $(415.75-219.98)+735.9=n$ 931.67

Solving Problems

Suppose that you can buy 3 cans of juice for \$.41 and that you want to know how much you would have to spend for 12 cans of the juice.

Call the price p. Then you can translate the problem into this proportion.

$$\frac{3}{.41}=\frac{12}{p}$$

Since 3 is contained in 12 exactly 4 times, p is a number in which .41 is contained 4 times. To find this number, multiply $\frac{3}{.41}$ by $\frac{4}{4}$. Why is this possible? Then

$$\frac{3}{.41}\times\frac{4}{4}=\frac{12}{p}$$
$$\frac{12}{1.64}=\frac{12}{p}$$

The fractions $\frac{12}{1.64}$ and $\frac{12}{p}$ have equal numerators. Such fractions are equal only if their denominators are equal. Therefore $p=1.64$ and the price for 12 cans is \$1.64.

Now suppose that Betty saves \$.15 out of each \$1.00 that she earns. If she earns \$12.00, how much of her earnings should she save?

If we call her savings s, we can translate the problem into the proportion $\frac{.15}{1.00}=\frac{s}{12.00}$. Study its solution.

$$\frac{.15}{1.00}\times\frac{12}{12}=\frac{s}{12.00} \quad \text{Why?}$$
$$\frac{.15\times 12}{1.00\times 12}=\frac{s}{12.00}$$
$$\frac{1.80}{12.00}=\frac{s}{12.00}$$

The fractions $\frac{1.80}{12.00}$ and $\frac{s}{12.00}$ have equal denominators. Therefore the numerators must be the same. Thus $s=1.80$. Betty should save \$1.80 of her earnings.

Read problems 1 through 5. Plan a solution for each one.

1. Don saved \$1.25 in 5 weeks. At this rate, how much can he plan to save in 15 weeks? $\frac{5}{1.25}=\frac{15}{n}$ \$3.75

2. Judy paid \$1.05 for cans of soup priced at 3 cans for \$.35. How many cans of soup did she buy?

$\frac{3}{.35}=\frac{n}{1.05}$ 9 cans

3. Arthur earned \$1.80 in 6 hours. At this rate, how much did he earn in 2 hours? $\frac{6}{1.80}=\frac{2}{n}$ \$.60

4. Dick spends \$.45 a day on lunches. Ann spends \$.35. How much money will Ann have spent for lunches during the time that Dick spends \$18.00? $\frac{.45}{18.00}=\frac{.35}{n}$ \$14.00

5. Mr. Davis paid \$5.25 for 15 gallons of gasoline. How much would 5 gallons of the gasoline cost?

$\frac{15}{5.25}=\frac{5}{n}$ \$1.75

Oral Tell the plan you will use in solving problems 1 through 5. Tell how you decided upon your plan.

Written Solve problems 1 through 5. Read each problem again to see if your answer seems reasonable. Write your answer.

MORE PRACTICE PAGE 312

Determining Equivalent Ratios

Suppose you want to discover whether or not two ratios are equivalent. For example, how do you know that $\frac{4}{6}=\frac{10}{15}$? One way to find out is to reduce both fractions as shown.

$$\frac{4}{6}=\frac{2}{3} \quad \text{and} \quad \frac{10}{15}=\frac{2}{3}$$

$$\text{Does } \frac{2}{3}=\frac{2}{3}? \qquad \text{Then } \frac{4}{6}=\frac{10}{15}.$$

A second way to prove that $\frac{4}{6}=\frac{10}{15}$ is to change to a common denominator as follows.

$$\frac{4}{6}=\frac{10}{15}$$

$$1\times\frac{4}{6}=1\times\frac{10}{15} \quad \text{Why?}$$

$$\frac{15}{15}\times\frac{4}{6}=\frac{6}{6}\times\frac{10}{15}$$

$$\frac{60}{90}=\frac{60}{90}$$

This proves again that $\frac{4}{6}=\frac{10}{15}$.

In the solution above observe that the denominator, 15, of $\frac{10}{15}$ was used to find $\frac{15}{15}$ as the multiplier for $\frac{4}{6}$. The denominator, 6, of $\frac{4}{6}$ was used to find $\frac{6}{6}$ as the multiplier for 15. You could do this because *any* fraction for 1 may be used. Choosing $\frac{15}{15}$ and $\frac{6}{6}$ as names for 1 was a good choice.

MORE PRACTICE PAGE 312

Recall that two equal fractions, or ratios, which have the same denominators must also have the same numerators. Therefore the test for finding whether or not two fractions are equivalent can be simplified. You need only to find out whether the two numerators will be the same when you multiply each numerator by the denominator of the other as shown below.

$$\frac{4}{6}=\frac{10}{15} \qquad \text{Think: } 15\times4=60$$
$$6\times10=60$$

The two numerators must be 60. Then the denominators will be identical. Why? Does the test prove that $\frac{4}{6}=\frac{10}{15}$?

Suppose you wanted to use the test to find if $\frac{4}{5}=\frac{5}{7}$ or if $\frac{4}{5}\neq\frac{5}{7}$. You can apply the test as follows.

$$\frac{4}{5}=\frac{5}{7} \qquad \text{Think: } 7\times4=28$$
$$\frac{4}{5}>\frac{5}{7} \qquad 5\times5=25$$
$$28>25$$

The two numerators are not the same. Therefore $\frac{4}{5}\neq\frac{5}{7}$.

Oral Tell how to decide which pairs of ratios below are equivalent.

See column 1 and above.

	a	*b*	*c*
1.	$\frac{8}{16}, \frac{4}{8}$ = 64 64	$\frac{3}{5}, \frac{12}{20}$ =	$\frac{2}{4}, \frac{4}{7}$ ≠
2.	$\frac{42}{14}, \frac{48}{16}$ =	$\frac{6}{3}, \frac{18}{10}$ ≠	$\frac{9}{12}, \frac{24}{32}$ =
3.	$\frac{17}{68}, \frac{19}{76}$ =	$\frac{58}{102}, \frac{18}{34}$ ≠	$\frac{20}{40}, \frac{52}{26}$ ≠
4.	$\frac{9}{12}, \frac{12}{16}$ =	$\frac{24}{8}, \frac{72}{24}$ =	$\frac{11}{16}, \frac{33}{48}$ =
5.	$\frac{32}{144}, \frac{10}{35}$ ≠	$\frac{10}{15}, \frac{50}{75}$ =	$\frac{32}{9}, \frac{8}{3}$ ≠

Written Write each pair of ratios above. Replace each comma with = or ≠, whichever is correct.

Solving Equations

Suppose you wish to find a number for n which will make $\frac{n}{3}=\frac{20}{5}$. You can make use of the numerator test which you have studied. Recall that this test consists of multiplying each numerator by the denominator of the other fraction. This test will produce the common denominator, 15, for both fractions. Why?

Study the way in which the numerator test is used below.

12

$$\frac{n}{3}=\frac{20}{5}$$

$$n\times5=20\times3$$

$$n\times5=60$$

Since division is the inverse of multiplication, $n\times5=60$ becomes $n=60\div5$ and $n=12$. Then $\frac{12}{3}=\frac{20}{5}$. The number you were seeking was 12. You can prove that $\frac{12}{3}=\frac{20}{5}$ by reducing both fractions.

Here is another example in which you find the number for a in $\frac{7}{21}=\frac{a}{12}$. If you wish, you may first rewrite this proportion as $\frac{a}{12}=\frac{7}{21}$ so you will have a on the left. Study the solution below.

$$a\times21=7\times12$$

$$a\times21=84$$

$$a=84\div21$$

$$a=4$$

Therefore $\frac{4}{12}=\frac{7}{21}$ and a must be 4.

Oral Tell the steps you would take in solving each equation below.

See column 1.

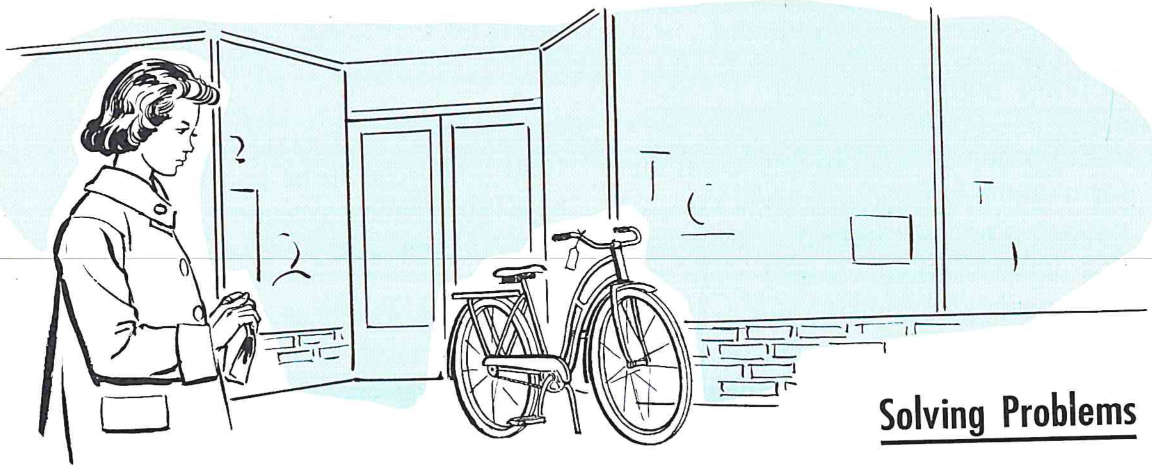
	a	*b*	*c*
1.	$\frac{20}{5}=\frac{x}{6}$ 24	$\frac{30}{15}=\frac{a}{20}$ 40	$\frac{48}{4}=\frac{72}{d}$ 6
2.	$\frac{9}{12}=\frac{p}{36}$ 27	$\frac{24}{6}=\frac{r}{4}$ 16	$\frac{15}{5}=\frac{s}{6}$ 18
3.	$\frac{s}{16}=\frac{7}{4}$ 28	$\frac{w}{30}=\frac{14}{21}$ 20	$\frac{40}{18}=\frac{6}{v}$ $2\frac{7}{10}$
4.	$\frac{9}{15}=\frac{f}{60}$ 36	$\frac{14}{g}=\frac{9}{16}$ $24\frac{8}{9}$	$\frac{k}{54}=\frac{22}{12}$ 99
5.	$\frac{96}{30}=\frac{a}{20}$ 64	$\frac{30}{36}=\frac{25}{u}$ 30	$\frac{a}{5}=\frac{20}{9}$ $11\frac{1}{9}$
6.	$\frac{10}{5}=\frac{a}{6}$ 12	$\frac{c}{6}=\frac{15}{9}$ 10	$\frac{12}{8}=\frac{b}{96}$ 144
7.	$\frac{6}{10}=\frac{9}{d}$ 15	$\frac{18}{6}=\frac{x}{10}$ 30	$\frac{t}{10}=\frac{18}{9}$ 20
8.	$\frac{24}{8}=\frac{6}{x}$ 2	$\frac{8}{10}=\frac{96}{r}$ 120	$\frac{9}{12}=\frac{x}{24}$ 18
9.	$\frac{27}{9}=\frac{2}{p}$ $\frac{2}{3}$	$\frac{25}{t}=\frac{5}{10}$ 50	$\frac{x}{2}=\frac{48}{6}$ 16
10.	$\frac{30}{6}=\frac{e}{8}$ 40	$\frac{n}{5}=\frac{32}{8}$ 20	$\frac{f}{6}=\frac{30}{5}$ 36

Written Copy. Solve each equation in rows 1 through 10 above. Check each answer.

A quick review Write only those pairs of ratios that are equivalent.

	a	*b*	*c*
1.	$\underline{\frac{32}{16}, \frac{6}{3}}$	$\frac{9}{25}, \frac{4}{10}$	$\underline{\frac{8}{10}, \frac{40}{50}}$
2.	$\underline{\frac{11}{22}, \frac{72}{144}}$	$\frac{32}{8}, \frac{4}{2}$	$\underline{\frac{20}{60}, \frac{200}{600}}$
3.	$\frac{17}{25}, \frac{9}{12}$	$\underline{\frac{24}{10}, \frac{36}{15}}$	$\underline{\frac{14}{60}, \frac{7}{30}}$
4.	$\frac{9}{15}, \frac{54}{75}$	$\frac{35}{50}, \frac{12}{16}$	$\frac{13}{16}, \frac{9}{10}$

MORE PRACTICE PAGE 313



Solving Problems

Many of the problems on these two pages can be solved by using ratios and proportions. As you read each problem, plan a solution. Reminder: More than one solution is possible. Write your plan, but do not solve the problems.

1. Linda plans to save $5 every 4 weeks to use in buying a bicycle that costs $40. How many weeks will it take her to save the money for the bicycle? $\frac{5}{4}=\frac{40}{n}$ 8 32 wk. 4×8

2. Mr. Sanders drove 95 miles in 2 hours. At this rate, how many hours will a trip of 380 miles take? $\frac{95}{2}=\frac{380}{n}$ 8 hr.

3. Mrs. Johnson invited 15 people to a party. She planned to buy 3 pounds of meat for each 8 guests. How many pounds of meat should she buy for the party? $\frac{3}{8}=\frac{n}{15}$ $5\frac{5}{8}$ lb.

4. Mr. Parks was paid $160 for a job that he finished in 28 hours. At this rate, how much should he be paid for a job that takes 42 hours? $\frac{160}{28}=\frac{n}{42}$ $240

5. A trucker hauled 7200 pounds of wheat in 2 loads. He still has 18,000 pounds of wheat to haul. How many loads, the same size as the first 2, will he have? $\frac{7200}{2}=\frac{18000}{n}$ 5 loads

6. Mrs. Cox paid $180 for 12 square yards of carpeting. She needs 5 more yards of the carpeting. How much will the 5 yards cost? $\frac{180}{12}=\frac{n}{5}$ $75

7. For every $10 that Bob saved, his father gave him $2 to add to the account. Bob saved $60. How much money did his father have to add to the account? $\frac{10}{2}=\frac{60}{n}$ $12

8. A jet plane made a flight between two cities in 1 hour 5 minutes. A small plane made the same trip in 4 hours. The speed of the jet plane was how many times the speed of the small plane? $\frac{65}{240}=\frac{1}{n}$ $3\frac{9}{13}$

9. A baseball team won 40 of its last 75 games. At this rate, how many of its scheduled 162 games should the team win? $\frac{40}{75}=\frac{n}{162}$ 86 games

10. One player made 32 hits in 80 times at bat. At this rate, how many hits would he expect to make after 10 more trips to the plate?

$\frac{32}{80}=\frac{n}{90}$ 36 hits

11. A gallon of paint covers 320 square feet of surface. Mr. Allyn is painting a fence having 800 square feet of surface. How many gallons of paint does he need?

$\frac{1}{320}=\frac{n}{800}$ $2\frac{1}{2}$ gal.

12. After paying for a pair of shoes that cost \$7.98, Mary had \$2.19 in her purse. How much money was in the purse before she paid for the shoes?

7.98+2.19=n \$10.17

13. In a test run, Car A averaged 27.325 miles per gallon of gasoline. Car B averaged 26.918 miles per gallon. How much better was the record of Car A than that of Car B?

27.325-26.918=n .407 mpg

14. Betty bought 4 cans of fruit juice for \$.75 and 10 pounds of potatoes for \$.79. The tax amounted to \$.06. How much change should she get from \$5.00?

500- [(4x75)+(10x79)+6]=n \$1.15

15. Mr. Baxter used 35.6 gallons of gasoline on a trip. His total cost for gasoline was \$13.14. How much, on the average, did he pay for each gallon? 13.14÷35.6=n \$.37

16. In 1960 Ray Norton tied the world's record of 9.3 seconds for the 100 yard race. At the same rate, how many seconds would it take him to run 220 yards?

$\frac{9.3}{100}=\frac{n}{220}$ 20.45 sec.

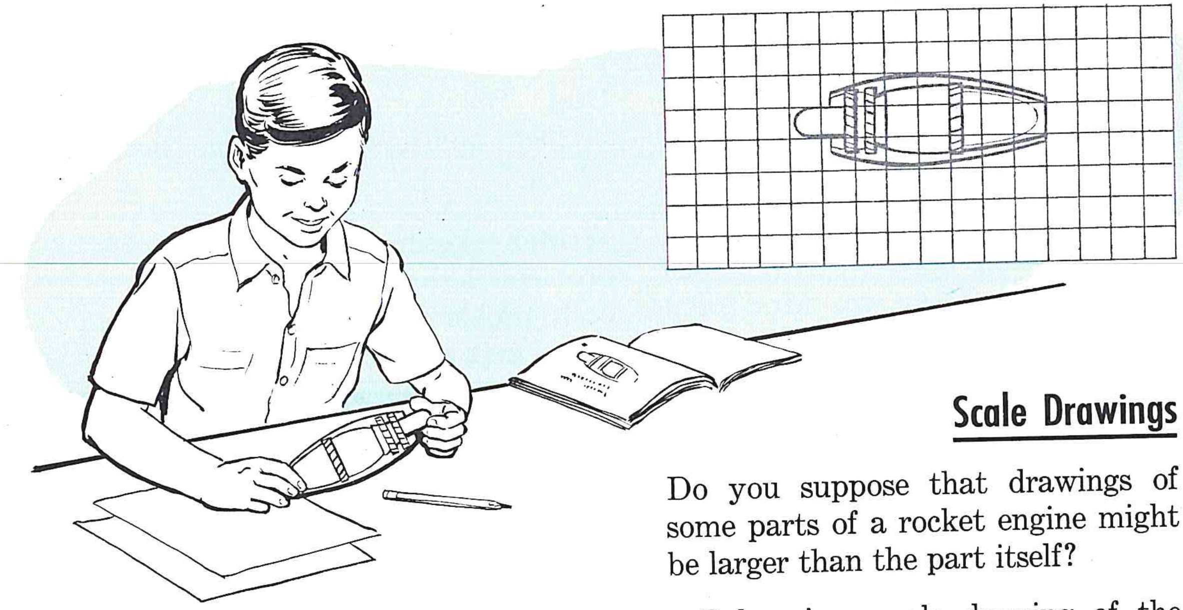
Oral Tell your plan for solving each problem and how you reached your decision.

Written Estimate the answers for problems 1 through 16. As you solve each problem, compare your computed and estimated answers. Then read the problem again to determine if your answer is reasonable. Write your answer.

Estimates will vary.

A quick review Solve for n.

	a	*b*
1.	$\frac{13}{16}+\frac{1}{4}=n$ $1\frac{1}{16}$	$\frac{9}{22}$ $n=1\frac{1}{2}\div3\frac{2}{3}$
2.	$\frac{17}{32}-\frac{3}{8}=n$ $\frac{5}{32}$	$\frac{5}{8}$ $n=2\frac{1}{4}-1\frac{5}{8}$
3.	$\frac{5}{6}\times\frac{9}{10}=n$ $\frac{3}{4}$	$6\frac{2}{3}$ $n=2\times3\frac{1}{3}$
4.	$\frac{4}{5}\div\frac{2}{3}=n$ $1\frac{1}{5}$	$3\frac{5}{9}$ $n=2\frac{2}{3}\div\frac{3}{4}$
5.	$.9+.6+.5=n$ 2.0	121.0 $n=69.3+51.7$
6.	$9.7-.9=n$ 8.8	4.39 $n=6.30-1.91$
7.	$.14\times.09=n$ .0126	.0445 $n=.89\times.05$
8.	$.28\div.14=n$ 2	.2 $n=.72\div3.6$
9.	$(3\frac{1}{6}+9\frac{9}{10})-4\frac{1}{2}=n$ $8\frac{17}{30}$	
10.	$(37\frac{2}{3}-15\frac{1}{4})+3\frac{7}{10}=n$ $26\frac{7}{60}$	
11.	$(9.6+3.17+4.13)-5=n$ 11.9	
12.	$(n-.3)\times.9=.72$ 1.1	
13.	$(357.12-257.08)+n=300.04$ 200	
14.	$(18\times1000)\div90=n$ 200	



Scale Drawings

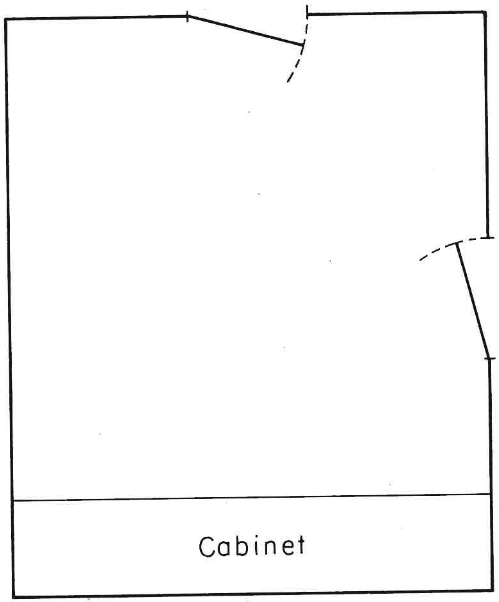
Bob is examining a model of a rocket engine. The ratio between the measures of each part of the model and each part of the engine is always the same. For that reason the model is called a *scale model*. As you would expect, the scale model has the same shape as the engine itself.

The designers of the engine began with an idea of how the engine would look. Then they made a rough drawing of it. Finally they made finished drawings of the engine and of each of its parts. In this drawing, they indicated the ratio of the measure of the drawing to the measure of the object. Drawings of this kind are called **scale drawings.**

The measure of a scale drawing may be smaller, equal to, or larger than the measure of the real object.

Do you suppose that drawings of some parts of a rocket engine might be larger than the part itself?

Below is a scale drawing of the floor plan of a dining room.



Scale: $\frac{1}{4}$ in. = 1 ft.

In this case the drawing is smaller than the room. The scale indicates that a length of $\frac{1}{4}$ inch on the drawing represents a length of 1 foot in the room itself. This scale may also be expressed as in the ratio at the right. Both numbers are in the same measure, inches.

$$\frac{\frac{1}{4}}{12}$$

Suppose you wish to use the scale in finding the width of the room in feet. The width of the drawing is $2\frac{1}{2}$ inches. Let m represent the measure of the room in inches. Then the problem can be translated into a proportion as follows.

$$\frac{\frac{1}{4}}{12}=\frac{2\frac{1}{2}}{m}$$

$$m\times\tfrac{1}{4}=12\times2\tfrac{1}{2}$$

$$m\times\tfrac{1}{4}=30$$

$$m=30\div\tfrac{1}{4}=120$$

The room is 120 inches, or 10 feet wide.

Now suppose you wished to use a proportion to find the length of the oom in inches and then in feet. The ength of the drawing is 3 inches. To find its length in inches you can use a proportion.

Study the solution below.

$$\frac{\frac{1}{4}}{3}=\frac{12}{m}$$

$$m\times\tfrac{1}{4}=36$$

$$m=36\times\tfrac{4}{1}=144$$

The room measures 144 inches or 2 feet in length.

What proportion would you use in finding the width in inches and then in feet and inches of the cabinet? Of each doorway?

Oral Tell whether you would use a scale that would make a scale drawing smaller, larger, or equal in measure to each object below.

1. A desk top smaller
2. A bridge smaller
3. A racing car smaller
4. A microbe larger

Tell the reason for the steps taken in finding a replacement for n.

5. $\frac{\frac{1}{6}}{12}=\frac{\frac{4}{6}}{n}$

$n\times\frac{1}{6}=12\times\frac{4}{6}$ numerator test

$n\times\frac{1}{6}=8$ multiply

$n=8\div\frac{1}{6}=48$ inverse

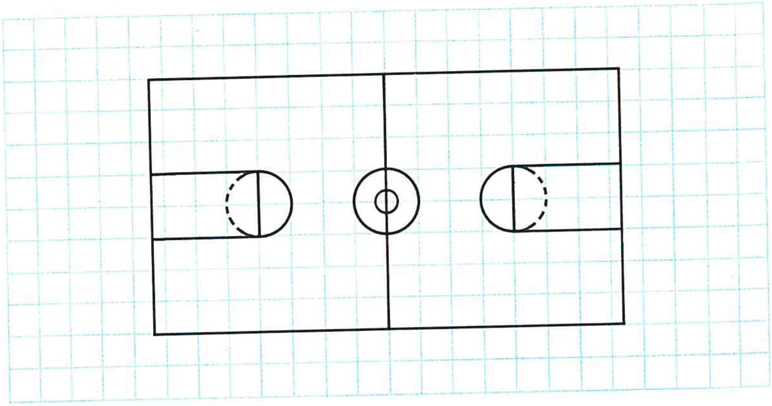
Written Solve for n.

	a	*b*	*c*
1.	$\frac{\frac{1}{4}}{12}=\frac{\frac{12}{4}}{n}$ 144	$\frac{\frac{1}{4}}{12}=\frac{\frac{4}{4}}{n}$ 48	$\frac{\frac{1}{4}}{12}=\frac{\frac{9}{4}}{n}$ 108
2.	$\frac{\frac{1}{8}}{12}=\frac{\frac{3}{8}}{n}$ 36	$\frac{\frac{3}{8}}{12}=\frac{\frac{6}{8}}{n}$ 24	$\frac{\frac{1}{8}}{12}=\frac{\frac{12}{2}}{n}$ 576
3.	$\frac{\frac{1}{2}}{12}=\frac{\frac{8}{2}}{n}$ 96	$\frac{\frac{1}{2}}{12}=\frac{\frac{6}{2}}{n}$ 72	$\frac{\frac{1}{2}}{12}=\frac{\frac{12}{2}}{n}$ 144
4.	$\frac{\frac{1}{16}}{12}=\frac{\frac{32}{16}}{n}$ 384	$\frac{\frac{1}{16}}{12}=\frac{\frac{9}{16}}{n}$ 108	$\frac{\frac{3}{16}}{12}=\frac{\frac{18}{16}}{n}$ 72

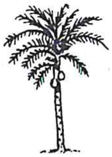
B

Basketball
Court

1 square=6 ft.



A



Date Palm
($\frac{1}{1000}$)

Using a Scale Drawing

In dictionaries you sometimes find the scale of a drawing indicated by a fraction as shown in A above. In A the ratio of the measure of the drawing to the measure of the tree is equal to the ratio of 1 to 1000.

The measure of the height of the tree is about $\frac{3}{4}$ in. To find the height of the tree you may use the solution shown below.

$$\frac{\frac{3}{4}}{h}=\frac{1}{1000} \quad \text{and} \quad h=1000\times\frac{3}{4}$$

The height of the palm tree is approximately 750 inches or $62\frac{1}{2}$ feet. What is the greatest spread of its branches?

Look at the scale drawing of a basketball court in B. The length of the court expressed in the number of sides of a square is approximately $14\frac{3}{4}$ squares. The side of each square represents a length of 6 feet. Since $6\times14\frac{3}{4}=88\frac{1}{2}$, the overall length of the court is approximately $88\frac{1}{2}$ feet. What is the measure of the width of the court in sides of a square? In feet?

MORE PRACTICE PAGE 313

Oral Use scale drawing B to help you answer questions 1 and 2.

1. What is the measure of the shortest distance between the free-throw line and the end of the court? About 20 ft.

2. What is the distance between the center line and the free-throw line? 24 ft.

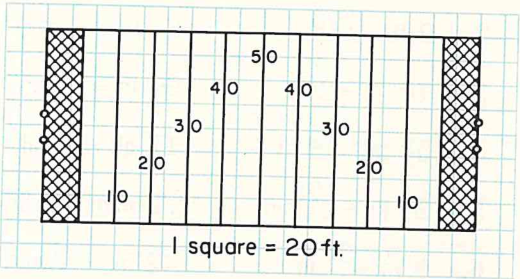
Answer questions 3 and 4.

3. In a dictionary a turtle was drawn $\frac{1}{4}$ actual size. The drawing was $1\frac{1}{2}$ inches long and $\frac{3}{4}$ inches high.

What was the length and the height of the turtle? Length: 6 in.; Height: 3 in.

4. A whale was drawn to a scale of $\frac{1}{300}$. The drawing was $2\frac{1}{4}$ inches in length. What was the length of the whale? 675 in. or $56\frac{1}{4}$ ft.

Written Use the scale drawing of a football field. Answer questions 1, 2, and 3.

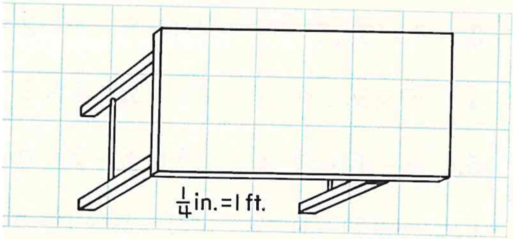


1. What is the length of the field in sides of a square? In feet? 18 sides; 360 ft.

2. What is the measure of the width of the field in sides of a square? In feet? 8 sides; 160 ft.

3. What is the distance between the 50-yard line and the goal posts? 180 ft.

Use the scale drawing of a table top. Answer questions 4, 5, and 6.

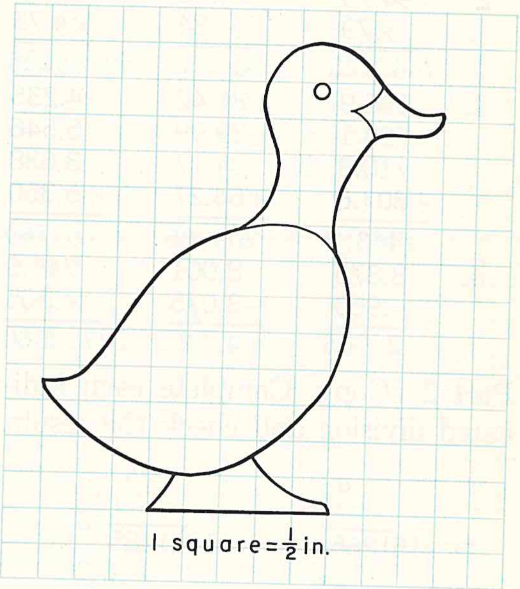


4. What is the measure of the length of the drawing of the table? $1\frac{1}{2}$ in.

5. What is the measure of the length of the table? 6 ft.

6. What is the distance around the outer edge of the table? 18 ft.

Use the scale model of the toy duck. Answer questions 7 and 8.



7. What is the length of the model duck in sides of a square? Of the duck in inches? 13 sides; $6\frac{1}{2}$ in.

8. What is the width of the body of the model duck in sides of a square? Of the duck in inches? 11 sides; $5\frac{1}{2}$ in.

Can you do this? Use paper ruled in $\frac{1}{4}$ in. squares. Make a scale drawing of the floor plan of your classroom.

Review and Practice

Part 1 Copy. Find each sum, difference, or product by performing the indicated operation.

	a	*b*	*c*
1.	95.47 +34.86 130.33	900.2 −587.6 312.6	5.935 −3.987 1.948
2.	94.75 ×79 7485.25	752.8 ×.84 632.352	8.435 ×4.75 40.06625
3.	196.9 728.1 710.8 +807.6 2443.4	31.42 29.99 84.17 +64.27 209.85	4.235 5.546 3.638 +5.350 18.769
4.	3.875 −.969 2.906	8.001 −3.075 4.926	745.6 ×.305 227.4080

Part 2 Copy. Complete each indicated division and check the result.

	a	*b*
1.	$\begin{array}{r} 57\frac{2}{3} \\ 15\overline{)865} \end{array}$	$\begin{array}{r} 91\frac{2}{13} \\ 13\overline{)1185} \end{array}$
2.	$\begin{array}{r} 63 \\ 25\overline{)1575} \end{array}$	$\begin{array}{r} 4089\frac{3}{7} \\ 21\overline{)85878} \end{array}$
3.	$\begin{array}{r} 324\frac{23}{356} \\ 356\overline{)115367} \end{array}$	$\begin{array}{r} 2143 \\ 348\overline{)745764} \end{array}$
4.	$\begin{array}{r} 120 \\ .003\overline{).36} \end{array}$	$\begin{array}{r} 24000 \\ .002\overline{)48} \end{array}$
5.	$\begin{array}{r} .4 \\ .27\overline{).108} \end{array}$	$\begin{array}{r} 2.5 \\ .12\overline{).30} \end{array}$
6.	$\begin{array}{r} .54 \\ 16\overline{)8.64} \end{array}$	$\begin{array}{r} 30 \\ 6.50\overline{)195} \end{array}$
7.	$\begin{array}{r} .90 \\ 1.6\overline{)1.440} \end{array}$	$\begin{array}{r} 18. \\ 9.25\overline{)166.50} \end{array}$

Part 3 Copy. Find a replacement for each blank so the sentences become true. Reminder: The sign = between two measures means that the measures are equivalent.

	a	*b*
1.	2 mi. = $\underline{10560}$ ft.	$4\frac{1}{2}$ mi. = $\underline{23760}$ ft.
2.	9 ft. = $\underline{108}$ in.	$8\frac{1}{4}$ ft. = $\underline{99}$ in.
3.	18 in. = $\underline{1\frac{1}{2}}$ ft.	96 in. = $\underline{8}$ ft.
4.	$5\frac{1}{2}$ gal. = $\underline{22}$ qt.	17 qt. = $\underline{4\frac{1}{4}}$ gal.
5.	$3\frac{1}{4}$ hr. = $\underline{195}$ min.	72 min. = $\underline{1\frac{1}{5}}$ hr.
6.	$\frac{5}{6}$ yd. = $\underline{30}$ in.	42 in. = $\underline{1\frac{1}{6}}$ yd.
7.	$\frac{1}{4}$ mi. = $\underline{1320}$ ft.	10 pt. = $\underline{5}$ qt.
8.	$5\frac{1}{2}$ bu. = $\underline{22}$ pk.	$6\frac{1}{2}$ hr. = $\underline{390}$ min.
9.	48 hr. = $\underline{2}$ da.	28 da. = $\underline{4}$ wk.

Part 4 Solve each of the following equations for n.

	a	*b*
1.	$(9\times6)\div2=\overset{27}{n}$	$50\times(25-9)=\overset{800}{n}$
2.	$(8-8)\times6=\overset{0}{n}$	$81\div(36\div9)=\overset{20\frac{1}{4}}{n}$
3.	$8\div(3\times6)=\overset{\frac{4}{9}}{n}$	$(96\div8)\div\overset{6}{n}=2$
4.	$(7\times3)\div\overset{3}{n}=7$	$\overset{1}{n}+(32\div4)=9$
5.	$\overset{48}{n}-(6\times8)=0$	$(54\div6)\div3=\overset{3}{n}$
6.	$(8+5)-\overset{4}{n}=9$	$(56-6)\div\overset{10}{n}=5$

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. A correspondence is determined by matching one set with another. (187)

2. A relation or correspondence between the numbers of two sets is called a ratio. (188)

3. A ratio may be expressed as a fraction. (189)

4. A proportion is a statement of equality between two ratios. (191)

5. A pair of ratios is equivalent when the two numerators are the same when you multiply each one by the denominator of the other fraction. (196).

6. The measure of a scale drawing may be smaller, equal to, or larger than the measure of the real object. (200)

Words to Know

1. One-to-one correspondence, two-to-three correspondence (187)
2. Ratio (188)
3. Equivalence row (190)
4. Proportion (191)
5. Scale drawing (200)

Questions to Discuss

See T205 for answers.

1. What is meant by a two-to-three correspondence? (187)

2. How is a proportion different from a ratio? (191)

3. How do you determine whether or not two ratios are equivalent ratios? (196)

4. How do you solve for n in $\frac{n}{6}=\frac{60}{5}$? (197) $\frac{12}{6}=\frac{\quad}{5}$

5. In what way does a scale drawing of an object differ from a picture of the object? (200)

Written Practice

Express each of the following as ratios in fractional form.

1. 2 runs in 5 innings (189) $\frac{2}{5}$
2. 8 riders for 2 cars (189) $\frac{8}{2}$ or $\frac{4}{1}$

Solve for n.

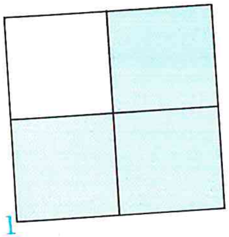
	a	*b*	
3.	$\frac{8}{5}=\frac{n}{20}$ 32	$\frac{10}{30}=\frac{n}{6}$ 2	(190)
4.	$\frac{9}{18}=\frac{20}{n}$ 40	$\frac{.30}{1.50}=\frac{n}{100}$ 20	(195)
5.	$\frac{20}{50}=\frac{n}{75}$ 30	$\frac{12}{4}=\frac{9}{n}$ 3	(197)
6.	$\frac{\frac{1}{8}}{12}=\frac{3}{n}$ 288	$\frac{\frac{1}{16}}{12}=\frac{6}{n}$ 1152	(201)

Self-Evaluation

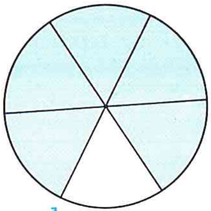
Part 1 Use a ratio to compare the white part of each drawing below to the part that is in blue.

1.

a $\frac{1}{3}$

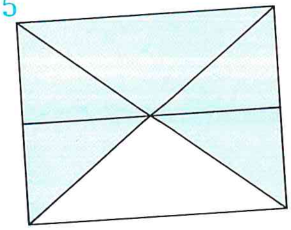


b $\frac{1}{5}$

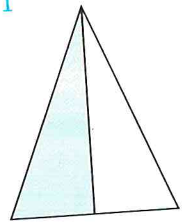


2.

$\frac{1}{5}$

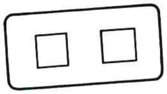


$\frac{1}{1}$

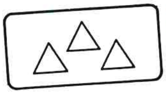


Use a ratio to compare the number of Set A to that of Set B in the drawings below.

3. Set A

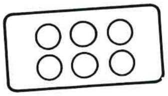


Set B

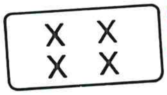


$\frac{2}{3}$

4. Set A

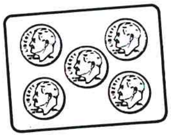


Set B

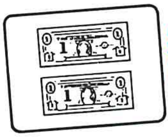


$\frac{6}{4}$ or $\frac{3}{2}$

5. Set A



Set B



$\frac{5}{2}$

Part 2 Replace each letter with a numeral to make the following equations true.

	a	*b*	*c*
1.	$\frac{18}{12}=\frac{54}{h}$ 36	$\frac{16}{32}=\frac{32}{a}$ 64	$\frac{16}{3}=\frac{n}{12}$ 64
2.	$\frac{22}{16}=\frac{c}{48}$ 66	$\frac{19}{57}=\frac{1}{r}$ 3	$\frac{60}{20}=\frac{x}{10}$ 30
3.	$\frac{38}{p}=\frac{8}{16}$ 76	$\frac{42}{n}=\frac{50}{70}$ 58.8	$\frac{50}{20}=\frac{n}{14}$ 35
4.	$\frac{54}{12}=\frac{36}{b}$ 8	$\frac{27}{18}=\frac{90}{t}$ 60	$\frac{95}{175}=\frac{38}{h}$ 70
5.	$\frac{30}{45}=\frac{v}{9}$ 6	$\frac{8}{100}=\frac{12}{s}$ 150	$\frac{25}{12}=\frac{n}{100}$ $208\frac{1}{3}$

Part 3 Rewrite. Replace each comma with = or ≠, whichever makes each statement true.

	a	*b*	*c*
1.	$\frac{10}{5}, \frac{30}{50}$ ≠	$\frac{5}{17}, \frac{6}{21}$ ≠	$\frac{9}{32}, \frac{3}{11}$ ≠
2.	$\frac{12}{27}, \frac{4}{9}$ =	$\frac{35}{45}, \frac{10}{13}$ ≠	$\frac{30}{10}, \frac{36}{12}$ =
3.	$\frac{51}{68}, \frac{3}{4}$ =	$\frac{24}{30}, \frac{8}{10}$ =	$\frac{106}{85}, \frac{6}{5}$ ≠

Part 4 Solve these problems.

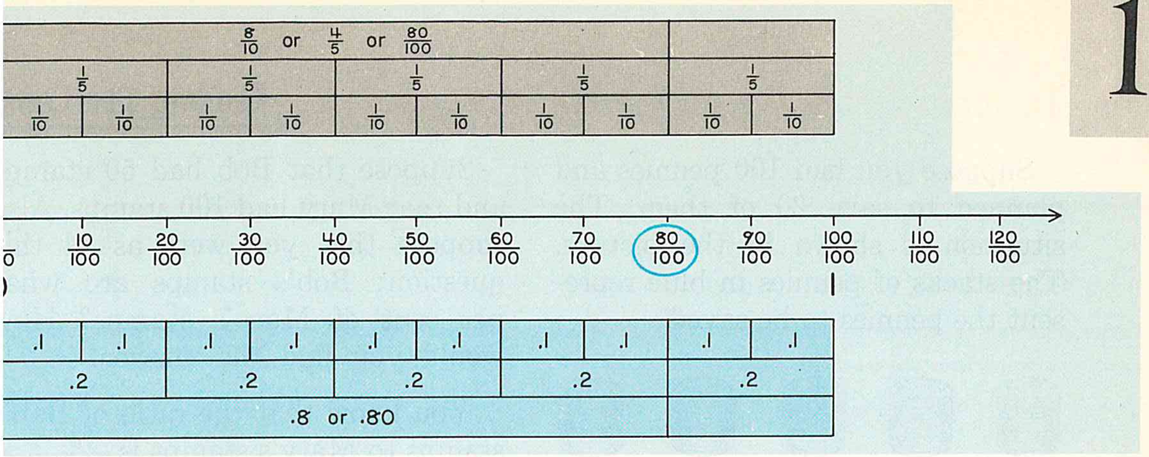
1. Mr. Cox spent 2 hours in making 5 boxes. At the same rate, how many boxes can he make in an 8-hour day? $\frac{2}{5}=\frac{8}{n}$ 20 boxes

2. Betty can buy 2 pounds of hamburger for 98¢. How much would 5 pounds of the hamburger cost?

$\frac{2}{98}=\frac{5}{n}$ $2.45

3. John had 18 words correct on a test of 20 words. At this rate, how many words might he expect to have correct in a 50-word test?

$\frac{18}{20}=\frac{n}{50}$ 45 words



$\frac{8}{10}$ or $\frac{4}{5}$ or $\frac{80}{100}$
$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$
$\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$
0
$\frac{10}{100}$ $\frac{20}{100}$ $\frac{30}{100}$ $\frac{40}{100}$ $\frac{50}{100}$ $\frac{60}{100}$ $\frac{70}{100}$ $\frac{80}{100}$ $\frac{90}{100}$ $\frac{100}{100}$ $\frac{110}{100}$ $\frac{120}{100}$
.1 .1 .1 .1 .1 .1 .1 .1 .1 .1
.2 .2 .2 .2 .2
.8 or .80

enaming Numbers

The figure in gray above the numer line was divided into 10 and hen into 5 parts of equal length. Iow many of the parts are $\frac{1}{10}$'s? Iow many are $\frac{4}{5}$'s or $\frac{8}{10}$'s? Do you ee that $\frac{8}{10}$, $\frac{4}{5}$, and $\frac{80}{100}$ are all names or the same number?

Look at the figure in blue that is elow the number line. How many f its parts are .1's? How many f its parts are .2's? How many 1's are equal to .80's? Do you see hat .8 and .80 are names for the umber that, in fractional form, is xpressed as $\frac{8}{10}$ or $\frac{80}{100}$? By using he figures in gray and in blue, you an see again that each number has any names. Use the above diagram o help you rename the following in ifferent ways as fractional numbers nd then as decimal numbers.

$\frac{20}{100}$, $\frac{30}{100}$, $\frac{40}{100}$, $\frac{50}{100}$, $\frac{60}{100}$, $\frac{70}{100}$

There is still another way to rename fractional or decimal numbers expressed in hundredths. You can use the Latin-derived word **per cent** which means *hundredths.*

For example, read:

$\frac{1}{100}$ or .01 as one per cent
$\frac{2}{100}$ or .02 as two per cent
$\frac{10}{100}$ or .10 as ten per cent

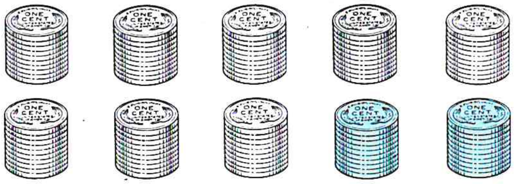
How would you expect to read $\frac{100}{100}$, $\frac{101}{100}$, and $\frac{110}{100}$ using per cent in place of hundredths?

Oral Read. Use *per cent* to replace *hundredths.* See above.

	a	*b*	*c*	*d*
1.	$\frac{5}{100}$	.05	$\frac{17}{100}$	.17
2.	$\frac{35}{100}$	.35	$\frac{99}{100}$	.99
3.	$\frac{102}{100}$	$\frac{120}{100}$	$\frac{110}{100}$	$\frac{115}{100}$

Reading Per Cents

Suppose you had 100 pennies and planned to save 20 of them. The situation is shown in the picture. The stacks of pennies in blue represent the pennies to be saved.



Do you see the 20 pennies out of 100 pennies which are to be saved? The comparison of pennies saved to pennies earned may be expressed by the ratio $\frac{20}{100}$. The ratio $\frac{20}{100}$ is written as a fraction, but it is read *twenty per cent.* How would you express a ratio of 30 pennies to 100 pennies as hundredths and then as per cent? What are some other ratios which have a denominator of one hundred? Express each one first as a fraction and then as a per cent.

In the arithmetic language, signs take the place of words. For example you use $\neq$ for *is not equal to.* You have probably seen the sign that takes the place of per cent. It is called the **per cent sign.** The sign is made as shown at the right. To read 20%, say *twenty per cent.*

%

Suppose that Bob had 50 stamps and that Mary had 100 stamps. Also suppose that you were asked this question: Bob's stamps are what per cent of Mary's stamps? How would you find the answer?

You know that the ratio of Bob's stamps to Mary's stamps is $\frac{50}{100}$, and that $\frac{50}{100}$ may be read and written as *50 per cent.* The number of Bob's stamps is 50% of Mary's stamps.

In the above problem, two completely different sets of stamps are being compared. That is, one of the sets is not a subset of the other. You will often use per cent to make a comparison of this kind.

Now suppose that a library had seating space for 100 pupils and that 110 pupils were in the library. What is the ratio of the number of pupils to the number of seats? Since $\frac{100}{100}$ would be 100%, the ratio of 110 to 100 must be more than 100%.

The ratio is shown below.

$$\frac{110}{100}\begin{array}{l}\text{Number of pupils}\\ \text{Number of seats}\end{array}$$

As a ratio, you read $\frac{110}{100}$ as *110 to 100.* As per cent you read $\frac{110}{100}$ like this: *110 per cent.* How would you use the sign % to express the ratio $\frac{110}{100}$?

Oral Read each statement below. Replace the sign % with the words *per cent.*

1. The shrinkage of this material is less than 1%.

2. Only 5% of the pupils were absent this morning.

3. The number of cars on a highway has increased by 200%.

Answer these questions about sentences 1 through 3 above.

4. Is a shrinkage of less than 1% more than, the same as, or less than a shrinkage of 1 inch out of every 100 inches? The same as

5. If 5% of the pupils were absent, how many pupils out of every hundred were absent? How many pupils out of every hundred were present? 5 pupils; 95 pupils

6. If the number of cars on the highway increased by 200% in ten years, how many cars are now on the highway for every 1 car that was on it ten years ago? For every 50 cars that were on it ten years ago? 3 cars; 150 cars

Read each statement below.

	a	*b*
7.	9 per cent	9%
8.	75 per cent	75%
9.	105 per cent	105%
10.	150 per cent	150%

Read each of the following as a ratio, then as a fraction, and then as a per cent.

	a	*b*	*c*	*d*
11.	$\frac{4}{100}$ 4%	$\frac{11}{100}$ 11%	$\frac{33}{100}$ 33%	$\frac{79}{100}$ 79%
12.	$\frac{81}{100}$ 81%	$\frac{67}{100}$ 67%	$\frac{17}{100}$ 17%	$\frac{59}{100}$ 59%
13.	$\frac{98}{100}$ 98%	$\frac{6}{100}$ 6%	$\frac{55}{100}$ 55%	$\frac{20}{100}$ 20%
14.	$\frac{5}{100}$ 5%	$\frac{8}{100}$ 8%	$\frac{10}{100}$ 10%	$\frac{15}{100}$ 15%
15.	$\frac{138}{100}$ 138%	$\frac{116}{100}$ 116%	$\frac{139}{100}$ 139%	$\frac{199}{100}$ 199%

Written Write each number in rows 11 through 15 above as per cent using the % sign. Express each of the following first as a ratio and then as a per cent.

16. 4 pennies out of 100 pennies $\frac{4}{100}$; 4%

17. 95 cents out of every 100 cents $\frac{95}{100}$; 95%

18. 22 boys out of every 100 boys $\frac{22}{100}$; 22%

19. 80 words out of 100 words $\frac{80}{100}$; 80%

20. 106 girls for 100 boys $\frac{106}{100}$; 106%

A quick review Solve for n.

	a	*b*
1.	39.90 $.42\times95=n$	10.80 $n=72\times.15$
2.	.065 $.52\div8=n$	.17 $34\div n=200$
3.	37.50 $150\times.25=n$	.12 $.96\div8=n$
4.	3 $.75\div n=.25$	2 $.20\times n=.40$
5.	20 $.50\times n=10$	.0875 $n=.05\times1.75$

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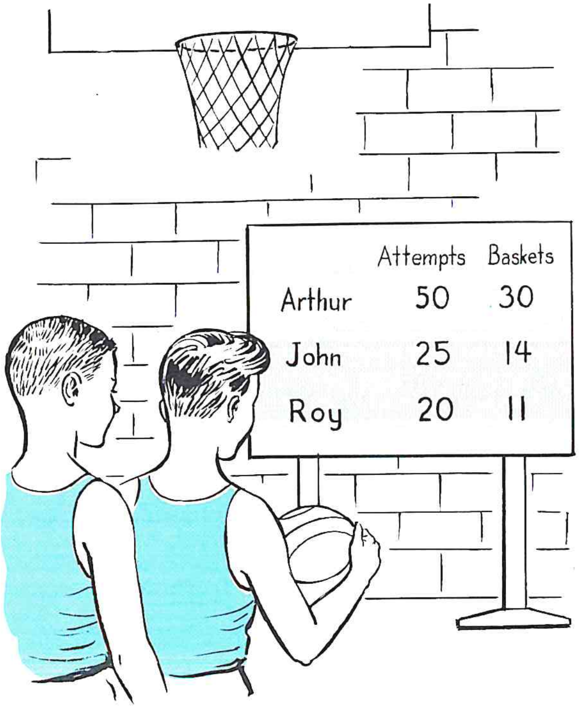

Using Per Cent in Comparison

Suppose that you wanted to know which boy made the best record. One way to find out is to express the ratio of baskets to attempts so that each ratio has the same denominator, 100. Study the examples below.

A $\frac{30}{50}=\frac{n}{100}$ $\quad \frac{30}{50}=\frac{2\times30}{2\times50}=\frac{60}{100}$

B $\frac{14}{25}=\frac{n}{100}$ $\quad \frac{14}{25}=\frac{4\times14}{4\times25}=\frac{56}{100}$

C $\frac{11}{20}=\frac{n}{100}$ $\quad \frac{11}{20}=\frac{5\times11}{5\times20}=\frac{55}{100}$

In A, the ratio of the number of baskets to the number of attempts is $\frac{30}{50}$. What is the ratio of the number of baskets to the number of attempts in B? In C? By renaming each ratio to give it a denominator of 100, the comparison is easy to make.

Look at A again. The denominator 100 is divisible by 50. Does $100\div50=2$? Then $\frac{2}{2}$ is chosen as a name for 1 and $\frac{30}{50}$ is multiplied by 2. Do you see that $\frac{30}{50}$ is equivalent to $\frac{60}{100}$? What steps were taken in B? In C?

The ratios $\frac{60}{100}$, $\frac{56}{100}$, and $\frac{55}{100}$ are expressed as per cents below.

$\frac{60}{100}$ is 60%

$\frac{56}{100}$ is 56%

$\frac{55}{100}$ is 55%

$60\% > 56\% > 55\%$

$56\% < 60\%$

$55\% < 56\% < 60\%$

Since 60% is greater than 56% and 55%, Arthur made the best record. Since 56% is between 55% and 60%, John's record was second best. Since 55% is less than 56% and 60%, Roy's record was poorest. Observe the use of the signs > and < in determining the best record.

Sometimes you will be working with ratios that have a denominator that will not divide 100. For example, suppose that a boy made 18 baskets in 40 attempts. The ratio of baskets to attempts is $\frac{18}{40}$. Does 40 divide 100 either 2 or 3 times? What might you expect to do so that you can express the ratio $\frac{18}{40}$ as a ratio having a denominator of 100?

Study the solution below.

$$\frac{18}{40}=\frac{18\div 2}{40\div 2}=\frac{9}{20}$$

$$\frac{9}{20}=\frac{n}{100} \qquad \frac{5\times 9}{5\times 20}=\frac{45}{100}$$

By reducing the ratio, it was possible to find an equivalent ratio with a denominator of 20, and 100 is divisible by 20. Then $\frac{18}{40}=\frac{9}{20}=\frac{45}{100}$. The boy made 45% of the baskets he attempted.

Oral Answer the following questions.

1. Why is it not so easy to determine whether $\frac{30}{50}$, $\frac{14}{25}$, or $\frac{11}{20}$ represents the best record for free throws?
The denominators are not alike.

2. In many cases why is it helpful to express all ratios as fractions with a denominator of 100? The ratios can easily be compared.

Determine which of the boys below had the best batting average. Keith

Boy	At Bat	Hits
3. Ronald	25	11
4. James	28	14
5. Roger	40	24
6. Keith	30	18

Read the statements in 7 through 11 as ratios. Tell the steps to take in finding an equivalent ratio with a denominator of 100.

7. 4 runs in 10 hits $\frac{4}{10}=\frac{10\text{x}4}{10\text{x}10}=\frac{40}{100}$

8. 1 red apple to 10 green apples $\frac{1}{10}=\frac{10\text{x}1}{10\text{x}10}=\frac{10}{100}$

9. 60 foreign stamps for 50 American stamps $\frac{60}{50}=\frac{2\text{x}60}{2\text{x}50}=\frac{120}{100}$

10. 12 prizes in each carton of 25 boxes $\frac{12}{25}=\frac{4\text{x}12}{4\text{x}25}=\frac{48}{100}$

11. 18 out of 20 words $\frac{18}{20}=\frac{5\text{x}18}{5\text{x}20}=\frac{90}{100}$

Written Express each of the following ratios as a per cent, using the % sign.

	a	*b*	*c*	*d*
1.	$\frac{9}{100}$ 9%	$\frac{25}{100}$ 25%	$\frac{100}{100}$ 100%	$\frac{110}{100}$ 110%
2.	$\frac{9}{20}$ 45%	$\frac{35}{50}$ 70%	$\frac{31}{25}$ 124%	$\frac{14}{10}$ 140%
3.	$\frac{16}{24}$ $66\frac{2}{3}$%	$\frac{32}{40}$ 80%	$\frac{45}{90}$ 50%	$\frac{60}{24}$ 250%

Write each of the following as a ratio having a denominator of 100.

	a	*b*	*c*	*d*
4.	4% $\frac{4}{100}$	60% $\frac{60}{100}$	85% $\frac{85}{100}$	99% $\frac{99}{100}$
5.	31% $\frac{31}{100}$	55% $\frac{55}{100}$	62% $\frac{62}{100}$	50% $\frac{50}{100}$

For each school below, write a ratio to make a comparison of wins to games played. Then express each ratio with a denominator of 100. Next express each ratio as per cent. See below.

School Team	Games Played	Losses
6. Franklin	8	2
7. Park View	10	4
8. Lincoln	6	3
9. Field Park	8	4
10. McKinley	10	7

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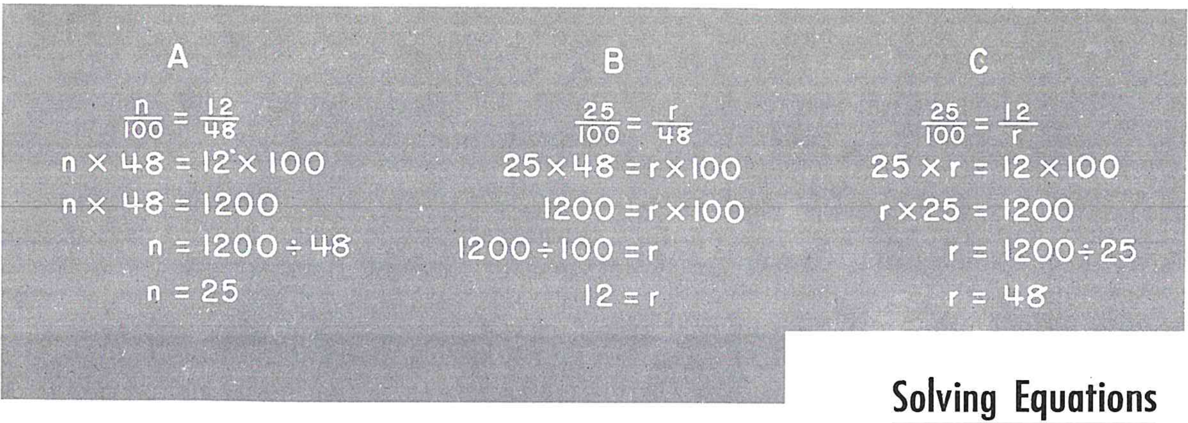
6. $\frac{6}{8}$; $\frac{75}{100}$; 75%

7. $\frac{6}{10}$; $\frac{60}{100}$; 60%

8. $\frac{3}{6}$; $\frac{50}{100}$; 50%

9. $\frac{4}{8}$; $\frac{50}{100}$; 50%

10. $\frac{3}{10}$; $\frac{30}{100}$; 30%



A

$$\frac{n}{100} = \frac{12}{48}$$
$$n \times 48 = 12 \times 100$$
$$n \times 48 = 1200$$
$$n = 1200 \div 48$$
$$n = 25$$

B

$$\frac{25}{100} = \frac{r}{48}$$
$$25 \times 48 = r \times 100$$
$$1200 = r \times 100$$
$$1200 \div 100 = r$$
$$12 = r$$

C

$$\frac{25}{100} = \frac{12}{r}$$
$$25 \times r = 12 \times 100$$
$$r \times 25 = 1200$$
$$r = 1200 \div 25$$
$$r = 48$$

Solving Equations

In each proportion on the board, one of the pair of equivalent ratios shows the relation of some number to 100. In A, you will know what this number is after you find a replacement for n. Which number is compared to 100 in B? In C? Do you recall that the ratio of any number to 100 may be expressed as per cent? In A you can read $\frac{n}{100}$ as n%. In both B and C how do you read $\frac{25}{100}$ as per cent?

Suppose now that you were to make up a problem that could be translated into equation A on the board. You might make up a problem such as the one that is given below.

Andy sold 12 rabbits out of the 48 rabbits he raised. What per cent of his rabbits did he sell?

In equation A, n=25. Then $\frac{n}{100} = \frac{25}{100}$. You use 25 as a replacement for n in n%. Then 12 is 25% of 48.

Andy sold 25% of his rabbits.

Suppose now that you are to make up a problem that could be translated into equation B. In this problem you will tell that you know the per cent. You will also tell that you know that 48 is the number to which an unknown number is to be compared. Here is a sample problem. What number are you asked to find?

Andy sold 25% of his 48 rabbits. How many rabbits did he sell?

In equation B, r=12; so 12 is the number being compared to 48. Then 25% of 48 is 12. Andy sold 12 rabbits.

Now see how you might make up a problem which could have been translated into equation C. Here you know the per cent, which is 25, or $\frac{25}{100}$. You also know that 12 is to be compared to an unknown number. Here is a sample problem.

Andy sold 12 rabbits. This was 25% of his rabbits. How many rabbits did he have before he sold any of them?

In equation C, r=48. Then 12 is 25% of 48. Andy had 48 rabbits before selling any.

Each of the sample problems involved the same three numbers, 25%, 12, and 48. Which of the three numbers was unknown in equation A, in equation B, and in equation C before the equations were solved? Do you see that if any one of the three numbers is missing, you can find it by solving an equation?

When you solve problems similar to those on this page, you can always write a proportion. One of the two ratios is the ratio of the two given numbers; the other is the number given in per cent form, expressed as a fraction with denominator 100.

Oral Tell the proportions you would use in solving each problem below.

1. Mary took a 50-word spelling test. She spelled 90% of the words correctly. How many words did she spell correctly? $\frac{n}{50}=\frac{90}{100}$

2. Ann gave 15 foreign stamps to Mary. This was 20% of Ann's collection of foreign stamps. How many stamps did Ann have before giving any of them to Mary? $\frac{15}{n}=\frac{20}{100}$

3. Sally had a box of paper that held 48 sheets. She used 8 of the sheets. What per cent of the paper had Sally used? $\frac{8}{48}=\frac{n}{100}$

Written Solve each equation.

	a	*b*	*c*
1.	$\frac{8}{40}=\frac{n}{100}$ 20	$\frac{15}{100}=\frac{f}{20}$ 3	$\frac{50}{100}=\frac{10}{a}$ 20
2.	$\frac{25}{100}=\frac{4}{b}$ 16	$\frac{12}{30}=\frac{n}{100}$ 40	$\frac{20}{100}=\frac{g}{30}$ 6
3.	$\frac{30}{100}=\frac{h}{50}$ 15	$\frac{30}{100}=\frac{6}{c}$ 20	$\frac{40}{80}=\frac{n}{100}$ 50
4.	$\frac{16}{400}=\frac{n}{100}$ 4	$\frac{85}{100}=\frac{k}{40}$ 34	$\frac{20}{100}=\frac{18}{d}$ 90
5.	$\frac{60}{100}=\frac{18}{e}$ 30	$\frac{24}{120}=\frac{n}{100}$ 20	$\frac{70}{100}=\frac{m}{20}$ 14
6.	$\frac{4}{20}=\frac{n}{100}$ 20	$\frac{30}{100}=\frac{a}{40}$ 12	$\frac{8}{x}=\frac{50}{100}$ 16
7.	$\frac{20}{40}=\frac{n}{100}$ 50	$\frac{48}{240}=\frac{r}{100}$ 20	$\frac{35}{100}=\frac{m}{10}$ 3.5
8.	$\frac{30}{100}=\frac{9}{s}$ 30	$\frac{72}{360}=\frac{n}{100}$ 20	$\frac{10}{100}=\frac{2}{a}$ 20

A quick review

1. $6205+2806+419=r$ 9430

2. $361\frac{1}{2}+717\frac{3}{4}+280\frac{2}{3}=t$ $1359\frac{11}{12}$

3. $.07+.85+.15+.90=x$ 1.97

4. $w=4.856+.908+.543$ 6.307

5. $c=5000-3001$ 1999

6. $735426-314289=m$ 421137

7. $a=(\frac{3}{4}+\frac{1}{8})-\frac{3}{4}$ $\frac{1}{8}$

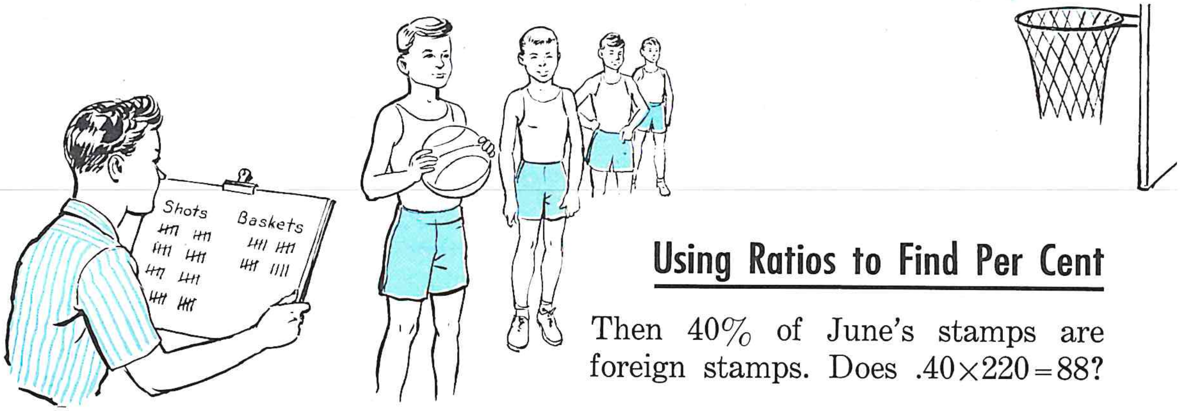
8. $d=(20+40\frac{1}{2})-28\frac{4}{5}$ $31\frac{7}{10}$

9. $(.300+.400)-.499=e$ .201

10. $82\times(\frac{1}{2}+\frac{3}{8})=d$ $71\frac{3}{4}$

11. $(1-\frac{1}{6})\times4=f$ $3\frac{1}{3}$

12. $g=685\times7235$ 4,955,975



Using Ratios to Find Per Cent

June had a collection of 220 stamps. Of these, 88 were foreign stamps. What per cent of June's stamps were foreign stamps?

One solution for the above problem involves using ratios. You are told the ratio of foreign stamps to the stamps in the entire collection. It is $\frac{88}{220}$. If you find the equivalent ratio with a denominator of 100, you will know the per cent. Let s stand for the number you are seeking. Then the ratio of s to 100 is expressed $\frac{s}{100}$.

Now the problem can be translated into the proportion $\frac{88}{220}=\frac{s}{100}$. It is solved below.

$$\frac{88}{220}=\frac{s}{100}$$

$220s=8800$ Why?

$s=\frac{8800}{220}$ Why?

$s=40$ Why?

You can replace s with 40. Is a ratio of $\frac{40}{100}$ equivalent to 40%? Then 40% of June's stamps are foreign stamps. Does $.40\times220=88$?

A basketball team made 19 baskets out of 40 shots at the basket. Suppose you are to find what per cent of the shots resulted in baskets. What comparison is made in the problem? Could the comparison be expressed by the ratio $\frac{19}{40}$? Then the problem can be translated into the proportion $\frac{19}{40}=\frac{b}{100}$. Study its solution.

$$\frac{19}{40}=\frac{b}{100}$$

$$40b=1900$$

$$b=\frac{1900}{40}$$

$$b=\frac{95}{2}=47\tfrac{1}{2}$$

The answer may be expressed either as $47\frac{1}{2}$% or as 47.5%.

To find to the nearer hundredth a number that makes the same comparison to 100 as 19 makes to 55, you might proceed as shown below.

$$\frac{19}{55}=\frac{b}{100}$$

$55b=1900$ Why?

$b=\frac{1900}{55}$ Why?

$b=34.54$ Why?

If you round 34.54 to the nearer tenth, you can replace b with 34.5. A ratio of 19 to 55 is equivalent to a ratio of $\frac{34.5}{100}$, which may be expressed as either $34\frac{1}{2}$% or 34.5%.

Oral For each problem below plan a solution in which you use a proportion. Be ready to tell your plan and why you decided upon it.

See page 214.

1. In a class of 34 pupils 18 of the pupils were girls. What per cent of the pupils were girls?

$\frac{18}{34}=\frac{n}{100}$ 52.9%

2. A farmer had 90 acres of land. He planted 12 acres of the land in wheat. What per cent of his land was planted in wheat? $\frac{12}{90}=\frac{n}{100}$ $13\frac{1}{3}$%

3. John took a spelling test of 50 words. He spelled 45 of the words correctly. What per cent of the words did he spell correctly?

$\frac{45}{50}=\frac{n}{100}$ 90%

4. A school had an enrollment of 685 pupils. One day 120 pupils had lunches in the school cafeteria. What per cent of the pupils ate in the school cafeteria? $\frac{120}{685}=\frac{n}{100}$ 17.5%

5. Bob weighed 85 pounds. His father weighed 165 pounds. Bob's weight is what per cent of the weight of his father? $\frac{85}{165}=\frac{n}{100}$ 51.5%

6. A satellite and its load weighed 3200 pounds. The weight of its load was 330 pounds. The load was what per cent of the total weight?

$\frac{330}{3200}=\frac{n}{100}$ 10.3%

7. Doris had a collection of 120 shells. Kim had a collection of 175 shells. Doris's collection of shells is what per cent of Kim's collection?

$\frac{120}{175}=\frac{n}{100}$ 68.6%

8. Larry made 8 hits in 22 times at bat. What was his batting average?

$\frac{8}{22}=\frac{n}{100}$ 36.4%

9. A pitcher won 19 of the 26 games that he pitched. What per cent of his games did he win?

$\frac{19}{26}=\frac{n}{100}$ 73.1%

10. A parking lot with 85 spaces had 62 cars parked in it. What per cent of the spaces were filled?

$\frac{62}{85}=\frac{n}{100}$ 72.9%

11. Bob had 75 tickets to sell. He sold all of them except 6. What per cent of the tickets did he sell?

$\frac{69}{75}=\frac{n}{100}$ 92%

Written Write an equation for each problem through 11. Solve each equation and write your answer to each problem. Express each per cent to the nearer tenth.

Solve each equation below.

	a	*b*	*c*
12.	$\frac{65}{130}=\frac{n}{100}$ 50	$\frac{23}{46}=\frac{n}{100}$ 50	$\frac{25}{125}=\frac{n}{100}$ 20
13.	$\frac{45}{15}=\frac{n}{100}$ 300	$\frac{39}{39}=\frac{n}{100}$ 100	$\frac{60}{150}=\frac{n}{100}$ 40
14.	$\frac{64}{400}=\frac{n}{100}$ 16	$\frac{35}{42}=\frac{n}{100}$ $83\frac{1}{3}$	$\frac{20}{300}=\frac{n}{100}$ $6\frac{2}{3}$
15.	$\frac{9}{18}=\frac{x}{4}$ 2	$\frac{2}{3}=\frac{10}{a}$ 15	$\frac{5}{n}=\frac{15}{36}$ 12
16.	$\frac{5}{4}=\frac{10}{x}$ 8	$\frac{171}{18}=\frac{n}{24}$ 228	$\frac{3}{a}=\frac{6}{15}$ $7\frac{1}{2}$
17.	$\frac{4}{c}=\frac{7}{28}$ 16	$\frac{7}{21}=\frac{3}{d}$ 9	$\frac{x}{4}=\frac{9}{8}$ $4\frac{1}{2}$
18.	$\frac{75}{m}=\frac{60}{1000}$ 1250	$\frac{x}{27}=\frac{17}{51}$ 9	$\frac{4}{9}=\frac{n}{12}$ $5\frac{1}{3}$

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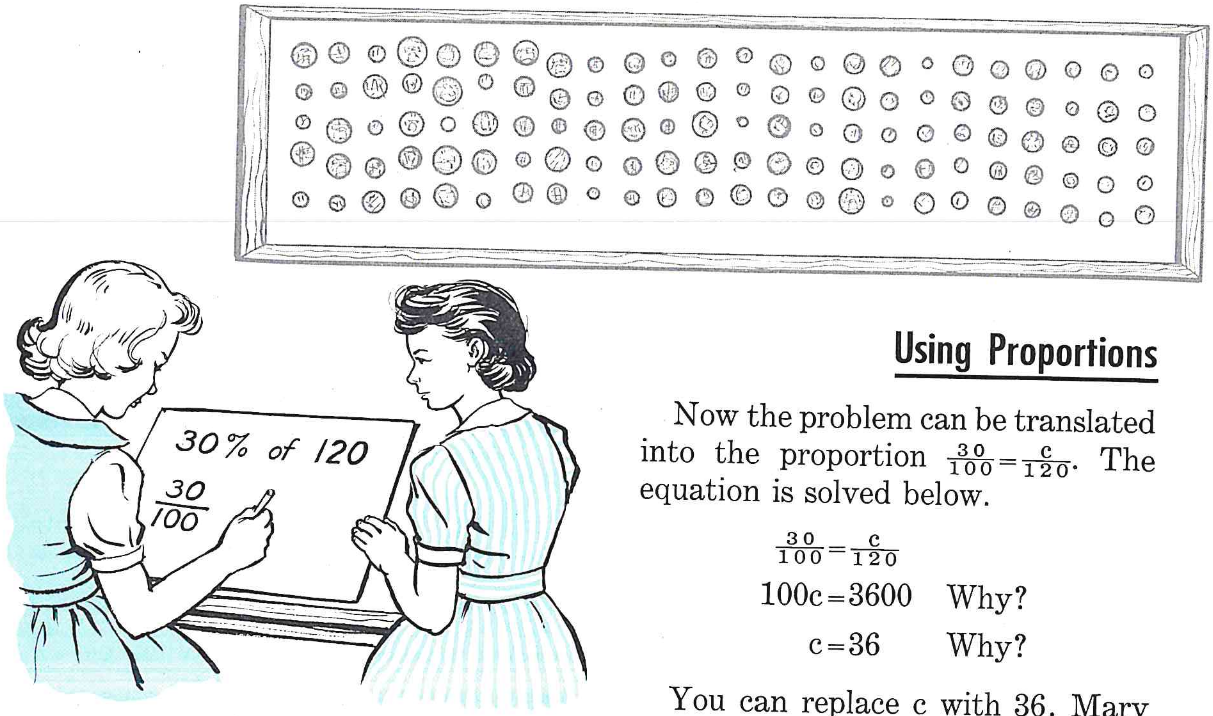


Using Proportions

Mary had 120 coins in her collection. She agreed to sell 30% of them to Ann. How many coins did she agree to sell to Ann?

Several solutions for the above problem are possible. However, since the problem makes comparisons, you may decide to use ratios in solving it. What ratio is represented by 30%? Is it $\frac{30}{100}$? How do you know?

Suppose now that you are to make a comparison of the number of coins Mary agreed to sell to the total number of coins.

Let c represent the number you are seeking. The comparison may be expressed by the ratio $\frac{c}{120}$.

Now the problem can be translated into the proportion $\frac{30}{100}=\frac{c}{120}$. The equation is solved below.

$$\frac{30}{100}=\frac{c}{120}$$

$100c=3600$ Why?

$c=36$ Why?

You can replace c with 36. Mary agreed to sell 36 coins to Ann.

In the problem, the per cent is 30. The number of the set of all of the coins is 120. You are to find a number having the same ratio to 120 as 30 has to 100.

Study the next problem. Which two comparisons are made?

Jack decided to sell 75% of his 48 rabbits. How many rabbits did he decide to sell?

If you let r stand for the rabbits Jack plans to sell, you can translate the problem into the equation shown below. Why?

$\frac{75}{100}=\frac{r}{48}$ $100r=3600$ $r=36$

Jack plans to sell 36 rabbits.

You can always recognize problems in which you are to find what per cent one number is of another. The number of per cent will be given, usually in % notation, so you can state the per cent ratio. A letter will represent one of the numbers in the other ratio used in the proportion for solving the problem.

Oral Decide how to use proportions in solving problems 1 through 10. Tell how you made your decision.

See page 216 and above

1. On a cold day 25% of the 36 pupils in a class were absent. How many of the pupils were absent?

$\frac{n}{36}=\frac{25}{100}$ 9 pupils

2. In one school the enrollment was 630 pupils. 80% of the pupils came to school in buses. How many of the pupils came on buses?

$\frac{n}{630}=\frac{80}{100}$ 504 pupils

3. A basketball team played 15 games and won 40% of them. How many games did the team win?

$\frac{n}{15}=\frac{40}{100}$ 6 games

4. In a class of 30 pupils, 60% were boys. How many of the pupils in the class were boys?

$\frac{n}{30}=\frac{60}{100}$ 18 boys

5. A football player completed 60% of the 15 passes attempted in one game. How many of the passes did he complete?

$\frac{n}{15}=\frac{60}{100}$ 9 passes

6. A junior high had an enrollment of 1200 pupils. 6% of the pupils were on the honor roll at the end of the year. How many of the pupils were on the honor roll?

$\frac{n}{1200}=\frac{6}{100}$ 72 pupils

7. If a 14-yard piece of material shrinks 1% when washed, how long will the material be after it has been washed? $\frac{n}{14}=\frac{1}{100}$ 13.86 yd.

8. A jet plane had a capacity of 108 passengers. On one trip the plane carried 75% of its capacity load. How many passengers were on the plane on that trip?

$\frac{n}{108}=\frac{75}{100}$ 81 passengers

9. The sixth graders made $40 selling tickets for a play. They planned to spend 25% of the money for records. How much money did they plan to spend for records?

$\frac{n}{40}=\frac{25}{100}$ $10

10. A salesman drove 1825 miles in a month. He estimated that 80% of his driving was for business purposes. How many miles did he drive on business during the month?

$\frac{n}{1825}=\frac{80}{100}$ 1460 mi.

Written Using proportions, translate problems 1 through 10 into equations. Solve each equation. Write each answer after rereading each problem.

Another way You can often find a per cent by using your knowledge of fractions. For example, to find 25% of 120 you may think as follows.

$25\%=\frac{1}{4}$ $\frac{1}{4}$ of $120=30$

Then 25% of $120=30$

Use the fraction method to check your computation in the preceding problems.

More Uses of Proportion

On a day after a storm only 300 pupils were present at Park View School. This number was 60% of the total enrollment. How many pupils were enrolled in the school?

In the above problem two comparisons are made. You know that 60% is the ratio of 60 to 100. The other comparison is 300 to some number representing the total enrollment. Suppose that you let e represent the number of pupils enrolled. Then the ratio of pupils present to those enrolled is $\frac{300}{e}$.

Now the problem can be translated into the equation $\frac{60}{100}=\frac{300}{e}$. The equation is solved below.

$$\frac{60}{100}=\frac{300}{e}$$

$30000=60e$ Why?

$500=e$ Why?

You can replace e with 500. 500 pupils were enrolled in the school.

You can check your answer by finding 60% of 500.

In the above problem, you knew the per cent. You also knew that 300 had the same relation to an unknown number as 60 has to 100. This number must be greater than 300. Why?

Now try to find n when 30% of n is equal to 50.

MORE PRACTICE PAGE 314

Oral After studying each problem below, tell the ratio you would use to express the per cent and the ratio you would use in comparing the number you know to the number you wish to find. Tell how you decided.

1. One day 75% of the pupils in a school took a special test. If 360 pupils took the test, how many pupils were enrolled in the school?

$\frac{360}{n}=\frac{75}{100}$ 480 pupils

2. Bob made a score of 80% on a test. He had 16 problems correct. How many problems were in the test? $\frac{16}{n}=\frac{80}{100}$ 20 problems

3. In one school 4% of the pupils made a perfect score on a test. If 8 pupils made a perfect score, how many pupils took the test?

$\frac{8}{n}=\frac{4}{100}$ 200 pupils

4. A school baseball team won 75% of its games in a season. The team won 12 games. How many games had the team played?

$\frac{12}{n}=\frac{75}{100}$ 16 games

5. Betty sold 17 tickets for a play. This was 10% of all tickets sold. How many tickets were sold?

$\frac{17}{n}=\frac{10}{100}$ 170 tickets

6. Tom made 15 baskets. He made baskets on 30% of his shots. How many shots did he make?

$\frac{15}{n}=\frac{30}{100}$ 50 shots

Written Write an equation for each problem above. Solve each equation and write each answer.

Solving Problems

Carefully read each problem at least twice. In the first reading try to get a picture of the entire situation. In the second reading look especially for the two ratios which you will need. Then write a proportion with those two ratios. Do not solve the equations until you have discussed them in class.

1. An airplane had 175 gallons of gasoline in its tanks at the beginning of a flight. At the end of its flight, 35 gallons of gasoline were in the tanks. What per cent of the gasoline was left?

$\frac{35}{175}=\frac{n}{100}$ 20%

2. Arthur had 50 chickens. He sold 30% of them. How many chickens did he sell? $\frac{n}{50}=\frac{30}{100}$ 15 chickens

3. A transcontinental bus was filled to 80% of its capacity. It was carrying 32 passengers. How many passengers could the bus carry when it was loaded to its capacity?

$\frac{32}{n}=\frac{80}{100}$ 40 passengers

4. A factory parking lot had spaces for 280 cars. 5% of the spaces were reserved for visitors. How many spaces were reserved for visitors?

$\frac{n}{280}=\frac{5}{100}$ 14 spaces

5. A salesman sold 420 cases of canned fruit. This was 35% of the cases he was expected to sell. How many cases of canned goods was he expected to sell? $\frac{420}{n}=\frac{35}{100}$ 1200 cases

6. Records in a fire department showed that 60 of the last 150 fires were caused by sparks from other fires. What per cent of the fires were caused by such sparks?

$\frac{60}{150}=\frac{n}{100}$ 40%

7. A basketball player made baskets on an average of 80% of his free throws. If he had 10 free throws in a game, how many baskets should he expect to make? $\frac{n}{10}=\frac{80}{100}$ 8 baskets

8. Mrs. Johnson drove 700 miles in March. Approximately 45% of her driving was within the city limits. How many miles did she drive within the city limits in March?

$\frac{n}{700}=\frac{45}{100}$ 315 mi.

9. Betty sold 12 of the 50 boxes of cards that she had. What per cent of the boxes of cards did she sell?

$\frac{12}{50}=\frac{n}{100}$ 24%

10. Walter sold 5 dozen eggs to a customer. This was 10% of the eggs he brought to the city. How many dozen eggs did he bring to the city?

$\frac{5}{n}=\frac{10}{100}$ 50 doz.

Oral State the proportion you would use in solving each problem 1 through 10. Tell how you decided upon the proportion to use.

Written Solve problems 1 through 10. Write your answer to a problem only after rereading it to make certain that your computed answer is reasonable. Check your computation for each problem.

Review and Practice

Part 1 Copy. Perform each operation as indicated.

	a	*b*	*c*
1.	1678 5784 7153 2360 +6759 23734	694.6 250.1 321.4 988.5 +427.9 2682.5	\$10.93 54.27 41.41 30.32 +83.26 \$220.19
2.	7214 −2387 4827	6200 −384 5816	680.2 −267.4 412.8
3.	594 ×36 21384	7.55 ×67 505.85	429.5 ×3.26 1400.170
4.	$6\frac{3}{4}$ $1\frac{1}{4}$ $+3\frac{1}{4}$ $11\frac{1}{4}$	$5\frac{1}{3}$ $2\frac{5}{6}$ $+1\frac{2}{3}$ $9\frac{5}{6}$	$3\frac{2}{5}$ $1\frac{1}{10}$ $+4\frac{3}{10}$ $8\frac{4}{5}$
5.	$5\frac{1}{3}$ $-3\frac{2}{3}$ $1\frac{2}{3}$	$4\frac{7}{8}$ $-1\frac{1}{4}$ $3\frac{5}{8}$	$3\frac{1}{4}$ $-1\frac{2}{3}$ $1\frac{7}{12}$
6.	20 $\times14\frac{1}{2}$ 290	$32\frac{1}{4}$ ×40 1290	50 $\times12\frac{1}{10}$ 605

Part 2 Copy. Perform each division.

	a	*b*
1.	$52\overline{)5782}$ $111\frac{5}{26}$	$86\overline{)\$73.10}$ \$ .85
2.	$240\overline{)24750}$ 103.125	$275\overline{)302300}$ $1099\frac{3}{11}$
3.	$.12\overline{)4800}$ 40000.	$95\overline{)3087.5}$ 32.5
4.	$.29\overline{)134.328}$ 463.2	$6.8\overline{)28260.8}$ 4156.

Part 3 Solve for n.

	a	*b*
1.	$8\times\frac{5}{6}=n$ $6\frac{2}{3}$	$n=6\times\frac{5}{9}$ $3\frac{1}{3}$
2.	$2\frac{1}{2}\times1\frac{4}{5}=n$ $4\frac{1}{2}$	$n=4\frac{2}{3}\times3\frac{1}{2}$ $16\frac{1}{3}$
3.	$\frac{5}{6}\div\frac{3}{4}=n$ $1\frac{1}{9}$	$n=2\frac{4}{5}\div\frac{7}{8}$ $3\frac{1}{5}$
4.	$4\frac{1}{6}\div5=n$ $\frac{5}{6}$	$n=3\frac{1}{2}\div2\frac{1}{3}$ $1\frac{1}{2}$
5.	$12\frac{7}{8}\times4=n$ $51\frac{1}{2}$	$n=8\frac{2}{5}\div4\frac{2}{3}$ $1\frac{4}{5}$
6.	$17\frac{1}{3}-6\frac{3}{4}=n$ $10\frac{7}{12}$	$n=37\frac{1}{8}+95\frac{6}{10}$ $132\frac{29}{40}$
7.	$42\frac{5}{6}+17\frac{1}{2}=n$ $60\frac{1}{3}$	$n=48-9\frac{5}{16}$ $38\frac{11}{16}$

Part 4 Replace each dot with =, >, or <, so that the following statements become true. Reminder: The sign = used in statements about measures means *equal in measure.*

1. 2 ft. 9 in. $\overset{=}{\bullet}$ 33 in.
2. 3400 lb. $\overset{=}{\bullet}$ 1 T. 1400 lb.
3. 15 gal. 3 qt. $\overset{>}{\bullet}$ 33 qt.
4. 9 hr. 15 min. $\overset{>}{\bullet}$ 550 min.
5. 2000 yd. $\overset{<}{\bullet}$ $1\frac{1}{2}$ mi.
6. 19.9×71.1 $\overset{>}{\bullet}$ 1400
7. $3\times(7\frac{1}{2}+3\frac{1}{4})$ $\overset{>}{\bullet}$ 30
8. $(2\frac{1}{2}+\frac{1}{4})+\frac{1}{4}$ $\overset{>}{\bullet}$ $\frac{6}{4}$
9. $4356.7\div9.9$ $\overset{>}{\bullet}$ $4350\div10$
10. $650.75\div\frac{3}{4}$ $\overset{<}{\bullet}$ 4×650.75
11. 10×95.98 $\overset{>}{\bullet}$ 9×100.50

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. The word *per cent* may be substituted for *hundredths* when you are reading two-place decimals or fractions with a denominator of 100. (207)

2. A per cent may be expressed in fractional or decimal form. (207)

3. Any ratio with a denominator of 100 may be read as per cent by naming the numerator and substituting *per cent* for the denominator. (208)

4. Comparisons are easier to make when the ratios being compared are all expressed with a denominator of 100. (210-211)

5. Proportions can be used to find either the per cent or the unknown number of two numbers being compared. (214, 215, 216, 218)

Words and Signs to Know

Per cent, % (207, 208)

Questions to Discuss

See page T221 for answers.

1. In which 3 forms can you name $\frac{80}{100}$? (207)

2. A per cent can name a number less than 1, equal to 1, or greater than 1. What can you tell about each kind? (208)

3. How would you express the comparison of 6 pennies out of 100 pennies as a ratio, as a fraction, and as a per cent? (209)

4. What ratios would you use to find what per cent 2 is of 6? (214)

5. What ratios would you use to find 30% of 120? To find 25% of 60? To find 10% of 8? (216)

6. What ratio would you use to find a number if you knew that 25% of it was equal to 40? (218)

Written Practice

See page T221 for answers.

1. Express $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{1}{3}$ first in fractional form with a denominator of 100, and then in per cent form. (210)

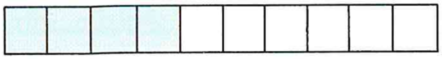
2. Express 75 out of 100 and 3 out of 100 first in fractional form with a denominator of 100 and then in per cent form. (210-211)

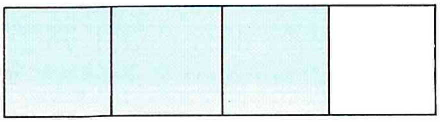
Copy. Replace each letter with the numeral that makes each of the following statements true.

	a	*b*	
3.	$\frac{44}{110}=\frac{s}{100}$ 40	$\frac{17}{56}=\frac{b}{100}$ $30\frac{5}{14}$	(214)
4.	$\frac{40}{100}=\frac{c}{200}$ 80	$\frac{18}{100}=\frac{c}{50}$ 9	(216)
5.	$\frac{30}{100}=\frac{150}{c}$ 500	$\frac{19}{100}=\frac{57}{d}$ 300	(218)

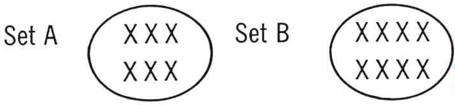
Self-Evaluation

Part 1 Write the numeral that tells approximately what per cent of each drawing is blue.

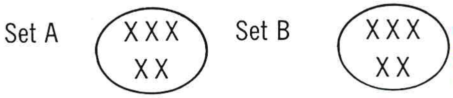
1.  40

2.  75

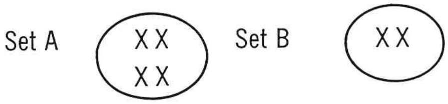
Write the ratio that compares the number of Set A to that of Set B in each problem. Then express the ratio in per cent form.

3. 

$\frac{3}{4}$; 75%

4. 

$\frac{5}{5}$; 100%

5. 

$\frac{4}{2}$; 200%

Now express the ratio of the number of Set B to the number of Set A in per cent form. $133\frac{1}{3}$%; 100%; 50%

Part 2 Express each ratio below in per cent form.

	a	*b*	*c*	*d*
1.	$\frac{9}{10}$ 90%	$\frac{6}{10}$ 60%	$\frac{3}{20}$ 15%	$\frac{4}{50}$ 8%
2.	$\frac{19}{25}$ 76%	$\frac{8}{2}$ 400%	$\frac{4}{1}$ 400%	$\frac{35}{50}$ 70%
3.	$\frac{17}{50}$ 34%	$\frac{32}{10}$ 320%	$\frac{45}{20}$ 225%	$\frac{9}{20}$ 45%

Part 3 Solve each equation.

	a	*b*	*c*
1.	$\frac{24}{400}=\frac{x}{100}$ 6	$\frac{54}{100}=\frac{27}{r}$ 50	$\frac{50}{100}=\frac{d}{25}$ $12\frac{1}{2}$
2.	$\frac{75}{100}=\frac{c}{25}$ $18\frac{3}{4}$	$\frac{44}{100}=\frac{22}{t}$ 50	$\frac{35}{100}=\frac{7}{p}$ 20
3.	$\frac{65}{100}=\frac{13}{w}$ 20	$\frac{4}{500}=\frac{n}{100}$ $\frac{4}{5}$	$\frac{19}{100}=\frac{n}{350}$ 66.5
4.	$\frac{14}{800}=\frac{m}{100}$ $1\frac{3}{4}$	$\frac{30}{100}=\frac{a}{80}$ 24	$\frac{22}{100}=\frac{11}{x}$ 50
5.	$\frac{20}{100}=\frac{w}{64}$ $12\frac{4}{5}$	$\frac{75}{100}=\frac{25}{n}$ $33\frac{1}{3}$	$\frac{8}{200}=\frac{n}{100}$ 4

Part 4 Solve each problem.

1. An automobile dealer sold 135 cars. 40% of the cars were new. How many new cars did he sell?

$\frac{n}{135}=\frac{40}{100}$ 54 cars

2. In one city the average yearly snowfall was 35 inches. In that city there were 11.5 inches of snow in December, 4.8 inches in January, and 3.7 inches in February. What per cent of the average yearly snowfall occurred in those 3 months?

$11.5+4.8+3.7=n$ $\frac{20}{35}=\frac{n}{100}$ $57\frac{1}{7}$%

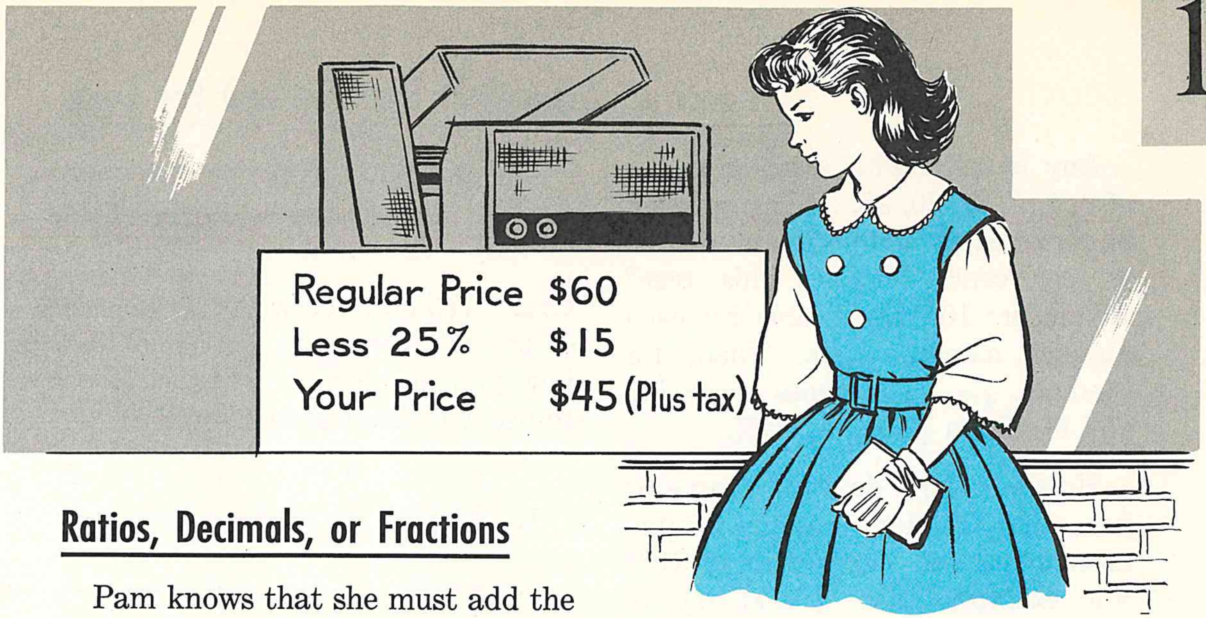
3. In June one city had 6.2 inches of rain. This was 15% of its average yearly rainfall. What was its average yearly rainfall? $\frac{6.2}{n}=\frac{15}{100}$ $41\frac{1}{3}$ in.

4. Bob spelled correctly 84% of the words on a 50-word test. How many words were spelled correctly?

$\frac{n}{50}=\frac{84}{100}$ 42 words

5. Nancy had 47 of the 50 words correct. What per cent of her words did she spell correctly? $\frac{47}{50}=\frac{n}{100}$ 94%

13



Ratios, Decimals, or Fractions

Pam knows that she must add the tax to the cost of the record player. How much will the tax be?

The tax is 4%, which is equivalent to $4 on every $100. The tax must be paid on $45. One way to find the tax is to find the number which has the same ratio to $45 as $4 has to $100. Let t represent the number. Then the ratio of the tax to the price is $\frac{t}{45}$.

The problem can be translated into the equation below.

$$\frac{4}{100}=\frac{t}{45} \qquad \text{Then} \quad 100t=180$$
$$\text{and} \quad t=1.80$$

The tax will be $1.80.

Other solutions for the problem are possible. Recall that $4\%=.04=\frac{1}{25}$. Suppose you wanted to use decimals in finding 4% of $45. You could use the equation $.04\times45=n$. If you wished to work with fractions, you could use the equation $\frac{1}{25}\times45=n$. Can you solve either equation mentally? Study the solutions.

$$.04\times45=n \qquad \frac{1}{25}\times45=n$$
$$1.80=n \qquad \frac{45}{25}=n$$
$$1\frac{20}{25}=n$$

Both 1.80 and $1\frac{20}{25}$, expressed as money, are $1.80. Would you use ratios, decimals, or fractions to find 4% of $45? Why?

Oral Tell how to find each result below by using the ratio, decimal, and fractional form of each per cent.

See column 1 and above.

	a	*b*
1.	20% of $40 $8	50% of $9 $4.50
2.	75% of $.50 $.37½	25% of $1.50 $.37½
3.	10% of $90 $9	90% of $65 $58.50

Equivalent Fractions and Per Cents

Any fraction with a denominator of 2, 4, 5, 10, 20, 25, or 50 can easily be renamed as hundredths and then as per cent. Why is this true? Reminder: 100 is divisible by each number named above. Then, for example, $\frac{1}{4}=\frac{n}{100}$ becomes $n \times 4=100$ and $n=25$; so $\frac{1}{4}=\frac{25}{100}$ or 25%.

However, how do you rename as per cents fractions having denominators that are not factors of 100? For example, take $\frac{1}{3}$ and try to express it as hundredths and then as per cent. Study the solution below.

$$\frac{1}{3}=\frac{n}{100}$$

$$\begin{aligned} &\text{Then} \quad n \times 3=100 \qquad \text{Why?}\\ &\text{and} \quad n=100 \div 3 \qquad \text{Why?}\\ &\qquad\quad n=33\tfrac{1}{3} \end{aligned}$$

Since n is $33\frac{1}{3}$, the ratio of n to 100 may be expressed in fractional form as shown. Then $\frac{1}{3}$ equals $33\frac{1}{3}$ hundredths or $33\frac{1}{3}$ per cent.

$$\frac{33\frac{1}{3}}{100}$$

Suppose you wish to express $\frac{2}{3}$ in per cent form. Since $\frac{2}{3}$ is $2 \times \frac{1}{3}$, $\frac{2}{3}=2 \times 33\frac{1}{3}\%$ or $66\frac{2}{3}\%$.

Take any other fraction having a denominator that is not a factor of 100. Convince yourself that the procedure suggested above will always produce a ratio with a denominator of 100.

Did you notice that the per cents are expressed as mixed fractions, for example $33\frac{1}{3}\%$, $66\frac{2}{3}\%$, and so on?

Oral Tell how you would write each of the following as a per cent. Tell how you decided upon the numeral to use with the per cent sign.

	a	*b*	*c*	*d*
1.	$\frac{1}{4}$ 25%	$\frac{2}{3}$ $66\frac{2}{3}\%$	$\frac{3}{4}$ 75%	$\frac{5}{5}$ 100%
2.	$\frac{1}{3}$ $33\frac{1}{3}\%$	$\frac{2}{4}$ 50%	$\frac{1}{2}$ 50%	$\frac{10}{10}$ 100%
3.	$\frac{4}{4}$ 100%	$\frac{3}{10}$ 30%	$\frac{1}{5}$ 20%	$\frac{4}{10}$ 40%
4.	$\frac{2}{5}$ 40%	$\frac{3}{5}$ 60%	$\frac{4}{5}$ 80%	$\frac{1}{7}$ $14\frac{2}{7}\%$
5.	$\frac{2}{7}$ $28\frac{4}{7}\%$	$\frac{1}{8}$ $12\frac{1}{2}\%$	$\frac{3}{8}$ $37\frac{1}{2}\%$	$\frac{5}{8}$ $62\frac{1}{2}\%$
6.	$\frac{7}{8}$ $87\frac{1}{2}\%$	$\frac{9}{10}$ 90%	$\frac{1}{12}$ $8\frac{1}{3}\%$	$\frac{1}{16}$ $6\frac{1}{4}\%$

Written Write an equivalent per cent for each fraction in rows 1 through 6 above.

Something to do You will save yourself time if you know the following equivalents.

$\frac{1}{2}=50\%$

$\frac{1}{3}=33\frac{1}{3}\%$

$\frac{2}{3}=66\frac{2}{3}\%$

$\frac{1}{4}=25\%$

$\frac{3}{4}=75\%$

$\frac{1}{5}=20\%$

$\frac{2}{5}=40\%$

$\frac{3}{5}=60\%$

$\frac{4}{5}=80\%$

$\frac{1}{6}=16\frac{2}{3}\%$

$\frac{5}{6}=83\frac{1}{3}\%$

$\frac{1}{8}=12\frac{1}{2}\%$

$\frac{3}{8}=37\frac{1}{2}\%$

$\frac{5}{8}=62\frac{1}{2}\%$

$\frac{7}{8}=87\frac{1}{2}\%$

Equivalent Decimals and Per Cents

You may recall that per cent may be thought of as another name for hundredths. Then 25%, $\frac{25}{100}$, $\frac{1}{4}$, and .25 are all names for the same number. Any number written in per cent form can also be expressed in fractional and in decimal forms. A few examples follow.

$50\% = \frac{50}{100} = .50$ $\qquad 75\% = \frac{75}{100} = .75$

$3\% = \frac{3}{100} = .03$ $\qquad 4\% = \frac{4}{100} = .04$

Look at the first example given above. Think of $\frac{50}{100}$ as a fraction rather than as a ratio. To change $\frac{50}{100}$ from fractional form to decimal form, you divide 50 by 100 to obtain .50. How do you change $\frac{3}{100}$, $\frac{75}{100}$, and $\frac{4}{100}$ to decimal form?

Choose several numbers written in per cent form. Express the numbers as hundredths, first in fractional and then in decimal form. Convince yourself that to change from per cent form to decimal form, you first think of the per cent as a fraction with a denominator of 100. Then you divide by $\frac{100}{100}$.

Suppose now that you want to change from decimal to per cent form. A few examples follow.

$.90 = 90\%$ $\qquad .435 = 43.5\%$

$.09 = 9\%$ $\qquad .06\frac{1}{2} = 6\frac{1}{2}\%$

Do you see that .90, .09, .435, and $.06\frac{1}{2}$ were changed to per cent form by multiplying each by $\frac{100}{100}$? Then the numerals were rewritten with the % sign.

Oral Tell how you would write each of the following per cents in decimal form.

	a	*b*	*c*	*d*
1.	35% .35	6% .06	76% .76	86% .86
2.	98% .98	13% .13	$33\frac{1}{3}\%$ $.33\frac{1}{3}$	$4\frac{1}{2}\%$ $.04\frac{1}{2}$
3.	47% .47	$6\frac{1}{2}\%$ $.06\frac{1}{2}$	$2\frac{1}{2}\%$ $.02\frac{1}{2}$	25% .25
4.	1.95% .0195	267% 2.67	49.5% .495	3.47% .0347

Tell how you would write each of the following in per cent form.

	a	*b*	*c*	*d*
5.	.15 15%	.37 37%	.75 75%	$.66\frac{2}{3}$ $66\frac{2}{3}\%$
6.	$.03\frac{1}{3}$ $3\frac{1}{3}\%$	.01 1%	$.87\frac{1}{2}$ $87\frac{1}{2}\%$	.50 50%
7.	.7 70%	.1325 $13\frac{1}{4}\%$	1.25 125%	1.5 150%
8.	2.2 220%	5.55 555%	3.67 367%	896% 8.96
9.	$.12\frac{1}{2}$ $12\frac{1}{2}\%$	$.37\frac{3}{8}$ $37\frac{3}{8}\%$	.40 40%	390% 3.90
10.	.17 17%	.99 99%	.60 60%	$15\frac{1}{2}\%$ .155
11.	$.16\frac{2}{3}$ $16\frac{2}{3}\%$	.70 70%	2.50 250%	90% .90

MORE PRACTICE PAGE 315

Written Write an answer for each exercise in rows 1 through 11 above.

Enough information is given in the picture to enable you to find an answer to these questions.

A 6 is what per cent of 20?
B What number is 30% of 20?
C 6 is 30% of what number?

You may recall how to use equivalent ratios to answer each of the questions. There is another and possibly shorter way to answer each question. This way consists of translating the questions into equations. Study the translation of question A into an equation as shown below.

6 is what per cent of 20?
$6 = n \times 20$

What symbol was used to replace *is?* To replace *what per cent?* To replace *of?*

Solving Equations

The equation $6 = n \times 20$ may be written $n \times 20 = 6$. Why is this possible? This equation is solved for you below. Study the solution.

$n \times 20 = 6$
$n = 6 \div 20$ Why?
$n = .30$ or 30%

Then 6 is *30%* of 20.

Question B can also be translated into an equation. The equation and its solution are shown below. Study the solution.

What number is 30% of 20?
$n = 30\%$ of 20
$n = .30 \times 20$
$n = 6$

What symbol was used to replace *what number?* To replace *is?* To replace *of?*

In the equation, .30 is used in place of 30%. Then *6* is 30% of 20.

Now study the translation of question C into an equation as shown below.

6 is 30% of what number?
$6 = 30\%$ of n
$6 = .30 \times n$

Do you see that = replaces *is*, .30 replaces *30%*, × replaces *of*, and n replaces *what number?*

The equation $6=.30\times n$ is solved for you below.

$6=.30\times n$
Then $.30\times n=6$
and $n\times .30=6$
and $n=6\div .30$
$n=20$

Then 6 is 30% of *20.*

Study the above solution again. You know that multiplication is commutative. This explains why $.30\times n=6$ becomes $n\times .30=6$. You also know that division is the inverse operation of multiplication. This explains why $n\times .30=6$ is solved as $n=6\div .30$. You may recall that you used this idea of inverse operations in solving equations with whole numbers as shown below.

$n-3=5$ $n\div 5=3$
$n=5+3$ $n=3\times 5$

Your understanding of the commutative property of multiplication together with your understanding of inverse operations will help you solve many equations.

Oral Tell the steps to take in translating each question below into an equation. Then tell how to find each answer. See page 226 and above.

1. 18 is what per cent of 30?
   18=nx30 60%
2. What number is 60% of 30?
   n=.60x30 18
3. 18 is 60% of what number?
   18=.60xn 30
4. 2.5 is what per cent of 5.0?
   2.5=nx5.0 50%

Tell the reason for each step taken in finding each answer below.

5. 9 is what per cent of 12?

$9=n\times 12$ Translation
$12\times n=9$ Prop. of equality & comm. prop. of mult.
$n=9\div 12$ Inverse
$n=.75$ or 75% Rename

6. What number is 25% of 60?

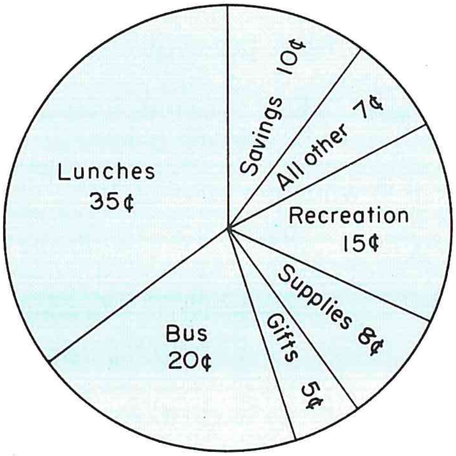
n is 25% of 60
$n=.25\times 60$ Translation
$n=15$ Rename

7. 14 is 25% of what number?

14 is 25% of n
$14=.25\times n$ Translation
$.25\times n=14$ Prop. of equality
$n\times .25=14$ Comm. prop.
$n=14\div .25$ Inverse
$n=56$ Rename

Written Express each of the following as an equation. Solve each equation.

1. 8 is what per cent of 96?
   8=nx96 $8\frac{1}{3}$%
2. What number is $33\frac{1}{3}$% of 96?
   n=.$33\frac{1}{3}$x96 32
3. 14 is 20% of what number?
   14=.20xn 70
4. 37 is what per cent of 148?
   37=nx148 25%
5. What number is 40% of 65?
   n=.40x65 26
6. 96 is 42% of what number?
   96=.42xn $228\frac{4}{7}$
7. What number is 10% of 50?
   n=.10x50 5



How Each Dollar Is Spent

You may find it worthwhile to make a plan showing how you will use your available money or time. A plan of this kind is called a **budget.**

One girl's budget is shown above. Look at the item *lunches*. The amount spent for lunches is what per cent of each dollar?

You can translate the question into an equation as shown below.

35 cents is what % of 100 cents

$$35 = n \times 100$$

Solve for n and you will find that 35 cents is 35% of 100 cents.

Oral Use the graph at the top of the page. Tell the equation to use in finding what per cent of each dollar was spent for each of the items that is shown. See above.

Using Per Cents for a Budget

Tell the steps you would take in translating problems 1 through 5 into equations. See column 1.

1. Betty plans on budgeting at least 50 minutes of each $5\frac{1}{2}$-hour school day on arithmetic. What per cent of each school day should she budget for arithmetic?

50=nx330 15.2%

2. Judy budgeted 14 hours out of each week for viewing television. What per cent of her time did she budget for this activity?

14=nx(7x24) $8\frac{1}{3}\%$

3. Mary planned to allow \$2 out of every \$5 for lunches. What per cent of her money did she allow for lunches?

2=nx5 40%

4. The members of a club decided to spend 80 cents out of each dollar for sports equipment. What per cent of the money was budgeted for sports equipment?

80=nx100 80%

5. The Johnsons planned to save at least \$75 out of every \$1000 of their earnings. What per cent of their earnings did the Johnsons plan to save?

75=nx1000 7.5%

Written Write the equations you would use in solving problems 1 through 5. Solve each equation. Reread the problems to see if your computed answers are reasonable. If so, write your answers.

BASEBALL

National League	*Won*	*Lost*	*Pct.*	*American League*	*Won*	*Lost*	*Pct.*
Pittsburgh	95	59	.617	New York	97	57	.630
Milwaukee	88	66	.571	Baltimore	89	65	.578
St. Louis	86	68	.558	Chicago	87	67	.565
Los Angeles	82	72	.532	Cleveland	76	78	.494

Using Per Cent in Sports

The four teams which led their leagues at the end of one baseball season are shown above. The column headed *Pct.* expresses in decimal form the ratio of games won to games played. To change from decimal form to per cent form, multiply by $\frac{100}{100}$. Thus .617 is 61.7%. Which of the teams listed above won more than 55% of the games played?

Suppose that Pittsburgh had won 96 of its 154 games. What per cent of its games would the team have won? The question means: 96 is what per cent of 154? You can translate this question into the equation $96 = n \times 154$. What steps are taken in doing this? To solve for n, you divide 96 by 154. Explain why. The division is carried out to 4 decimal places, like this: .6233. Then the quotient is rounded to .623. Finally the answer is expressed as 62.3%.

Oral Tell the equation to use in finding the per cent of games won by each team listed at the top of this page. Tell the steps to take in solving each equation. See below.

Written Write the equation you would use to find the per cent of games won by each team listed below. Solve each equation. Express the answers, correct to the nearest tenth of 1 per cent.

National League	*Won*	*Lost*
1. San Francisco	79	75
79=nx(79+75)	51.3%	
2. Cincinnati	67	87
67=nx(67x87)	43.5%	
3. Chicago	60	94
60=nx(60+94)	39.0%	
4. Philadelphia	59	95
59=nx(59+95)	38.3%	
American League	*Won*	*Lost*
5. Washington	73	81
73=nx(73+81)	47.4%	
6. Detroit	71	83
71=nx(71+83)	46.1%	
7. Boston	65	89
65=nx(65+89)	42.2%	
8. Kansas City	58	96
58=nx(58+96)	37.7%	

Pittsburgh	95=nx(95+59)	New York	97=nx(97+57)
Milwaukee	88=nx(88+66)	Baltimore	89=nx(89+65)
St. Louis	86=nx(86+68)	Chicago	87=nx(87+67)
Los Angeles	82=nx(82+72)	Cleveland	76=nx(76+78)

Using Decimals or Fractions to Find Per Cent

Suppose you were asked to find a number which is 25% of 64. Also suppose you were to choose between the fractional and the decimal form of 25% in finding the number. Since $25\% = \frac{1}{4}$, you should be able to use 25% and $\frac{1}{4}$ interchangeably. See if this is so. Study solutions A and B.

A	**B**
n is 25% of 64	n is $\frac{1}{4}$ of 64
$n = .25 \times 64$	$n = \frac{1}{4} \times 64$
$n = 16$	$n = 16$

Which solution is more likely to require using pencil and paper? Which solution would you choose?

Suppose though that you were to find a number which is 75% of 315. Multiplying 315 either by .75 or by $\frac{3}{4}$ would require using pencil and paper. Neither solution has any advantage over the other.

When the per cents are expressed as mixed fractions, you will be likely to use the fractional form of the per cent. For example, study C and D, which show two ways of finding a number which is $33\frac{1}{3}\%$ of 198.

C	**D**
n is $33\frac{1}{3}\%$ of 198	n is $\frac{1}{3}$ of 198
$n = .33\frac{1}{3} \times 198$	$n = \frac{1}{3} \times 198$
$n = 66$	$n = 66$

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In C the computation almost certainly required the use of pencil and paper. In D, finding a replacement for n consisted of dividing 198 by 3.

Before attempting to find a number which is a known per cent of another, decide whether to use the fractional or decimal equivalent of the per cent. Choose the form which will be fastest for you.

Oral Tell whether you would use the fractional or decimal form of each per cent to find the number equal to each of the following.

Answers will vary.

	a	*b*
1.	50% of 11 $5\frac{1}{2}$	25% of 96 24
2.	50% of 148 74	25% of 119 $29\frac{3}{4}$
3.	$33\frac{1}{3}\%$ of 69 23	$33\frac{1}{3}\%$ of 146 $48\frac{2}{3}$
4.	$12\frac{1}{2}\%$ of 64 8	47% of 18 8.46
5.	52% of 87 45.24	$87\frac{1}{2}\%$ of 24 21
6.	$62\frac{1}{2}\%$ of 72 45	65% of 80 52
7.	15% of 85 12.75	29% of 16 4.64

Written Write your answer for each exercise in rows 1 through 7 above. Use whichever method seems easiest for you. Check by using a different method.

Using Per Cents in Solving Problems

A girls' club was allowed to keep 25% of all money collected at a bazaar. The members collected $80. How much money did they earn for their club?

You are to find a number that is equal to 25% of 80; so you use the formula n = 25% of 80. Recall that 25% = .25 = $\frac{1}{4}$. Then the problem can be translated into either one of the two equations below.

$n = .25\times 80$ or $n = \frac{1}{4}\times 80$

Is 20 the correct replacement for n in each equation above? Why? Then the girls earned $20 for their club.

Read problems 1 through 6. Translate each problem into two different equations. Be ready to tell which one of the two equations you would use. Fractional form shown only.

1. Jane sold $36 worth of magazines. She was allowed to keep 35% of the money as her share. How much money was she allowed to keep?
$n=\frac{7}{20}x36$ $12.60

2. Betty can buy records for 50% of their regular price. The regular price is $2.98. How much money must Betty pay for records?
$n=\frac{1}{2}x2.98$ $1.49

3. Bob bought a used bicycle for $\frac{3}{5}$ of its original price of $39.98. How much did he pay?
$n=\frac{3}{5}x39.98$ $23.99

4. Ann had to pay a 4% sales tax on a sweater that cost $3.98. How much did she have to pay as tax?
$n=\frac{1}{25}x3.98$ $.16

5. Mary bought a pair of shoes for $66\frac{2}{3}\%$ of the regular price of $9.75. How much did she have to pay?
$n=\frac{2}{3}x9.75$ $6.50

6. Ann had 90% of 50 spelling words correct. How many of the words were correct?
$n=\frac{9}{10}x50$ 45 words

Oral Tell the equation you would use in solving each of the problems.

Written Translate problems 1 through 6 into equations. Solve each problem. Check your computation by using a different method.

Finding the Number to Which Another Is Compared

A basketball team won 75% of its games. The team won 9 games. How many games did the team play?

You know that 9 is 75% of some number and that $9 = .75 \times n$. Since $75\% = \frac{3}{4}$, the formula can be translated into either equation A or B below. Study the solution of each equation.

A	B
$9 = .75 \times n$	$9 = \frac{3}{4} \times n$
$.75 \times n = 9$	$\frac{3}{4} \times n = 9$
$n \times .75 = 9$	$n \times \frac{3}{4} = 9$
$n = 9 \div .75$	$n = 9 \div \frac{3}{4}$
$n = 12$	$n = 12$

In both solutions the following steps were taken. First the equations were rewritten so as to place the letter n at the left of the = sign. Then the commutative property of multiplication was used in rewriting the equations. Finally the equations were solved by using division, the inverse operation of multiplication. Do you obtain the same result when you express per cent in the fractional and in the decimal form? Which of the two solutions seems easier?

Suppose you had to decide whether to use the fractional or decimal form of a per cent in translating a problem into an equation. How would you decide which form to use?

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Oral Tell the reasons for the steps taken in finding each answer below.
See column 1.

a	*b*
60% of n is 18	$33\frac{1}{3}\%$ of n is 42
$\frac{3}{5} \times n = 18$	$\frac{1}{3} \times n = 42$
$n \times \frac{3}{5} = 18$	$n \times \frac{1}{3} = 42$
$n = 18 \div \frac{3}{5}$	$n = 42 \div \frac{1}{3}$
$n = 30$	$n = 126$

Written Find a suitable replacement for n in each statement below.

	a		*b*	
1.	25% of $n = 80$	320	20% of $n = 85$	425
2.	40% of $n = 12$	30	10% of $n = 15$	150
3.	$33\frac{1}{3}\%$ of $n = 15$	45	$16\frac{2}{3}\%$ of $n = 12$	72
4.	$87\frac{1}{2}\%$ of $n = 49$	56	$62\frac{1}{2}\%$ of $n = 35$	56
5.	$66\frac{2}{3}\%$ of $n = 64$	96	50% of $n = 95$	190

A quick review Write an equation to use in answering each question below. Solve each of your equations.
Fractional form shown only.

1. What number is 25% of 24?
$n = \frac{1}{4} x 24$ 6
2. 6 is what per cent of 24?
$6 = n x 24$ 25%
3. 6 is 25% of what number?
$6 = \frac{1}{4} x\ n$ 24
4. 15% of 75 is what number?
$n = \frac{3}{20} x 75$ 11.25
5. 16 is what per cent of 80?
$16 = n x 80$ 20%
6. 80 is 25% of what number?
$80 = \frac{1}{4} x\ n$ 320
7. 9 is what per cent of 25?
$9 = n x 25$ 36%

Making Estimates with Per Cent Forms

Suppose you wish to estimate an answer to this question: What number is 23% of \$360? You would use the statement: n is 23% of \$360. Why?

Which fractional equivalent, $\frac{1}{4}$ or $\frac{1}{5}$, is nearer to 23%? It is $\frac{1}{4}$. Therefore in estimating your answer to $n=.23\times360$, you could use the equation given below.

$$\begin{aligned} n &= \tfrac{1}{4}\times360 \\ &= 90 \end{aligned}$$

The number that is 23% of \$360 must be less than \$90. Could it be as little as \$72? Think: $\frac{1}{5}\times360=72$.

Now suppose you were to estimate an answer to the question: 34 is what per cent of 120? The equation to use is $34=n\times120$. Since $\frac{1}{3}\times120=40$, and $\frac{1}{4}\times120=30$, then n must be a number between $\frac{1}{3}$ and $\frac{1}{4}$. In per cent form the answer must be greater than 25% but less than $33\frac{1}{3}$%.

You may also find it worthwhile to use the decimal form of a per cent in estimating an answer to such questions as this: 19 is 65% of what number? The equation to use is $19=.65\times n$.

Think: 19 is close to 20 and .65 is close to $.66\frac{2}{3}$ or $\frac{2}{3}$. What equation would you use in making an estimate? Would 30 be a good estimate?

Oral Tell the equation you would use in estimating an answer to each question below. Tell your estimate.

Estimates will vary.

1. What number is 49% of 224?
   $n=.49\times224$ 109.76
2. 36 is what per cent of 70?
   $36=n\times70$ 51.43%
3. What number is 24% of 989?
   $n=.24\times989$ 237.36
4. 2 is 98% of what number?
   $2=.98\times n$ 2.04
5. What number is 9% of 115?
   $n=.09\times115$ 10.35
6. 29 is what per cent of 93?
   $29=n\times93$ 31.18%
7. What number is 63% of 47?
   $n=.63\times47$ 29.61
8. 5 is 21% of what number?
   $5=.21\times n$ 23.81
9. What number is 19% of 254?
   $n=.19\times254$ 48.26
10. What number is 53% of 35?
    $n=.53\times35$ 18.55

Written Write the equation to use in finding the answer to each question above. Solve each equation. Compare your estimated and computed answers.

A quick review Solve for n. Do not use pencil and paper.

	a	*b*
1.	65 $10\times6.5=n$	.6 $n=10\times.06$
2.	405 $100\times4.05=n$	590 $n=100\times5.9$
3.	5965 $1000\times5.965=n$	4200 $n=1000\times4.2$
4.	9.6 $96\div10=n$	.005 $n=.05\div10$

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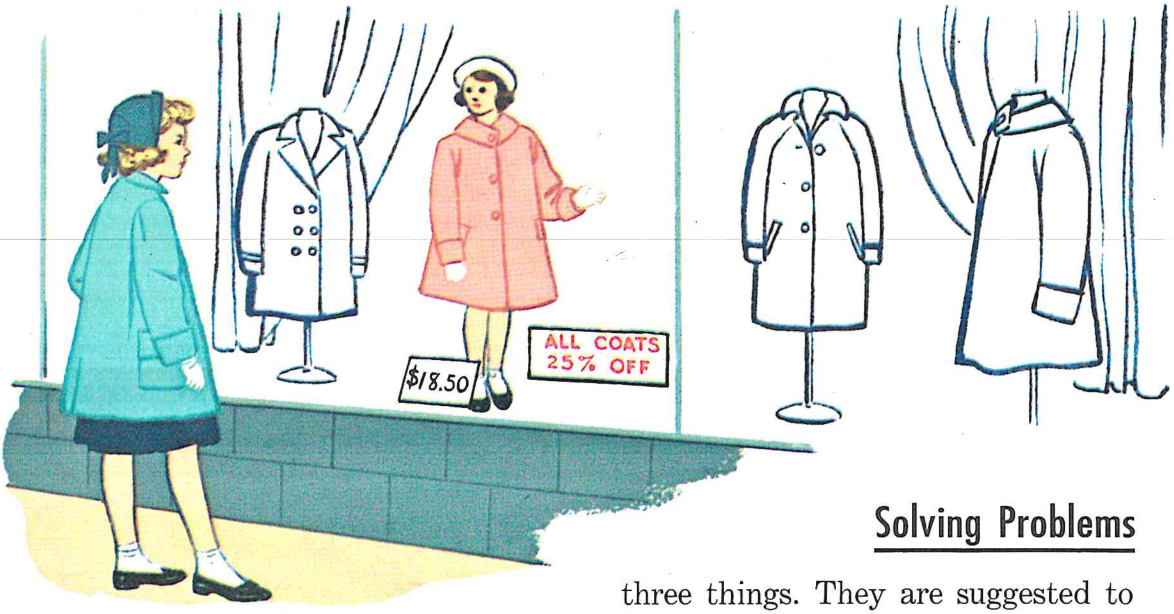


Solving Problems

Julie had $16 to spend on the coat she saw in the window. Did she have enough money to buy the coat?

In many of your problems, as in the one above, you must get information from more than one source. From what sources will you get the numbers to use in solving the above problem?

Before attempting to solve the above problem or any other problem, read it at least twice. In the first reading try to get a general idea of the problem situation. In this reading also determine the numbers that will be used in solving the problem.

In the second reading decide what you are to find. In the problem which Julie had, you are to find at least three things. They are suggested to you by the three questions below.

By how much was the price reduced?
What was the new price?
Was the new price greater or less than $16?

Why must you answer the first two questions above before you can answer the third question?

After the second reading, you should be ready to translate the problem into a brief statement as follows: Cost equals Regular Price less 25% of Regular Price. Then translate the problem into an equation which you solve as shown.

$$\begin{aligned} c &= 18.50 - (.25 \times 18.50) \\ &= 18.50 - 4.625 \\ &= 13.875 \end{aligned}$$

The coat would cost $13.88. Julie had enough money to buy the coat.

Careful planning is required when you are solving problems involving the use of per cent. You will find it worthwhile to begin your solution by writing an equation.

Oral Tell the equation you would use in solving each of these problems.

1. A $15 dress could be bought on sale for $3.75 less than the regular price. By how many per cent was the price reduced?

3.75=nx15 25%

2. One summer Ann saved $30. She said that this was 40% of all the money she had earned. How much money had she earned?

30=.40xn $75

3. Larry had $2.50 in his pocket after buying a ball. The ball was on sale for $33\frac{1}{3}\%$ off its regular price of $1.50. How much money did he have before buying the ball?

2.50+[1.50-($\frac{1}{3}$x1.50)]=n $3.50

Written Write an equation for each problem above. Then solve problems 4 through 14.

4. Ronald paid $2.80 for a book that usually sold for $3.50. What per cent of the regular price did he save?

3.50-2.80=nx3.50 20%

5. Joan bought a watch regularly priced at $28.50 for 20% off. How much more or less than $20 did she have to pay?

28.50-(.20x28.50)=20+n $2.80

6. Ann had to pay a 4% state sales tax plus a 10% federal tax on a watch priced at $25. How much did she have to pay for the watch and the tax together? 25+(.04x25)+(.10x25)=n $28.50

7. Mary said that she had spent 25% of her savings. She had spent $8. How much money did she have in savings before spending any?

8=.25xn $32

8. Mr. Kohl paid a 4% sales tax on a car priced at $2950. The price did not include the tax. What was the price of the car including the tax?

2950+(.04x2950)=n $3068

9. Betty had $5 and spent $3.50 of her money on a new purse. What per cent of her money did she spend?

3.50=nx5 70%

10. John spent 16% of his $40 savings for new clothes. How much money did he spend?

n=.16x40 $6.40

11. A new car that cost $2985 lost 30% of its value in its first year. How much was the car worth at the end of its first year?

2985-(.30x2985)=n $2089.50

12. A bicycle had a sale price of $39.60. This was 75% of the original price. What was the original price of the bicycle?

39.60=.75xn $52.80

13. A man who saved 5% of his salary saved $212.50 in a year. How much money did he earn as salary in that year?

212.50=.05xn $4250

14. Mr. Karls pays $115 a month for rent. His take-home pay is $460 per month. What per cent of his take-home pay goes for rent?

115=nx460 25%

Review and Practice

You should be able to do the work on this page without any help.

Part 1 Copy. Perform the indicated operations.

	a	*b*	*c*
1.	8. .37)2.96	30. .175)5.25	.9 1.9)1.71
2.	926.7 31.8 517.3 +6.4 1482.2	65.93 92.46 46.87 +79.52 284.78	\$20.95 37.24 22.56 +19.25 \$100.00
3.	705.80 −299.98 405.82	420.004 −234.506 185.498	36.412 −6.833 29.579
4.	3.249 ×40.2 130.6098	.904 ×.513 .463752	67.431 ×24.5 1652.0595
5.	$412\frac{1}{3}$ $+395\frac{1}{2}$ $807\frac{5}{6}$	709 $-458\frac{1}{8}$ $250\frac{7}{8}$	$546\frac{3}{8}$ $+275\frac{5}{6}$ $822\frac{5}{24}$
6.	92 $\times45\frac{3}{4}$ 4209	$45\frac{1}{2}$ ×29 1319.5	$38\frac{3}{8}$ $\times15\frac{1}{2}$ $594\frac{13}{16}$

Part 2 Solve for n.

	a	*b*
1.	$(72\div9)\times10=\overset{80}{n}$	$21=(54\div\overset{9}{n})+15$
2.	$36\div(8\times7)=\overset{.643}{n}$	$\overset{81}{n}=(90-14)+5$
3.	$(42\div7)\times\overset{5}{n}=30$	$\overset{2.857}{n}=40\div(19-5)$
4.	$\overset{7}{n}\times(50\div5)=70$	$8=100-(4\times\overset{23}{n})$
5.	$(8\times7)\div80=\overset{.7}{n}$	$49=\overset{42}{n}+(42\div6)$

Part 3 Replace each dot with the symbol =, +, −, >, or <, whichever makes each statement below true.

	a	*b*
1.	$(5\times19)+3\overset{<}{\bullet}100$	$(32\overset{\div}{\bullet}8)+6=10$
2.	$\frac{3}{4}\div(\frac{1}{2}\times\frac{1}{8})\overset{=}{\bullet}12$	$(7\overset{\times}{\bullet}4)\times3=84$
3.	$(46\times2)\overset{-}{\bullet}2=90$	$45=(3\overset{\times}{\bullet}9)+18$
4.	$99\overset{<}{\bullet}(6\times12)+30$	$87\overset{=}{\bullet}(20+9)\times3$
5.	$(62\overset{\div}{\bullet}2)+9=40$	$(5+18)\overset{\times}{\bullet}0=0$

Part 4 Solve each problem.

1. The distance between 2 cities by way of the new highway is $7\frac{4}{10}$ miles which is $1\frac{4}{5}$ miles less than by way of the old highway. What was the distance by way of the old highway?

$7\frac{4}{10}+1\frac{4}{5}=n$ $9\frac{1}{5}$ mi.

2. Al worked for $10\frac{1}{4}$ hours at \$.75 an hour. How much money did he earn? 10.25x.75=n \$7.69

3. Bob must pay a 4% tax on a \$2.25 glove. What is the price of the glove including tax?

2.25+(2.25x.04)=n \$2.34

4. Sally took a test having 30 questions. She had them all right except 6. What per cent of the questions did Sally have right?

(30-6)=n x30 80%

5. A football team played 12 games. It lost 3 of the games and won the rest. What per cent of its games did the team win?

(12-3)=n x12 75%

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. To change a ratio from its fractional to its per cent form, express the ratio as a fraction with a denominator of 100. Then rename this fraction as per cent. (224)

2. Numbers expressed in per cent form may also be expressed in fractional or decimal form. (224-225)

3. To change a number from its per cent form to its decimal form, think of the per cent as a fraction with a denominator of 100. Then divide by $\frac{100}{100}$. (225)

4. To change a number from its decimal form to its per cent form, multiply the number in decimal form by $\frac{100}{100}$. (225)

5. In per cent problems any of three numbers may be unknown at the outset. The unknown number can be determined by solving an equation. (226-227)

Questions to Discuss

See page T237 for answers.

1. What equation would you use in answering these questions: 5 is what per cent of 20? What number is 25% of 20? 5 is 25% of what number? 3 is what per cent of 100? 9 is $33\frac{1}{3}\%$ of what number? (226)

2. Tell the reason for each step you would take in solving the equations you would use in answering the questions in exercise 1. (227)

3. How can you decide whether to use the fractional or decimal form of a per cent when translating a question into an equation? (230)

4. How would you estimate an answer to these questions: What number is 19% of 325? What number is 52% of 816? (233)

Written Practice

1. Write the per cent equivalent of $\frac{1}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, and $\frac{5}{6}$. (224) 20%, $12\frac{1}{2}\%$, $37\frac{1}{2}\%$, $83\frac{1}{3}\%$

2. Write the decimal equivalent of 80%, 45%, 20.8%, and $4\frac{1}{2}\%$. (225)
.80, .45, .208, .045

Express each statement below in equation form. Solve. (226-227)

3. 9 is what per cent of 36?
$9=n\times36$ 25%

4. 35 is what per cent of 49?
$35=n\times49$ $71\frac{3}{7}\%$

5. 12 is 40% of what number?
$12=.40\times n$ 30

Find the number equal to each of the following. (230)

6. $12\frac{1}{2}\%$ of 480 60

7. 63% of 575 $362\frac{1}{4}$

Self-Evaluation

Part 1 Express the following fractions in per cent form.

	a	*b*	*c*	*d*
1.	$\frac{1}{2}$ 50%	$\frac{1}{4}$ 25%	$\frac{3}{4}$ 75%	$\frac{1}{5}$ 20%
2.	$\frac{2}{5}$ 40%	$\frac{1}{6}$ $16\frac{2}{3}\%$	$\frac{5}{6}$ $83\frac{1}{3}\%$	$\frac{1}{8}$ $12\frac{1}{2}\%$
3.	$\frac{1}{10}$ 10%	$\frac{3}{8}$ $37\frac{1}{2}\%$	$\frac{7}{8}$ $87\frac{1}{2}\%$	$\frac{4}{5}$ 80%

Express the following per cents in decimal form and then in fractional form. Reduce fractions to lowest terms. See below.

	a	*b*	*c*	*d*
4.	75%	$33\frac{1}{3}\%$	$12\frac{1}{2}\%$	50%
5.	20%	25%	$66\frac{2}{3}\%$	40%
6.	80%	$37\frac{1}{2}\%$	$16\frac{2}{3}\%$	30%

Part 2 Answer each question below.

1. 6 is what per cent of 18? $33\frac{1}{3}\%$

2. What number is 40% of 320? 128

3. 9 is 20% of what number? 45

4. What number is $66\frac{2}{3}\%$ of 24? 16

5. 14 is what per cent of 35? 40%

6. 51 is 19% of what number? $268\frac{8}{19}$

7. 12 is 21% of what number? $57\frac{1}{7}$

8. What number is 18% of 45.5? 8.19

9. 35 is what per cent of 210? $16\frac{2}{3}\%$

10. What number is 80% of 200? 160

4. .75, $\frac{3}{4}$; .$33\frac{1}{3}$, $\frac{1}{3}$; .125, $\frac{1}{8}$; .50, $\frac{1}{2}$

5. .20, $\frac{1}{5}$; .25, $\frac{1}{4}$; .$66\frac{2}{3}$, $\frac{2}{3}$; .40, $\frac{2}{5}$

6. .80, $\frac{4}{5}$; .375, $\frac{3}{8}$; .$16\frac{2}{3}$, $\frac{1}{6}$; .30, $\frac{3}{10}$

Part 3 Solve each problem.

1. Janet collected \$14.50 for her work at a cooky sale. Her club was allowed to keep 25% of the money. How much money did she earn for her club? $n=\frac{1}{4}$x14.50 \$3.62

2. Ronald made 17 baskets out of 51 shots at the basket. What per cent of his shots at the basket resulted in baskets? 17=nx51 $33\frac{1}{3}\%$

3. June saved 20% of the cost of a \$5.75 sweater bought on sale. How much money did June save on the sweater? $n=\frac{1}{5}$x5.75 \$1.15

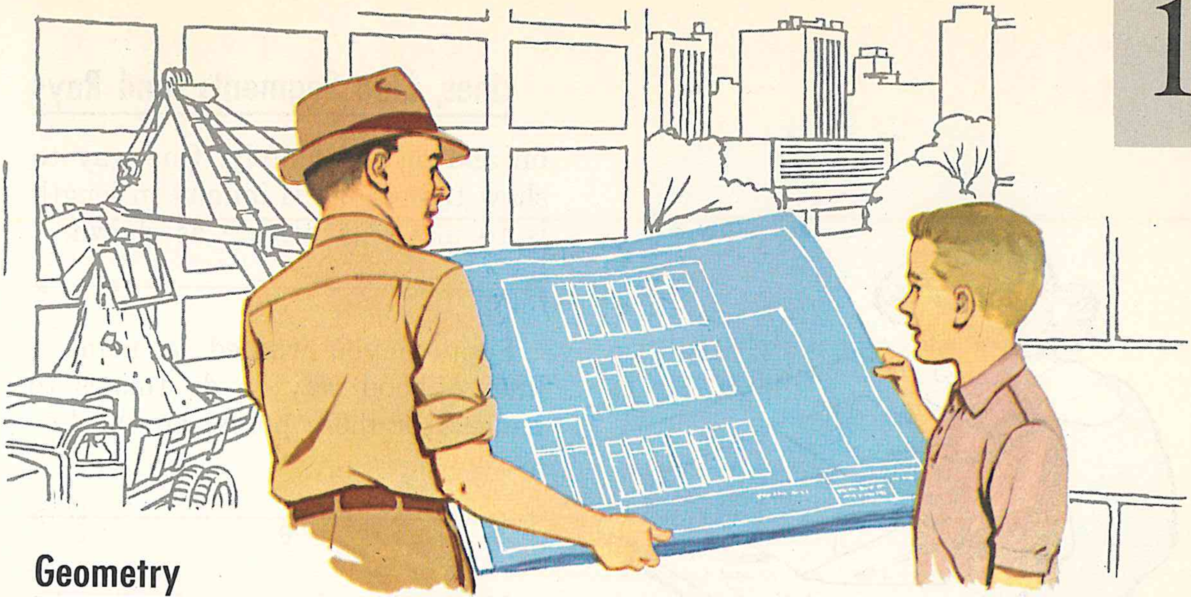
4. Bob paid \$7.20 for shoes. This was 80% of the regular price. How much was the regular price? 7.20=.80xn \$9.00

5. Jane paid 18¢ for a notebook at a sale where everything was reduced in price by 10%. What was the regular price of the notebook? 18=(1.00-.10)xn 20¢

6. Andy saved 15% of his earnings of \$35. How much money did he save? n=.15x35 \$5.25

7. In a class of 35 pupils, 60% of the pupils were girls. How many girls were in the class? n=.60x35 21

8. A baseball team won 69 out of 121 games. What per cent of its games did the team win? (Express the answer to the nearest tenth of 1 per cent.) 69=nx121 57.0%



Geometry

Choosing the correct location is an important step in planning for a new building. The location of the building is marked on a map in the city engineer's office. The location of the excavation is marked by stakes in the ground. The locations of the footings for the building are marked on the blueprints. In fact the location of every single part of the building is recorded somewhere.

Anyone responsible for determining the location of anything on earth or in space is using a branch of mathematics called **geometry.** Do you use geometry? Think of the last time you tried to determine the location of something before answering the question.

The mathematician is interested in exact locations. Locations of this kind are considered as **points.** You may think of a point as an exact location in space. All it has is a fixed place; so it cannot be measured in regard to length, width, or thickness. A point has no dimensions. No instruments exist that enable you to measure a point, because a point is only an idea.

Suppose you wish to make a picture of a point and to give it a name. One way to do this is to make a dot and to label it with a capital letter of the alphabet.

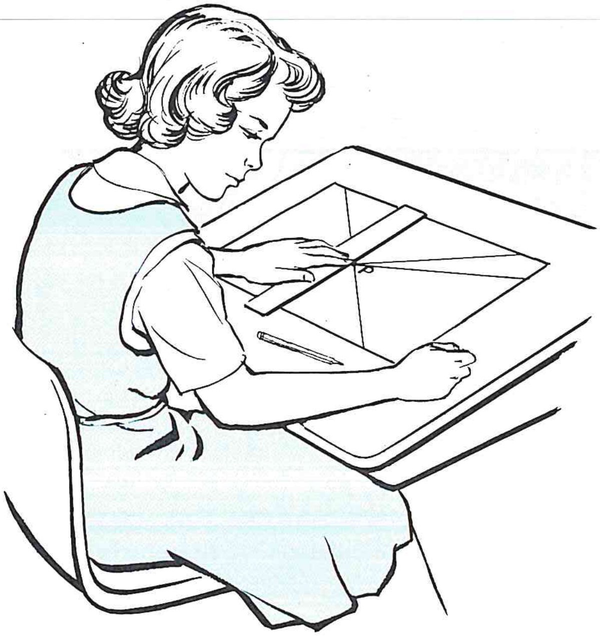
Oral Answer the questions below.

See above.

1. Why would it always be possible to count dots and impossible to count points? Dots are pictures; points are ideas.

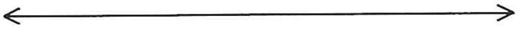
2. What is the advantage of being able to name a picture of a point? To distinguish it from other points.

Lines, Line Segments, and Rays

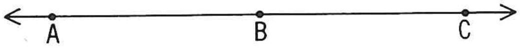


Joan is showing the correct way to draw a picture of a straight line that passes through point P. Her only tools are a sharp pencil and a **straightedge,** which could be any piece of wood, plastic, or metal with a straight edge. She will move her pencil along the straightedge from one edge of the paper to the other. The size of the paper or drawing surface limits the length of the picture of the straight line she can draw.

By definition a straight line is **infinite** in length in both directions. This means that it could be extended on and on indefinitely. One way to show that a line is infinite in length is to make its picture as shown.



Suppose you wanted to name a line. A good way to do this is to name 2 or more points on the line as shown.

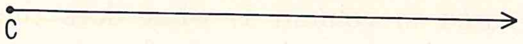


Now you can name the line by stating the names of any two of its points in either order. For example, the line may be named AB, BA, AC, CB, and so on.

You must keep in mind that a dot is a picture of a point and *not* a point. A straight line drawn on your paper is only a picture of a line because a line is only a collection or set of points. The *picture* of a line can be measured in regard to length, width, and thickness. The line itself has only one dimension, length.

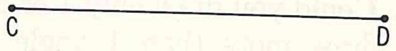
Suppose that John lights a candle out of doors on a dark night, and that he wants to draw a picture to illustrate his idea of one of the rays of light. Could he use a dot to show the point from which the light was coming? How could he show the direction of the ray?

One of the possible drawings is shown below.



The picture is different from that of a line, because it is infinite in length in only one direction. A drawing such as that above is called a **ray.** A ray is a part of a straight line that has one endpoint and extends indefinitely in the other direction. In what way does a ray differ from a line segment?

Suppose that the beam of light was halted at a definite point. Then you could use the picture below to show your idea of the situation.

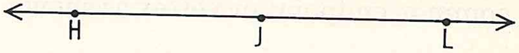


The portion or piece of the line between C and D is called a **straight-line segment,** or **line segment.** The endpoints of a line segment, in this case C and D, give their names to the segment. The line segment is named CD or DC. Line segment CD includes points C and D and all of the points between them.

Oral Answer these questions.

1. Why is it important to give a line a name? So one can distinguish it from other lines

2. How many different names could you give to the picture of a line shown below?
HJ, HL, JH, JL, LH, LJ



3. How is a picture of a straight line different from the picture of a ray? How is it different from the picture of a line segment? See page 240 and column 1.

4. What is the least number of points necessary to determine a straight line? To determine a ray? To determine a line segment?
2; 2; 2

Written On your paper, perform the following exercises.

1. Draw pictures of 3 straight lines that would not intersect, or cross, if your paper were of infinite extent.

2. Draw pictures of 3 straight lines that intersect at a point labeled P.

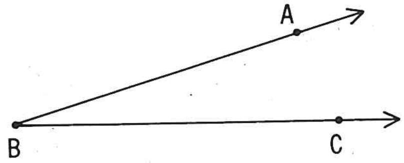
3. Draw pictures of 6 rays that have point A as a common starting point. Label point A.

4. Mark points A, B, and C, so that not all 3 of them lie on the same straight line. Draw line segments AB, BC, and AC. Label the endpoints of the segments.

5. Draw a line segment EF. Extend it to make a picture of a line. Label the line segment.

Angles

Suppose that you wanted to name a figure formed by 2 rays having a common endpoint or **vertex** as shown.



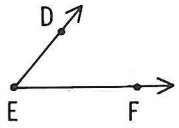
The name of the figure is **angle.** A short symbol for angle is ∠. To enable you to identify and talk about an angle, you should name it. You can do this by using the letters of the points that denote the rays and the single letter that indicates their common vertex. The vertex letter is in the middle of the name. The angle pictured above may be named as follows.

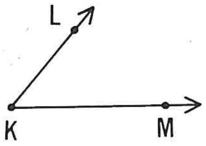
Angle ABC or ∠ABC

Angle CBA or ∠CBA

Rays AB and BC of ∠ABC are called the sides of the angle. Why would it be impossible to measure their length?

Now look at the pictures of ∠DEF and ∠LKM below. Which angle appears to be the larger?





In the angles DEF and LKM pictured in column 1, what does the arrowhead at the end of each ray tell you? Do you notice that the length of the rays has nothing whatever to do with the size of the angle? The angles DEF and LKM have the same measure. You can state this by writing ∠DEF = ∠LKM. The = sign between angles means equal in measure.

Oral Answer these questions.

1. How many rays are needed to draw an angle? 2

2. Why is it reasonable to have the vertex letter in the middle when you are naming an angle?
To identify the angle

3. Could you draw only 1 or could you draw more than 1 angle with the same vertex? Explain.
See below.

Written Follow these directions.
See column 1.

1. On your paper, draw 3 points, F, G, and H, not on a straight line. Use rays to draw ∠FGH.

2. Draw an angle greater in measure than ∠FGH.

3. Draw an angle smaller in measure than ∠FGH.

4. On your paper place points P, Q, R, S, T, U. Use rays to draw 5 angles all having P as the vertex.

3. More than 1 angle; rays may extend many directions from it.

Planes

The page you are looking at, your desk top, and the floor are a few examples of flat surfaces. Name others if you can. All surfaces of this kind should have a common name. You cannot call the surfaces *points* or *lines*, because points have no dimensions and lines have only one dimension. The name **plane** is given to the surfaces you have named.

You may recall that a straight line is defined as having infinite length. Try to think of the surface of a table as being extended infinitely in all directions. The *idea* which comes to you as a result of this thought is called a *plane*. Thus it is possible to think of such limited flat surfaces as a piece of paper or a desk top as a **part of a plane.**

In geometry we think of a plane as a continuous set of points such as that suggested by an enlarged table top. A part of a plane differs from a plane because it has boundaries. Trace the boundaries of your desk. Is the surface of your desk part of a plane or is it a plane?

Take a piece of paper and name two points on it as A and B. Draw a line through the points. Now suppose that you could extend the boundaries of the paper infinitely in all directions to make a plane. Would you expect all of line AB to be in that plane? Why?

You may recall that two points determine a straight line. A straight line has only one dimension. You need at least three points, not on the same line, to determine a plane.

Oral Answer these questions.
See column 1 and above.

1. Name some objects in your room that represent parts of planes.

2. Why is it impossible to tell how many points are contained either in a plane or in part of a plane?

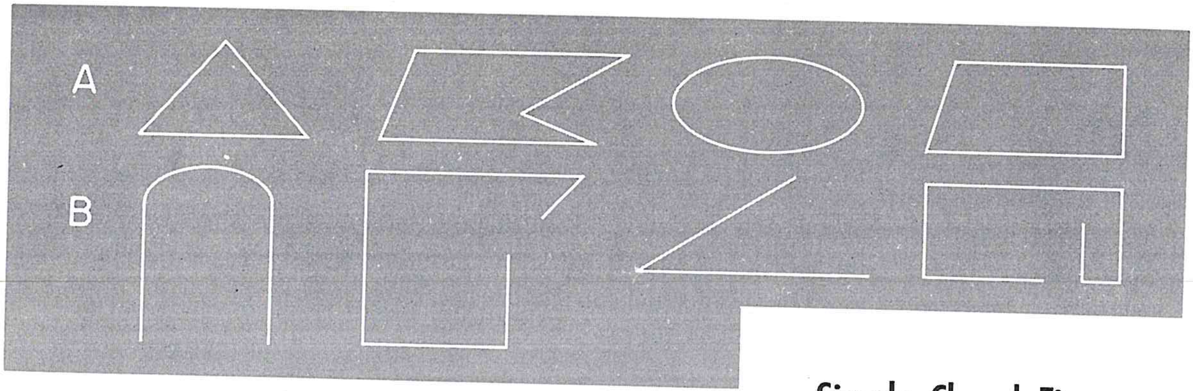
3. Can you tell how many lines are contained in a plane? Why or why not?

Written On your paper perform exercises 1 and 2 and indicate whether the resulting picture is a picture of a line, a line segment, or a plane. Then do exercise 3.

1. Draw 3 points which will not fall on the same straight line. Connect these points with line segments.

2. Draw 3 points which will fall on the same straight line. Connect the points.

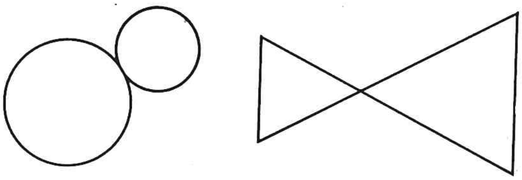
3. Name four objects whose surface is part of a plane.



Simple Closed Figures

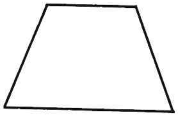
Look at each figure in row A above. Trace completely around each one and return to the point where you began your tracing. Is this possible? Is a similar experiment possible with each figure in row B above? Each figure in A is called a **closed figure.** Since you can trace around the complete figure without having the figure cross or **intersect** itself, the figures are called **simple closed figures.** What name might you give to the figures in B?

The drawings below are closed figures but not simple closed figures. How can you tell?



In this chapter we will be most concerned with simple closed figures formed by line segments. Figures of this kind are called **polygons.**

Take a piece of paper. Think of it as part of a plane and draw a polygon as shown at the right. The figure divides the plane into three distinct sets of points. See if you can find them by answering these questions: Are some of the points *inside* the polygon? Are other points *on* the polygon? Are other points *outside* the polygon?



Oral What objects in your room could you trace to obtain a simple closed figure? Which of the figures would be polygons? See column 1 and above.

Written Follow these directions.

1. Draw 3 points which do not lie on the same straight line. Connect the points with line segments to make a polygon.

2. Follow the instructions given in 1 but begin with 4 points and then with 5 points. You may have 2 of the points on a straight line.

Triangles

Jane made a polygon in which she connected three points A, B, and C that were not on the same straight line. The figure she drew is called a **triangle.** It has 3 sides, named AB, BC, and CA, and 3 angles. Name each angle. The prefix *tri* means *three.* Why is triangle a good name for figure ABC?

Look at the triangle the girl drew. It may be named in any of the ways shown below. Note: The symbol △ is read *triangle.*

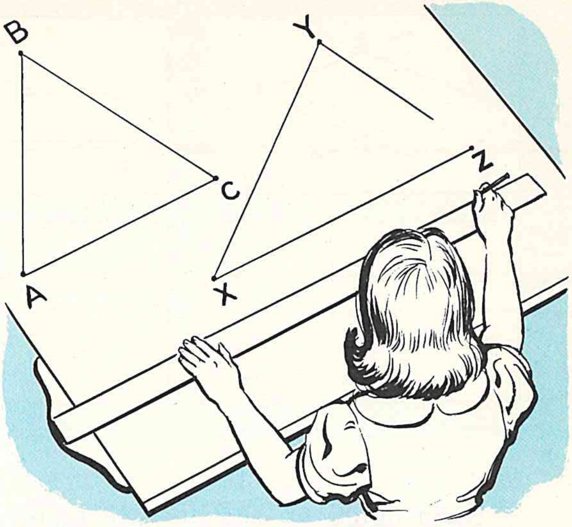
Triangle ABC (△ABC)
Triangle BCA (△BCA)
Triangle CAB (△CAB)
Triangle ACB (△ACB)
Triangle CBA (△CBA)
Triangle BAC (△BAC)

Look at figure XYZ. It has 3 straight sides. However, it is *not* a closed figure; so it is not a polygon. How many angles does it have? Is figure XYZ a triangle?

Oral Answer the questions below.

1. What are 4 objects that have the shape of a triangle?

2. In how many ways can you name a triangle having line segments that intersect at points R, S, and T?
RST, RTS, SRT, STR, TRS, TRS

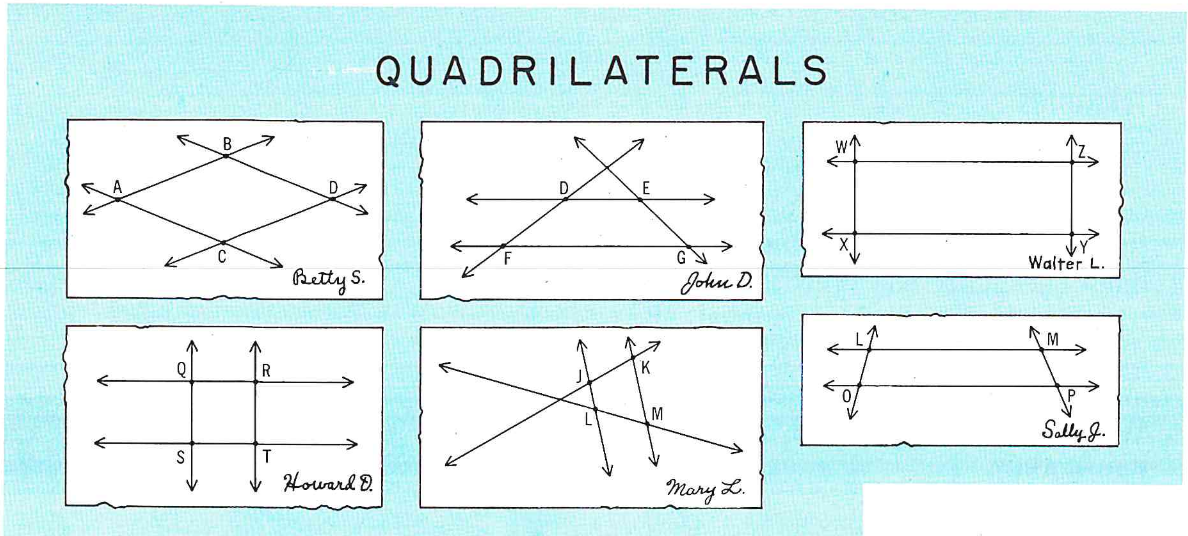


Written On your paper perform the following exercises.

1. Draw 3 points, not on the same straight line. Label them with capital letters and connect them with line segments. Name the polygon.

2. Draw 3 points on a straight line. Label them A, B, and C. Draw another point D not on the same straight line with A, B, and C. Draw all the possible straight-line segments between points A, B, C, and D. How many triangles have been formed? Name all of them, using the letters in counter-clockwise order.

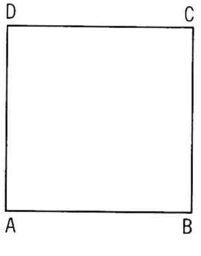
3. Draw a rectangle LKMN. Connect points L and M and points K and N with straight-line segments so they intersect at a point P. Name as many triangles as you can find.



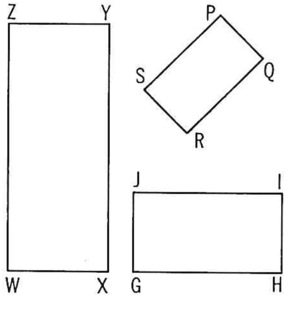
Quadrilaterals

Each figure on the bulletin board is a simple closed figure. Polygons such as those above are called **quadrilaterals.** *Quad* means *four* and *lateral* refers to *side;* so a quadrilateral indicates 4 sides. Does a quadrilateral also have 4 angles?

Sometimes all of the angles of a quadrilateral are equal in measure. Quadrilaterals of this kind are so common that they have been given special names as shown below.

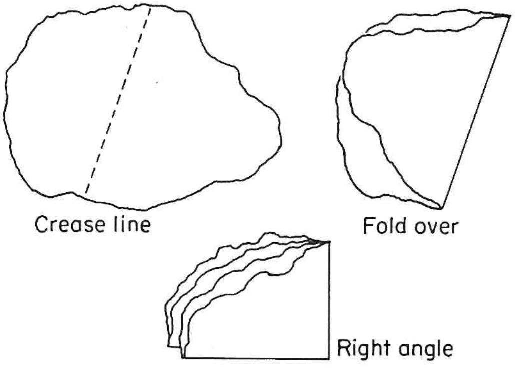


Square



Rectangles

All 4 of the angles of either a square or a rectangle are called right angles. There is an easy way to make a right angle to use in checking the measure of an angle. The drawing below shows you how you may begin with a piece of paper of *any* shape.



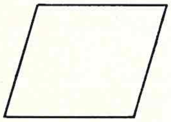
A square is a special type of rectangle. Like any rectangle, a square has 4 right angles, but it also has 4 sides of equal measure.

A quadrilateral is named by stating the letters which label the endpoints of its sides. The letters are named as they appear in clockwise or counter-clockwise direction around the figure. Two ways to read the name of the square are ABCD and ADCB. Can you name each rectangle?

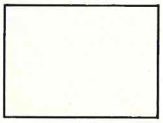
Look at square ABCD on page 246. Would sides AD and CB intersect if you extended them in either direction? Would sides AB and CD intersect? Lines AD and CB are in the same plane. Two lines in a plane may or may not intersect. If they do not intersect they are called **parallel lines.** Lines through AD and CB are parallel lines. Line segments AD and BC are called **parallel-line segments.** Are WX and ZY parallel-line segments? Which parallel-line segments do you see on rectangles SPQR and GHIJ?

One very important class of quadrilaterals includes all of those whose opposite sides are parallel in pairs regardless of whether or not the angles are right angles. Quadrilaterals of this class are called **parallelograms.** All of the figures below belong to this general class.

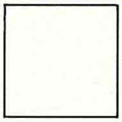
A



B



C

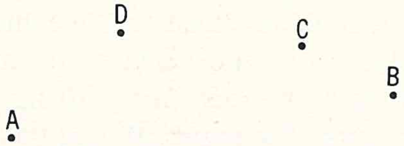


Figures B and C have special names: rectangle and square. Figure A, which has less practical use, is given the general name parallelogram.

Oral What objects in your room have the shape of a rectangle? Of a square? Of a parallelogram which is neither a rectangle nor a square?

Written On your paper perform the following exercises.

1. Draw 4 points A, B, C, and D, no 3 of which lie on a straight line, as shown.



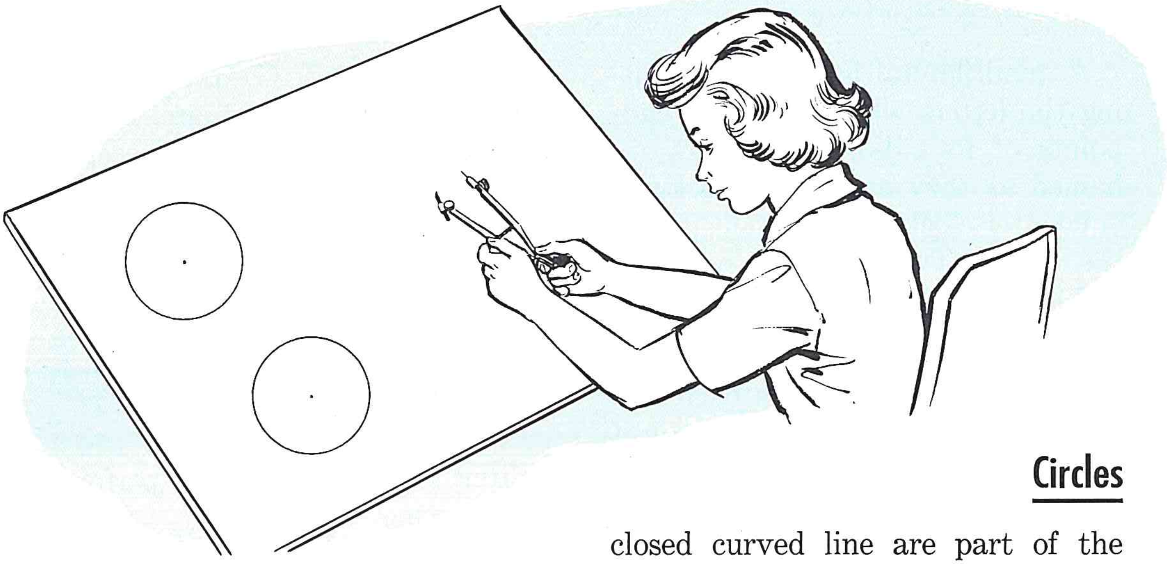
Use straight lines to connect AB, BC, CD, and DA. Tell why the figure is a quadrilateral.

It has 4 sides.

2. Draw 4 points E, F, G, H so that the lines through EF and through GH are parallel. Make segment GH longer than segment EF.

Use a right angle measure at E and draw a line segment from E to the line through H and G. Call the point of intersection C. Do the same at F and call the point of intersection with the line through H and G point D. What kind of figure did you draw?

Rectangle



Circles

Alice is drawing pictures of simple closed figures. She is using an instrument called a **compass.** The point of the compass must be held firmly in place, and the curved line must be drawn without once lifting the pencil from the paper. Would you expect all of the points on the curved line to be **equidistant,** or at the same distance, from the point at which you placed the steel point of the compass? Why? This point is called the **center.**

A closed curved line such as Alice has drawn is called a **circle.** The simple closed curve is part of a plane. It divides the plane into 3 distinct sets of points as follows.

a. All points *inside* the curve

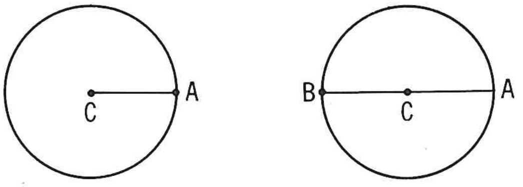
b. All points *on* the curve

c. All points *outside* the curve

Only those points that are *on* the closed curved line are part of the circle. Is the point called the center *on* the curve or is the point *inside* the curve?

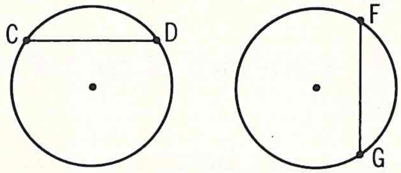
Any line segment which joins the center to any point on the circle is called a **radius.** The plural of radius is **radii** (rā′·dĭ·ī). The compass opening determines the radius of the circle. If the compass opening remains unchanged, all of the circles drawn with that compass will be the same size, or equal in measure. What happens to the size of a circle as the compass opening is increased? Decreased?

Look at this picture of two circles whose radii are of equal length.



Look at the pictures in column 2 of page 248. In one, the straight-line segment CA connects the center of the circle with point A on the circle. In the other, the straight-line segment AC passes through the center of the circle but has its endpoints on the circle. Line segment BA is called a diameter of the circle. Observe that BC and CA are segments of the same straight line. Do you see that any given circle can have many radii and many diameters?

Suppose you drew a line segment to connect any 2 points on a circle. For example, see line segments CD and FG on the circles below.



The line segment joining any two points on a circle is called a **chord.** Could any circle have many chords? Would a chord through the center of a circle be a diameter?

Oral Answer each question below.

1. How might you construct 4 circles having the same center but different radii? By changing the compass opening; same center

2. What is the ratio of a radius to a diameter of the same circle? 1 to 2

3. What is the ratio of the longest chord of a circle to the diameter of the circle? Longest chord is a diameter; 1 to 1

Written Do the following exercises.

1. Draw 2 circles of equal measure which intersect. In how many points do they intersect? 1 or 2

2. Draw 3 circles having the same center but different radii. Will the diameter of the largest circle intersect each of the smaller circles? In how many points? Yes; 2

3. Draw 3 circles with unequal radii. Label a radius of each circle.

4. Draw points C and P. Use line segment CP as a radius of a circle you draw.

5. Draw a circle with a center at A. Draw two radii of the circle, but not in a straight line. Label the endpoints of these radii B and C. Draw straight line segment BC. What kind of figure did you draw? Which two sides are equal in length? Why? Triangle; AB and AC, radii of same circle

6. Draw a circle with a center at A. Draw a radius AB. Leave the opening of the compass unchanged. Draw a second circle with its center point at B. Label one point where the circles intersect as point C. Draw straight line segments AC and BC. What kind of figure did you draw? Are its sides equal in length? Triangle; yes

Review and Practice

Part 1 Copy. Find each answer by performing the indicated operation.

	a	*b*	*c*
1.	$19\frac{1}{5}$ $+27\frac{4}{5}$ 47	$25\frac{1}{3}$ $+39\frac{1}{4}$ $64\frac{7}{12}$	$96\frac{3}{4}$ $+41\frac{1}{5}$ $137\frac{19}{20}$
2.	$27\frac{9}{16}$ $-12\frac{5}{16}$ $15\frac{1}{4}$	$80\frac{5}{6}$ $-27\frac{1}{3}$ $53\frac{1}{2}$	$56\frac{7}{16}$ $-19\frac{11}{12}$ $36\frac{25}{48}$
3.	.356 .7792 +.178 1.3132	.096 .432 +.075 .603	14.9 35.62 +49 99.52
4.	2.463 4.7 5.930 +7.81 20.903	12.952 91.87 76.4 +93.934 275.156	78.017 2.1 14.934 +85.0 180.051
5.	98.54 −39.28 59.26	8.924 −2.587 6.337	.7762 −.3775 .3987
6.	85.75 ×65 5573.75	9.273 ×8.4 77.8932	.437 ×.69 .30153

Part 2 Copy. Find each quotient to the nearer tenth.

	a	*b*
1.	50.0 $.16\overline{)8.0}$	.24 $9\overline{)2.16}$
2.	5.0 $1.4\overline{)7.0}$	8.0 $3.8\overline{)30.4}$
3.	.25 $5.8\overline{)1.45}$	2.4 $3.50\overline{)8.4}$
4.	4.0 $24.7\overline{)98.8}$	30.0 $.175\overline{)5.25}$

Part 3 Copy. Solve each of the following equations.

	a	*b*
1.	4 $n\times\frac{2}{5}=\frac{8}{5}$	$\frac{3}{8}$ $2\times n=\frac{6}{8}$
2.	$\frac{77}{96}$ $\frac{7}{8}\times\frac{11}{12}=n$	14 $(\frac{1}{2}\times12)+n=20$
3.	$9\frac{3}{8}$ $n=2\frac{1}{2}\times3\frac{3}{4}$	$\frac{5}{6}$ $n-(\frac{3}{4}\times\frac{5}{6})=\frac{5}{24}$
4.	$1\frac{4}{5}$ $n=\frac{9}{16}\div\frac{5}{16}$	8 $(\frac{4}{5}\div\frac{2}{5})+n=10$
5.	$4\frac{1}{2}$ $3\frac{3}{8}\div\frac{3}{4}=n$	$9\frac{2}{5}$ $(6\frac{5}{8}\div2\frac{1}{2})+6\frac{3}{4}=n$
6.	110 $n=.66\div.006$	100 $(72\div.08)-n=800$
7.	$2\frac{1}{8}$ $8\frac{1}{2}\div4=n$	$19\frac{2}{5}$ $25-(14\div2\frac{1}{2})=n$
8.	$8\frac{4}{27}$ $27\frac{1}{2}\div3\frac{3}{8}=n$	25.75 $n=37-(12.5\times.9)$

Part 4 Solve each of the following problems.

1. At a sale Betty saved 30% of the cost of a $2.50 record. How much did she save?

.30x2.50=n $.75

2. Karen had $.90, which was $\frac{1}{3}$ of the money she needed for a book. What was the price of the book?

3x.90=n $2.70

3. Bob's mother ordered a $1.15 luncheon. Bob ordered a luncheon that cost $1.25. There was a 4% tax. What was the total bill? (1.15+1.25)+[.04x(1.15+1.25)]=n $2.50

4. Jack wasted $2\frac{1}{4}$ feet of a 14-foot board. What per cent of the board did he waste?

2.25÷14=n $16\frac{1}{14}$ per cent

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. A point cannot be measured, for it has no dimensions. (239)

2. A straight line is infinite in length in both directions. (240)

3. Rays and line segments are parts of lines. (240-241)

4. The measure of an angle does not depend on the lengths of the sides, but on the amount of opening between the sides. (242)

5. Every simple closed figure determines 3 distinct sets of points: points on the line, points inside the figure, and points outside the figure. (244, 248-249)

6. Circles having the same radius are equal in measure. (248-249)

Words and Symbols to Know

1. Geometry, point (239)

2. Line, line segment, infinite, ray (240-241)

3. Vertex, angle, ∠ (242)

4. Plane, part of a plane (243)

5. Closed figure, simple closed figure, intersect, polygon (244)

6. Triangle, △ (245)

7. Quadrilateral, rectangle, square, parallel lines, parallelogram, polygon (246-247)

8. Equidistant, circle, radius, radii, diameter, chord (248-249)

See T251 for answers.

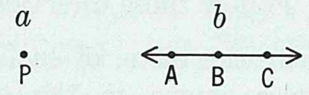
Questions to Discuss

1. How does a picture of a point differ from a point? (239)

2. Why is it impossible to measure either a point, a line, or a ray? (239, 240-241)

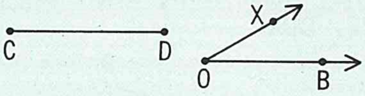
What kind of figure is shown by each drawing below? How would you name it?

3.



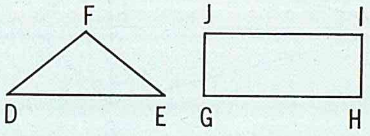
(240-241)

4.



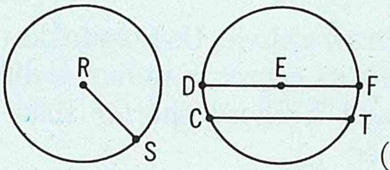
(242-243)

5.



(245, 247)

6.



(248-249)

Self-Evaluation

Part 1 Write the word or words in ( ) that make each of the following statements true.

1. A line has (0, 1, 2, more than 2) endpoints.

2. A point has (0, 1, 2, 3) dimensions.

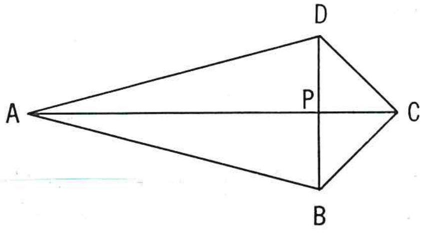
3. A triangle has (more, fewer, the same number of) angles as a quadrilateral.

4. A circle has (1, 2, 3, any number of) radii.

5. A diameter is (longer than, shorter than, the same length as) the longest chord of a circle.

Part 2 Follow these directions.

1. Write the name of each of the 8 triangles shown in the drawing below. APD, ACD, DPC, DBC, CPB, CAB, BPA, BDA



2. Draw a circle. Use capital letters to name its center, a radius, a diameter, and a chord shorter than its diameter.

Part 3 On your paper perform the following exercises.

1. Draw a line through points labeled A and B.

2. Draw a ray from a point named D.

3. Draw an angle named EFG.

4. Draw a triangle named HIJ.

5. Draw a square named KLMN.

6. Draw a rectangle named STUV.

7. Draw a parallelogram that is neither a rectangle nor a square.

8. Draw a figure having 4 sides, which is not a simple closed figure.

Part 4 Solve for n.

1. $37.46+14+85.9=\overset{137.36}{n}$

2. $\overset{53.5}{n}+46.5=100$

3. $(3\times.09)\div\overset{1}{n}=.27$

4. $(\overset{2}{n}\times1.5)+10=13$

5. $\overset{100}{n}=(45\div.5)+10$

6. $\overset{2.10}{n}=(1.95+.65)-.50$

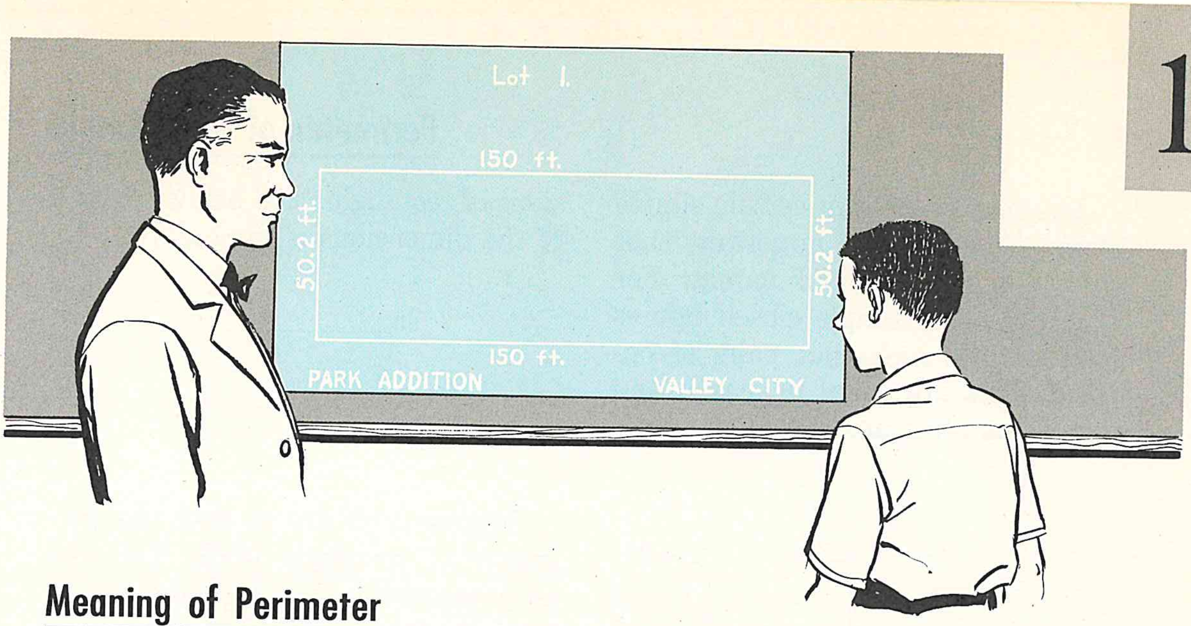
7. $(10\times\overset{2}{n})-15=5$

8. $(6\frac{1}{2}\times2\frac{2}{3})\div2=\overset{8\frac{2}{3}}{n}$

9. $\overset{354.166...}{n}=(16+5\frac{1}{4})\div.06$

10. $(40\div2\frac{1}{2})\times6=\overset{96}{n}$

15


Lot 1.
150 ft.
50.2 ft.
50.2 ft.
150 ft.
PARK ADDITION
VALLEY CITY

Meaning of Perimeter

The measures of the sides of a city lot are given on the blueprint. What is the measure of the length of all of the sides together?

You can find the answer by setting up an equation in which p represents the total length. Study the equation.

$$50.2+150+150+50.2=p$$

Since $p=400.4$, the answer to the question is 400.4 feet.

In the above problem, the measure of 400.4 feet is called the **perimeter** of the lot. *Peri* means *around* and *meter* means *measure*. Thus *perimeter* means the measure around a simple closed figure.

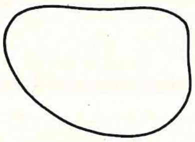
Look at the figure formed by the surface of your desk. How would you find the perimeter of the figure? Do the figures formed by each window and door in your room have a perimeter? Does the figure formed by any one of our coins have a perimeter?

Oral Answer each question below.

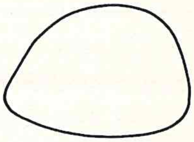
1. Why might you need to know the perimeter of the figure formed by the surface of an object?
Answers will vary.

2. How could you use a piece of string and then a ruler to find the perimeter of each simple closed figure below? See below.

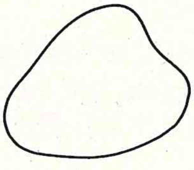
a



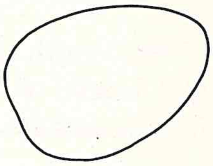
c



b



d

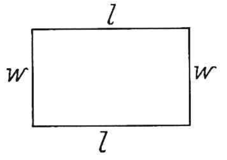


2. Place the string along the line of the figure; then measure the length of the string with a ruler.

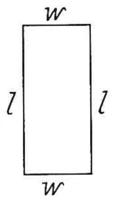
Perimeter of a Rectangle

You may recall that certain simple closed figures are so important that they are given special names. For example, some simple closed figures have 4 sides such that pairs of opposite sides are equal in length and parallel as illustrated below.

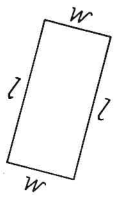
A



B



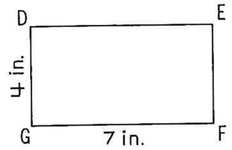
C



Each of these figures represents a rectangle. Notice the letters *l* and *w* on each rectangle. They represent the *measures* of the length and width.

The measure of the longer sides of a rectangle as in A is usually called the *length* of the rectangle. The measure of each side in the pair of shorter sides is usually called the *width* of the rectangle. However when a rectangle is in the position chosen in B and C, the lower horizontal side, regardless of its length, is often called the **base.** Then the vertical side is called the **height.**

The length and width, or the base and height of a rectangle are called its **dimensions.** What are the dimensions of rectangle DEFG below? Which of the dimensions is the base?



Suppose you are to find the perimeter of rectangle DEFG. You can let p represent perimeter and set up the following equation.

$$p=7+4+7+4$$
$$p=22$$

Since $p=22$, and the unit of measure is inches, the perimeter of rectangle DEFG is 22 inches.

Study the following solution. See if you can develop a formula to use in finding the perimeter of a rectangle.

$$\begin{aligned} p&=7+4+7+4 \\ &=(7+7)+(4+4) && \text{Assoc. property} \\ &=(2\times7)+(2\times4) && \text{Distr. property} \\ &=2\times(7+4) \end{aligned}$$

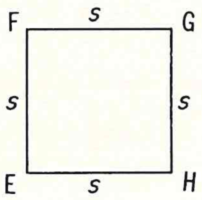
Since 7 represents the measure of the length and 4 represents the measure of the width, you may substitute l for 7 and w for 4 in the equation $p=2\times(7+4)$. Translate the equation into English like this: *Perimeter equals 2 times the sum of the measures of the length and the width.*

In abbreviated form, the statement can be translated into this formula:

$$p = 2 \times (l + w) \qquad \text{or} \qquad p = 2(l + w)$$

Before using the formula, be sure that the numbers all express measures in the same unit. If you wish to know the perimeter in feet, for example, the measures of both the length and the width must be in feet.

Suppose you are to find the perimeter of a square. You may recall that a square is a rectangle having 4 sides of the same measure, as shown in figure EFGH below. Observe that each side is named s.



Suppose that in square EFGH, s is $5\frac{1}{2}$ inches long. How would you find the perimeter?

You can find the answer by setting up an equation as follows:

$$\begin{aligned} p &= 5\tfrac{1}{2} + 5\tfrac{1}{2} + 5\tfrac{1}{2} + 5\tfrac{1}{2} \\ &= 4 \times 5\tfrac{1}{2} \\ &= 22 \end{aligned}$$

Since $p = 22$ and the unit of measure is feet, the perimeter of square EFGH is 22 feet.

Do you see that the perimeter of a square is equal to 4 times the measure of one side? Then

$$p = 4 \times s \qquad \text{or} \qquad p = 4s$$

Think of $4s$ as 4 times s even though the times sign is omitted.

Oral Answer these questions.

1. How would you find the perimeter of the figure formed by the floor of your room? p=2(l+w) or p=4s

2. Why might you sometimes wish to express a perimeter in more than one unit of measure, for example, in feet *and* inches? To avoid using fractions

Written Set up an equation to use in finding the perimeter of each of the following. Solve each equation.

	Figure	*Width*	*Length*
1.	$p=2x(6+9)$ 30 ft. rectangle	6 ft.	9 ft.
2.	$p=2x(4\frac{1}{4}+7\frac{1}{2})$ $23\frac{1}{2}$ ft. rectangle	$4\frac{1}{4}$ ft.	$7\frac{1}{2}$ ft.
3.	$p=4x6\frac{3}{8}$ $25\frac{1}{2}$ in. square	$6\frac{3}{8}$ in.	
4.	$p=2x(5+15)$ 13 yd. 1 ft. rectangle	5 ft.	5 yd.
5.	$p=2x(9+15)$ 4 ft. rectangle	9 in.	$1\frac{1}{4}$ ft.
6.	$p=4x5\frac{1}{4}$ 21 yd. square	$5\frac{1}{4}$ yd.	
7.	$p=2x(2.5+4.6)$ 14.2 mi. rectangle	2.5 mi.	4.6 mi.
8.	$p=4x19.5$ 78 mi. square	19.5 mi.	
9.	$p=2x(36+65)$ 202 rd. rectangle	36 rd.	65 rd.
10.	$p=2x(25+62\frac{1}{2})$ 175 mi. rectangle	25 mi.	$62\frac{1}{2}$ mi.

MORE PRACTICE
PAGE
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Suppose that you knew that the perimeter of a rectangle was 20 feet and that its width was 4 feet. How could you find its length?

You can find the length by using the formula you used in finding the perimeter of a rectangle. Study the solution below.

$p=2\times(l+w)$

Use known values

$20=2\times(l+4)$

Divide by 2

$\frac{20}{2}=\frac{2}{2}\times(l+4)$

Perform operation

$10=1\times(l+4)$

$10=l+4$

To obtain an equation which contains l on one side, follow the steps given below.

$$10-4=l+(4-4)$$
$$6=l+0$$
$$6=l$$

The rectangle is 6 feet long.

See now if you can develop a formula to use in finding the unknown dimension of any rectangle when its perimeter and one other dimension are known. Study the formulas below.

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$p=2\times(l+w)$	$p=2\times(l+w)$
$\frac{p}{2}=l+w$	$\frac{p}{2}=l+w$
$\frac{p}{2}-l=w$	$\frac{p}{2}-w=l$

How would you express $\frac{p}{2}-l=w$ and $\frac{p}{2}-w=l$ in the English language?

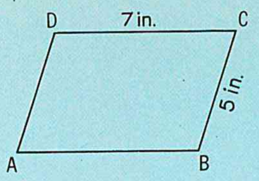
From the formula $p=4s$, or $4s=p$, it is possible to develop the formula $s=\frac{p}{4}$ to use in finding the length of a side of a square when its perimeter is known. What does the formula $s=\frac{p}{4}$ mean? How would you use the formula?

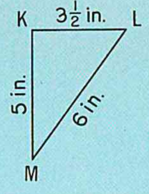
Oral Tell the equation to use in finding the unknown dimension in the figures below.

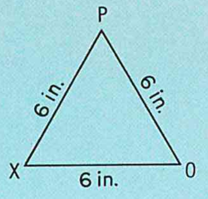
	Figure	Perimeter	Known Dimension
	$w=\frac{18}{2}-5$	4 ft.	
1.	rectangle	18 ft.	length, 5 ft.
	$l=\frac{27}{2}-6$	$7\frac{1}{2}$ in.	
2.	rectangle	27 in.	width, 6 in.
	$s=42\div4$	10.5 yd.	
3.	square	42 yd.	——
	$w=\frac{38}{2}-14$	5 yd.	
4.	rectangle	38 yd.	length, 14 yd.
	$l=\frac{40.5}{2}-9$	$11\frac{1}{4}$ ft.	
5.	rectangle	$40\frac{1}{2}$ ft.	width, 9 ft.
	$[40\frac{1}{2}-(9\text{x}2)]\div2=l$	$11\frac{1}{4}$ ft.	

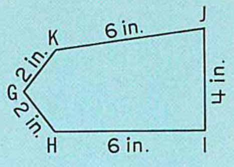
Written Set up an equation for each exercise in rows 1 through 5. Solve each equation. Then solve for l, w, or s in each equation below.

	a	b
6.	$90=2\times(15+l)$ 30	$70=2\times(9+l)$ 26
7.	$14=4\times s$ $3\frac{1}{2}$	$55=2\times(8+l)$ $19\frac{1}{2}$
8.	$2\times(w+35)=80$ 5	$2\times(w+17)=64$ 15
9.	$4s=108$ 27	$4s=82.4$ 20.6









Perimeters of Other Polygons

Parallelogram ABCD has opposite sides which are equal in length and are also parallel. Why could you use the formula $p=2\times(l+w)$ to find the perimeter of a parallelogram?

Triangle KLM has sides that are all different in length. To find its perimeter, you would add the measures of its three sides.

$$p=5+6+3\tfrac{1}{2}$$
$$p=14\tfrac{1}{2}$$

Since $p=14\frac{1}{2}$, and the unit is inches, the perimeter of △KLM is $14\frac{1}{2}$ in.

Triangle XOP has sides that are equal in measure. A triangle whose sides are equal in measure is called an **equilateral triangle.** See if you can tell why. To find the perimeter of △XOP, you could use the formula $p=3\times l$ or $p=3l$. The perimeter is 18 in.

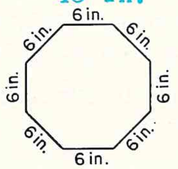
Look at the dimensions of figure GHIJK. Why would you add 6, 6, 4, 2, and 2 to find its perimeter? What is the perimeter of figure GHIJK in inches? In feet and inches?

Oral Answer these questions.

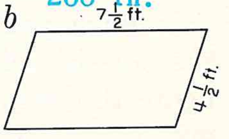
1. If you use *a*, *b*, and *c* to name the sides of a triangle, what formula can you develop to use in finding its perimeter? p=a+b+c

2. To the nearest inch, what is the perimeter of each polygon below?

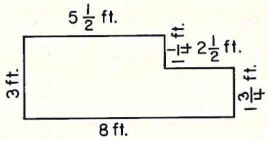
a 48 in.



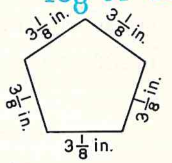
b 288 in.



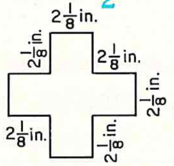
c 264 in.



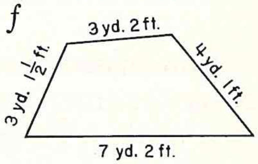
d $15\frac{5}{8}$ or 16 in.



e $25\frac{1}{2}$ or 26 in.



f 690 in.



Written Write an equation to use in finding the perimeter of each figure in exercise 2. Solve each equation.

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Equations only. See below.

a. $6\times8=n$

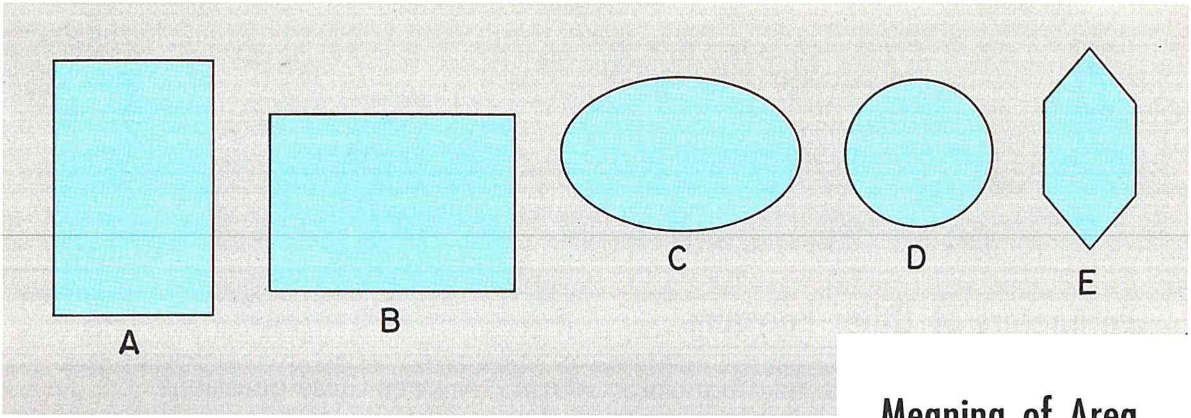
b. $12\times[2\times(7\frac{1}{2}+4\frac{1}{2})]=n$

c. $12\times[(8+3)+(5\frac{1}{2}+2\frac{1}{2})+1\frac{3}{4}+1\frac{1}{4})]=n$

d. $5\times3\frac{1}{8}=n$

e. $12\times2\frac{1}{8}=n$

f. $36\times(3\frac{2}{3}+4\frac{1}{3}+7\frac{2}{3}+3\frac{1}{2})=n$

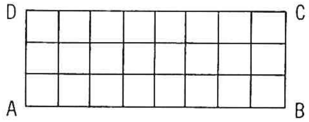


Meaning of Area

Each simple closed figure above encloses part of a plane. The part of any plane enclosed by a simple closed figure is called its **interior.** The interior of each figure above is shown in blue color.

Suppose you wanted to measure the interior of either figure A or figure B. Could you use linear measure? Why or why not?

See what happens when you use small unit squares to measure the interior of rectangle ABCD.



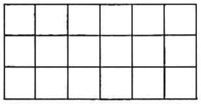
How many squares are needed to cover the interior? The smaller part of a plane (the square) is contained in the larger part of a plane (rectangle ABCD) 24 times. Why? The measure of the interior of rectangle ABCD is 24 unit squares.

As you have seen, it is possible to use some part of a plane, as for example a square, to measure the interior of a plane surface. The measure of a part of a plane or of a surface is called **area.** Do figures C, D, and E at the top of this page enclose an area? Can you measure them with unit squares?

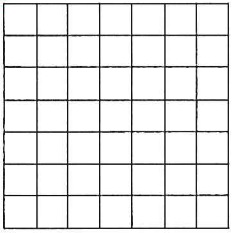
Oral What is the measure in unit squares of each figure below?

1.

a 18 unit sq.

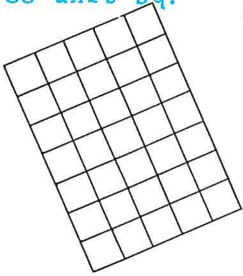


b 49 unit sq.

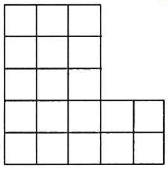


2.

35 unit sq.

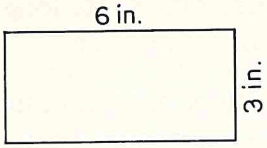


19 unit sq.



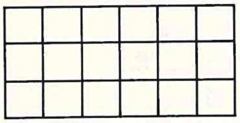
Finding the Area of a Rectangle

Suppose you wish to measure the interior of the rectangle below.



You can do the measuring by finding how many times a smaller part of a plane, called an inch square, is contained in the rectangle. The area enclosed by this inch square is called a **square inch.** It is a unit of area measure.

One way to find the area of the rectangle is to cover it with square inches and count them.



How many square-inch units are shown in each row? How many rows are there? You can also find the area by multiplying 6 and 3. Why? What is the area measure of the surface?

The area measure of a rectangle is equal to the product of the linear measure of the length and width of the rectangle. If you let A stand for the number of square units in the area, you can express the formula for finding the area measure, or area of a rectangle as follows.

$$A = l \times w \qquad \text{or} \qquad A = lw$$

In the rectangle on this page $l = 6$ and $w = 3$.

$$\begin{aligned} \text{Since} \quad & A = lw \\ \text{Then} \quad & A = 6 \times 3 \\ \text{And} \quad & A = 18 \end{aligned}$$

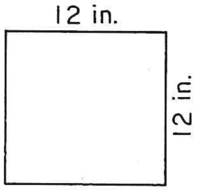
Since the unit of measure is square inches, the area is 18 square inches. How would you use the formula to find the area of a rectangle with dimensions of 4 in. by 6 in.?

Oral What would be a suitable unit of area measure to use in measuring a rectangle with dimensions expressed in feet? In yards? Square foot; square yard

Written Find the area of rectangles with these dimensions.

	a	*b*
1.	35 sq.in 5 in. by 7 in.	432 sq.in. 12 in. by 36 in.
2.	152 sq.ft. 8 ft. by 19 ft.	2520 sq.ft. 45 ft. by 56 ft.
3.	147 sq.yd. 7 yd. by 21 yd.	798 sq.yd. 19 yd. by 42 yd.
4.	76 sq.mi. 4 mi. by 19 mi.	$12\frac{1}{4}$ sq.mi. $1\frac{3}{4}$ mi. by 7 mi.
5.	$23\frac{5}{8}$ sq.in. $2\frac{5}{8}$ in. by 9 in.	$82\frac{7}{8}$ sq.ft. $8\frac{1}{2}$ ft. by $9\frac{3}{4}$ ft.
6.	$52\frac{111}{128}$ sq.in. $6\frac{5}{16}$ in. by $8\frac{3}{8}$ in.	$32\frac{83}{100}$ sq.mi. $4\frac{9}{10}$ mi. by $6\frac{7}{10}$ mi.
7.	$35\frac{3}{4}$ sq.rd. $5\frac{1}{2}$ rd. by $6\frac{1}{2}$ rd.	$80\frac{19}{100}$ sq.mi. $8\frac{1}{10}$ mi. by $9\frac{9}{10}$ mi.

Suppose you wanted to find the area of the interior of a square that measured 12 inches on each side. Since a square is a special kind of rectangle, you could use the formula $A = lw$ as shown.



$$A = lw$$
$$A = 12 \times 12$$
$$A = 144 \text{ sq. in.}$$

If you let s represent the length of each side, you can shorten the formula $A = lw$ as follows.

$A = lw$ becomes $A = s \times s$.

Then $A = s^2$.

The symbol s^2 is read *s squared*. The small numeral ^{2} tells you to use the length measure of one side twice as a factor.

Oral Tell the area of the interior of squares having these measures.

	a	*b*
1.	4 sq.in. 2 inches	9 sq.in. 3 inches
2.	16 sq.in. 4 inches	25 sq.in. 5 inches
3.	36 sq.in. 6 inches	49 sq.in. 7 inches
4.	64 sq.in. 8 inches	1 sq.ft. 1 foot
5.	144 sq.in. 12 inches	81 sq.ft. 9 feet

Written Solve these problems.

1. A ball diamond is a square whose sides are 90 ft. long. What is the area of its interior?
90x90=n 8100 sq.ft.

2. The floor of a room in the shape of a square has sides that are $10\frac{1}{2}$ ft. in length. What is the area of the interior of the square?
$10\frac{1}{2}$x$10\frac{1}{2}$=n $110\frac{1}{4}$ sq.ft.

3. The square-shaped floor of a tent had sides of 6 ft. What was the area of the floor?
6x6=n 36 sq.ft.

4. A square-shaped field measured 96.5 ft. on each side. What was the area of the field? 96.5x96.5=n
9312.25 sq.ft.

A quick review Copy. Replace the blanks with numerals that make each statement true. (= means equal in measure.)

1. 27 in. = $\underline{2\frac{1}{4}}$ ft.

2. 4 ft. = $\underline{48}$ in.

3. 6 yd. = $\underline{18}$ ft.

4. 5 ft. + $\frac{1}{2}$ ft. = $\underline{66}$ in.

5. 15,840 ft. = $\underline{3}$ mi.

6. 33 ft. + 6 ft. = $\underline{13}$ yd.

7. 73 yd. = $\underline{219}$ ft.

8. 7 mi. + .4 mi. = $\underline{39072}$ ft.

9. 2 mi. + .5 mi. = $\underline{4400}$ yd.

10. 10,560 yd. = $\underline{6}$ mi.

Other Units of Measure

Mr. Andrews put a fence around a rectangular field that had dimensions of 40 rods by 60 rods. What was the area of the field?

By using the formula $A = lw$, you can find the area of the field in square rods as shown below.

$$A = lw$$
$$A = 40 \times 60$$
$$A = 2400$$

The field has an area of 2400 **square rods.** (sq. rd.). A square rod measures 1 rod or $16\frac{1}{2}$ ft. on each side.

The square rod is rarely used in measuring area. A much more common unit of area measure used in measuring larger surfaces is the **acre.** The area measure of a football field is approximately 1 acre. An acre is a square surface that measures about 210 ft. on each side. Is your playground more or less than 210 ft. in length? In width? Is it more or less than 1 acre? An acre is equal in measure to 160 square rods.

Another unit of area measure is the **square mile.** (sq. mi.). It is the area measure of a square surface which measures 1 mile or 5280 ft. on each side. The measure of an acre is contained in the measure of a square mile 640 times.

A table of square measure is given below for reference. The sign = means equal in measure.

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
160 sq. rd. = 1 acre (A.)
640 acres = 1 sq. mi.

Oral What formula would you expect to use in converting 3 square feet to square inches? In converting 288 square inches to square feet? Explain how to use each formula.
See below.

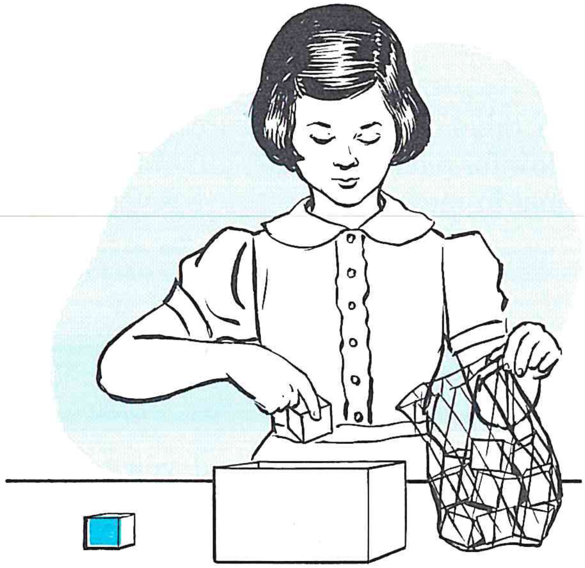
Written Copy. Replace each blank with the numeral that makes each statement below true.

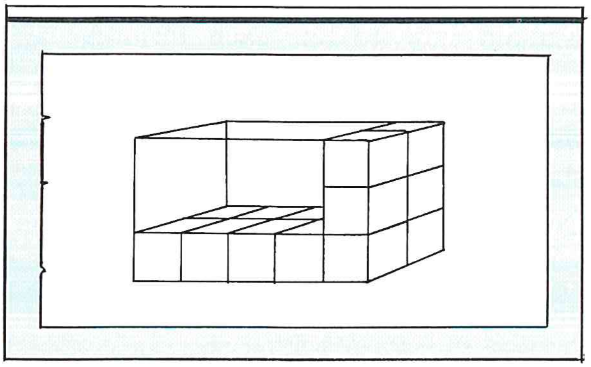
1. 5 sq. yd. = 45 sq. ft.
2. 32 sq. ft. = $3\frac{5}{9}$ sq. yd.
3. 2 sq. ft. = 288 sq. in.
4. 432 sq. in. = 3 sq. ft.
5. 6 sq. yd. = 7776 sq. in.
6. 3200 A. = 5 sq. mi.
7. 4 sq. mi. = 2560 A.
8. 96 sq. in. = $\frac{2}{3}$ sq. ft.
9. $1\frac{3}{4}$ sq. mi. = 1120 A.
10. 2592 sq. in. = 2 sq. yd.
11. 5.9 sq. mi. = 3776 A.
12. 48.6 sq. ft. = 5.4 sq. yd.

MORE PRACTICE PAGE 317

```
sq.in.=144xsq.ft.   n=144x3     432 sq.in.
sq.ft.=sq.in.÷144   n=288÷144     2 sq.ft.
```

Meaning of Volume

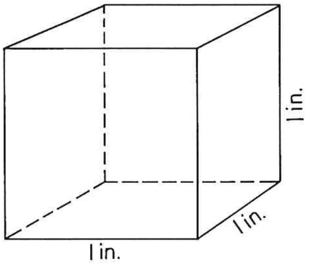




Sometimes you measure line segments. They have only one dimension, length; so you choose a line segment, perhaps an inch or a foot, as the unit of measure for measuring their length. Sometimes you measure a part of a plane. Then you are measuring a figure having two dimensions. As a measuring unit you choose another part of a plane having two dimensions. For example, you choose a square inch or a square foot.

Suppose you are to measure the space inside a box. A box has three dimensions: length, width, and height. Is it reasonable to expect to measure the space inside a box by using a measuring unit having three dimensions? Jane is using blocks, which are 3-dimensional units, to measure the space inside the box in the illustration at the left. One of the outer surfaces, or faces, of one block is shown in blue. The block has how many identical faces?

You are very familiar with blocks such as those Jane is using. A picture of one of them is shown below. How many dimensions does it have? Name them.



A geometric object with 3 dimensions is often called a **solid,** even if it is not solid in a physical sense. Are both the box and the block called solids? See if you can construct a model of a solid by using 12 sticks and glue or 6 squares of cardboard and gummed tape.

A rectangular solid with six faces of equal shape and measure is called a **cube.** A cube is a solid of which each face is a square. The interior measure of a cube that is 1 inch long, 1 inch wide, and 1 inch high is called a **cubic inch.**

The cubic inch is one of the measuring units used to measure the space inside a rectangular solid. The measure of the space inside a solid is called its **volume.** What is the volume expressed in cubic inches of a box 5 inches long, 2 inches wide, and 3 inches high?

Oral Answer these questions about the box near the top of page 262.

1. How many cubes has Jane put in one layer? 10

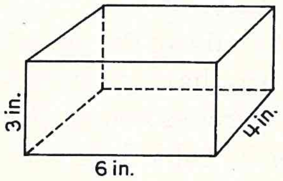
2. How many layers of cubes will fit into the box? 3

3. How many 1-inch cubes will fill the box? 30

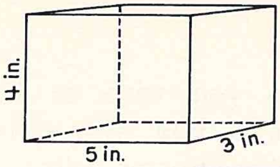
4. What is the measure of the box in cubic inches? 30 cu. in.

Give the 3 dimensions of boxes A, B, and C.

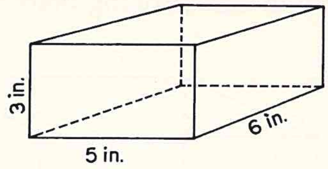
A



B



C



Written Write the answer to each question below.

1. In box A, how many cubes would make 1 layer? 2 layers? 3 layers? 24; 48; 72

2. What is the volume of box A in cubic inches?
6x4x3=n 72 cu. in.

3. In box B, how many cubes would make 1 layer? 2 layers? 4 layers? 15; 30; 60

4. What is the volume of box B in cubic inches?
5x3x4=n 60 cu. in.

5. In box C how many layers would fill the box? How many blocks would make one layer? 3; 30

6. What is the volume of box C in cubic inches?
6x5x3=n 90 cu. in.

A quick review Solve for n.

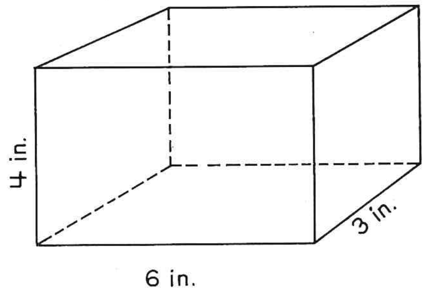
3456
1. $144\times(3\times2\times4)=n$

82944
2. $n=1728\times(4\times3\times4)$

2677.5
3. $n=9\times(12.5\times6.8\times3.5)$

Using a Formula in Finding Volume

Suppose you wish to find the volume of a box with dimensions shown in the drawing below.



The box is 6 inches long, 3 inches wide, and 4 inches high. How many cubic inch units are needed to make one layer of blocks in it? You can find the answer by multiplying 3 and 6. Is 18 cubic inches the measure of 1 layer? How many layers of cubes would be needed to fill the box? Why? The volume of the box equals 4×18 or 72 cubic inches.

The volume of this box was found by multiplying the numbers 3, 6, and 4 used in expressing the measures of its edges. Let V stand for the number of cubic units in the volume, and let l, w, and h represent the measure of the length, width, and height. Then you can express the formula for finding the volume of a rectangular solid as follows.

$$V = lwh$$

In the box on this page, $l=6$, $w=3$, and $h=4$.

Since $V=lwh$
Then $V=6\times3\times4$
And $V=72$

The volume is 72 cubic inches.

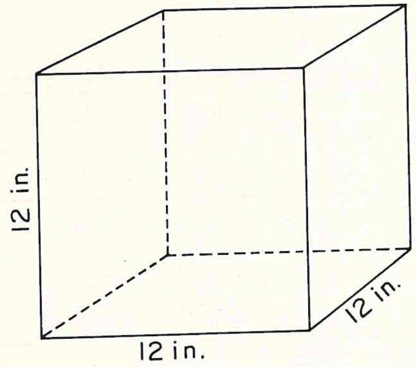
Oral Tell how to use the formula $V=lwh$, in finding the volume of boxes having the dimensions given in each exercise below.

	Length	*Width*	*Height*
1.	8 in.	3 in.	4 in.
	V=8x3x4	96 cu.in.	
2.	7 in.	5 in.	2 in.
	V=7x5x2	70 cu.in.	
3.	9 in.	8 in.	5 in.
	V=9x8x5	360 cu.in.	
4.	11 in.	9 in.	6 in.
	V=11x9x6	594 cu.in.	
5.	17 in.	8 in.	8 in.
	V=17x8x8	1088 cu.in.	
6.	$3\frac{1}{2}$ in.	2 in.	2 in.
	$V=3\frac{1}{2}x2x2$	14 cu.in.	
7.	$5\frac{3}{16}$ in.	4 in.	3 in.
	$V=5\frac{3}{16}x4x3$	$62\frac{1}{4}$ cu.in.	
8.	$4\frac{1}{4}$ in.	$2\frac{1}{2}$ in.	3 in.
	$V=4\frac{1}{4}x2\frac{1}{2}x3$	$31\frac{7}{8}$ cu.in.	
9.	$5\frac{1}{16}$ in.	$4\frac{3}{8}$ in.	4 in.
	$V=5\frac{1}{16}x4\frac{3}{8}x4$	$88\frac{19}{32}$ cu.in.	
10.	$6\frac{1}{8}$ in.	$3\frac{1}{8}$ in.	$4\frac{1}{4}$ in.
	$V=6\frac{1}{8}x3\frac{1}{8}x4\frac{1}{4}$	$81\frac{89}{256}$ cu.in.	
11.	$8\frac{1}{4}$ in.	$3\frac{5}{8}$ in.	$2\frac{1}{2}$ in.
	$V=8\frac{1}{4}x3\frac{5}{8}x2\frac{1}{2}$	$74\frac{49}{64}$ cu.in.	

Written Write the volume of each of the boxes having dimensions given in the exercises above. (Express each answer in cubic inches.)

Other Measures of Volume

The figure below represents a unit of volume measure.



How many faces would you find on a rectangular solid such as that shown above? What are the dimensions of each face? Since all of its faces are equal in measure, each face is a square. The measure of each edge of a face is 12 inches or 1 foot, and the interior measure of the solid is a **cubic foot.**

How many cubic inches are contained in a cubic foot? Suppose you could construct a cube having faces that measured 3 feet, or 1 yard, on each edge. The interior measure of the cube would be a **cubic yard.** Why? How many cubic feet are contained in a cubic yard? Try to draw this cube.

Oral Would you choose a cubic inch, a cubic foot, or a cubic yard as the unit of measure for measuring the space inside of a box with dimensions of 4 in., 3 in., and 2 in.? For measuring the amount of gravel in a large pile of gravel? Why?

Written Find the volume of the interior of each rectangular solid having the dimensions given below. The length of each edge is given.

1. 1 ft., 1 ft., 1 ft. 1 cu.ft.
2. 2 ft. 2 ft., 2 ft. 8 cu.ft.
3. 3 yd., 2 yd., 2 yd. 12 cu.yd.
4. 6 yd., 6 yd., 6 yd. 216 cu.yd.
5. 3 ft., 2 ft., 3 ft. 18 cu.ft.
6. 5 yd., 3 yd., 4 yd. 60 cu.yd.
7. $3\frac{1}{2}$ ft., 4 ft., 3 ft. 42 cu.ft.

Answer each question below.

8. How many cubic inches are contained in 3 cubic feet? In $4\frac{1}{2}$ cubic feet? In 6.5 cubic feet? 5184 cu.in.; 7776 cu.in.; 11,232 cu.in.

9. How many cubic feet are contained in 4 cubic yards? In $7\frac{1}{2}$ cubic yards? In 10 cubic yards? See below.

10. How many cubic feet are contained in 576 cubic inches? $\frac{1}{3}$ cu.ft.

11. How many cubic yards are contained in 72 cubic feet? $2\frac{2}{3}$ cu.yd.

MORE PRACTICE PAGE 317

9. 108 cu.ft.; $202\frac{1}{2}$ cu.ft.; 270 cu.ft.

Because the procedure in using this page parallels that used on page T250, no page T266 is included.

Review and Practice

Part 1 Copy. Find the correct answer for each exercise by performing the indicated operation.

	a	*b*	*c*
1.	$6\overline{)42.6}$ 7.1	$.3\overline{)8.55}$ 28.5	$38\overline{)80.94}$ 2.13
2.	$.24\overline{)5.4}$ 22.5	$.65\overline{)6.24}$ 9.6	$3.2\overline{)7.20}$ 2.25
3.	26.88 94.32 18.76 +65.4 205.36	37.495 23.657 5.426 +6.2 72.778	37.555 15.941 64.0 +84.76 202.256
4.	62.84 −35.17 27.67	76.324 −5.867 70.457	98.475 −39.686 58.789
5.	6.14 ×.3 1.842	.721 ×.05 .03605	8.271 ×.25 2.06775

Part 2 Copy. Solve each equation.

	a	*b*
1.	$n=\frac{3}{4}\times\frac{1}{2}$ $\frac{3}{8}$	$\frac{4}{5}\times\frac{5}{8}=n$ $\frac{1}{2}$
2.	$2\frac{1}{2}\times\frac{3}{10}=n$ $\frac{3}{4}$	$4\frac{3}{8}\times3\frac{1}{5}=n$ 14
3.	$n=\frac{5}{6}\div\frac{5}{8}$ $1\frac{1}{3}$	$n=2\div4\frac{1}{2}$ $\frac{4}{9}$
4.	$6\frac{2}{3}\div4\frac{2}{3}=n$ $1\frac{3}{7}$	$n=8\frac{4}{5}\div3\frac{1}{7}$ $2\frac{4}{5}$
5.	$12-(2\frac{2}{3}+4)=n$ $5\frac{1}{3}$	$n=(7\frac{1}{8}+4\frac{3}{4})-3\frac{1}{2}$ $8\frac{3}{8}$
6.	$n=20-(5\frac{7}{10}+8\frac{3}{5})$ $5\frac{7}{10}$	$(9\frac{2}{3}+3\frac{1}{2})-n=13$ $\frac{1}{6}$
7.	$n-(7\frac{2}{3}+6\frac{3}{4})=6$ $20\frac{5}{12}$	$8\frac{1}{4}-(1\frac{3}{4}+n)=0$ $6\frac{1}{2}$
8.	$(n\times6)+0=54$ 9	$6\frac{1}{2}-(3\times n)=6\frac{1}{2}$ 0

Part 3 Copy. Find the correct sum or difference.

	a	*b*	*c*	*d*
1.	$7\frac{1}{10}$ $-5\frac{7}{10}$ $1\frac{2}{5}$	$8\frac{5}{6}$ $+2\frac{1}{3}$ $11\frac{1}{6}$	$6\frac{1}{4}$ $-2\frac{5}{8}$ $3\frac{5}{8}$	$5\frac{3}{4}$ $+9\frac{5}{12}$ $15\frac{1}{6}$
2.	$16\frac{3}{8}$ $-9\frac{2}{3}$ $6\frac{17}{24}$	$5\frac{3}{4}$ $+7\frac{4}{5}$ $13\frac{11}{20}$	$11\frac{5}{16}$ $+9\frac{9}{10}$ $21\frac{17}{80}$	$21\frac{1}{12}$ $-9\frac{3}{5}$ $11\frac{29}{60}$
3.	$7\frac{2}{3}$ $4\frac{2}{3}$ $+9\frac{1}{3}$ $21\frac{2}{3}$	$8\frac{1}{2}$ $7\frac{3}{4}$ $+5\frac{1}{2}$ $21\frac{3}{4}$	$6\frac{2}{3}$ $5\frac{1}{2}$ $+8\frac{5}{6}$ 21	$9\frac{3}{4}$ $2\frac{1}{2}$ $+5\frac{1}{3}$ $17\frac{7}{12}$

Part 4 Copy. Write a numeral in each blank so that the statements below become true. (The = sign means equal in measure.)

1. 48 in. = 4 ft.

2. 66 ft. = 4 rd.

3. 80 ft. = 4 rd. 14 ft.

4. 29 ft. = 9 yd. 2 ft.

5. 132 ft. = 44 yd. 0 ft.

6. 18 pt. = $2\frac{1}{4}$ gal.

7. 12 gal. = 96 pt.

8. $1\frac{3}{4}$ hr. = 105 min.

9. $3\frac{1}{2}$ mi. = 18480 ft.

10. 13,824 cu. in. = 8 cu. ft.

11. 38 pt. = 19 qt.

12. 9.5 cu. ft. = 16416 cu. in.

Because the procedure in using this page parallels that of previous pages of *Checkup Time* (see T251), no page T267 is included.

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. The perimeter of a rectangle or of a parallelogram is equal to 2 times the sum of the measures of its length and width. (254, 257)

2. The area measure of a rectangle is equal to the product of the linear measures of its length and its width. (259, 260)

3. When finding the area measure of a large rectangle, express its measure in square rods, in acres, or in square miles. (261)

4. The volume measure of a rectangular solid is equal to the product of the linear measures of its length, width, and height. (264)

Words to Know

1. Perimeter (253)
2. Base, height, dimensions (254)
3. Equilateral triangle (257)
4. Interior, area (258)
5. Square inch, square rod, acre, square mile (259, 261)
6. Solid, cube, cubic inch, volume (262-263)
7. Cubic foot, cubic yard (265)

Questions to Discuss

1. What are some units of measure used in measuring area? In measuring volume? (259, 261, 262-263) Sq.ft., sq.mi., acre; cu.in., cu.yd., cu.rd.

2. What formula do you use in finding the volume of a rectangular solid? (264) $V=\ell wh$

Written Practice

Find the perimeter and then the area of rectangles having these dimensions.

1. width, 7 inches; length, 9 inches (254, 259) $p=2\text{x}(9+7)=32$ in.
$A=9\text{x}7$ 63 sq.in.

2. width, 5 feet; length, 3 yards (255) See below.

3. width, $3\frac{1}{10}$ miles; length, $3\frac{1}{10}$ miles (255, 260) See below.

Answer these questions.

4. What is the length and what is the area of a rectangle 6 inches wide if the perimeter is 22 inches? (256, 259) $22=2\text{x}(\ell+6)$ 5 in.
$A=5\text{x}6$ 30 sq.in.

5. What is the perimeter of an equilateral triangle measuring $8\frac{1}{2}$ ft. on each side? (257) $3\text{x}8\frac{1}{2}=n$ $25\frac{1}{2}$ ft.

6. What is the volume of a rectangular box measuring 20 in. by 15 in. by 8 in.? (264)
$20\text{x}15\text{x}8=n$ 2400 cu.in.

2. $p=2\text{x}(9+5)$ 28 ft.; $A=9\text{x}5$ 45 sq.ft.

3. $p=4\text{x}3\frac{1}{10}$ $12\frac{2}{5}$ mi.; $A=3\frac{1}{10}\text{x}3\frac{1}{10}$ $9\frac{61}{100}$ sq.mi.

Self-Evaluation

Part 1 Follow these directions.

1. Draw a line segment equal in length to the perimeter of a rectangle with dimensions of $1\frac{1}{4}$ in. by $1\frac{1}{2}$ in.

2. Draw a sketch of a rectangle with dimensions of 3 in. by 4 in. Show its area by drawing unit squares in its interior.

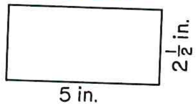
3. Write the numeral that tells how many 1-inch cubes you would use to fill the inside of a rectangular box measuring 9 inches by 6 inches by 3 inches. 162

Part 2 Find the perimeter of each polygon below.

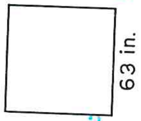
a *b*

1.

15 in.

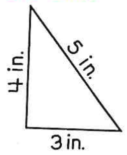


252 in.

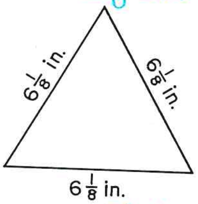


2.

12 in.

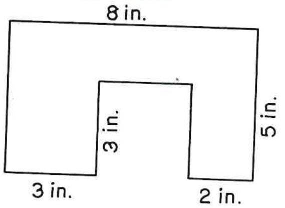


$18\frac{3}{8}$ in.

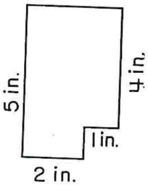


3.

32 in.



16 in.



Part 3 Find the area of the interior of rectangles whose sides have the measures given below.

	a	*b*
1.	24 sq. ft. 4 ft. by 6 ft.	$30\frac{1}{4}$ sq. in. $5\frac{1}{2}$ in. by $5\frac{1}{2}$ in.
2.	304 sq. ft. 16 ft. by 19 ft.	$97\frac{1}{8}$ sq. ft. $9\frac{1}{4}$ ft. by $10\frac{1}{2}$ ft.
3.	$16\frac{2}{3}$ sq. yd. $3\frac{1}{3}$ yd. by 5 yd.	$64\frac{97}{100}$ sq. mi. $7\frac{3}{10}$ mi. by $8\frac{9}{10}$ mi.

Find the volume of rectangular solids whose dimensions are given below.

	Length	*Width*	*Height*
4.	7 in. $7 \times 5 \times 4 = n$	5 in. 140 cu. in.	4 in.
5.	$3\frac{1}{3}$ yd. $3\frac{1}{3} \times \frac{2}{3} \times 1 = n$	2 ft. $2\frac{2}{9}$ cu. yd.	1 yd.
6.	$1\frac{1}{2}$ ft. $1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} = n$	$1\frac{1}{2}$ ft. $3\frac{3}{8}$ cu. ft.	$1\frac{1}{2}$ ft.
7.	2 ft. 6 in. $2\frac{1}{2} \times 1\frac{1}{3} \times 1 = n$	1 ft. 4 in. $3\frac{1}{3}$ cu. ft.	1 ft.

Part 4 Copy. Replace the blanks with numerals that make each statement true. (= means equal in measure.)

1. 48 in. = $\underline{4}$ ft.

2. $4\frac{1}{2}$ ft. = $\underline{54}$ in.

3. 576 sq. in. = $\underline{4}$ sq. ft.

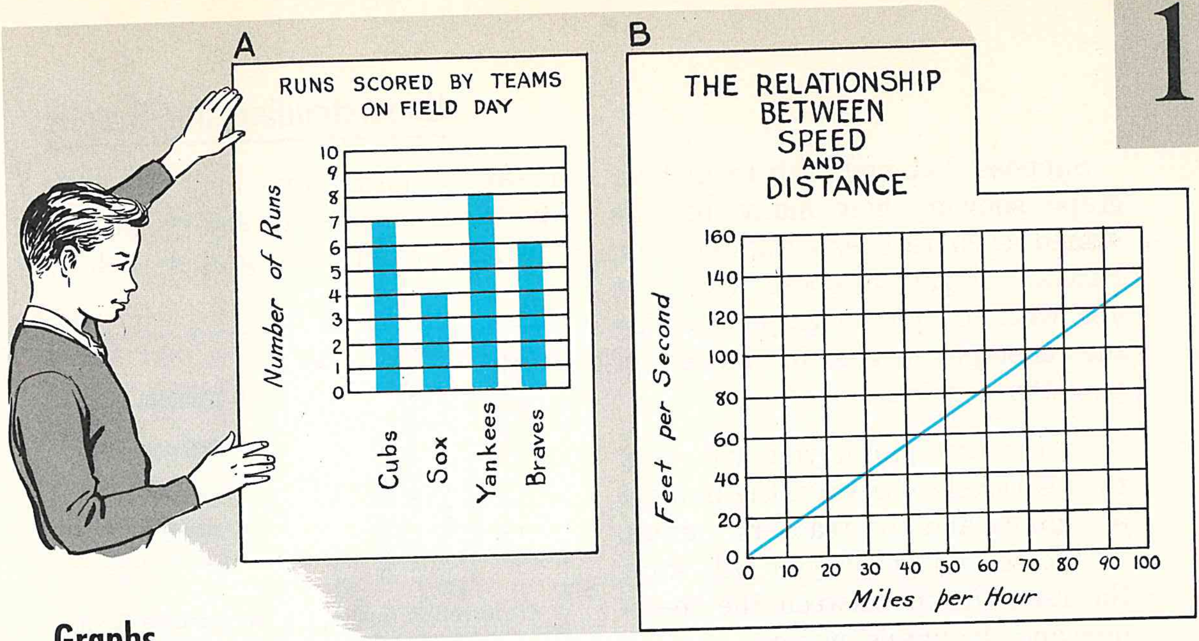
4. 15 sq. ft. = $\underline{2160}$ sq. in.

5. 8 sq. yd. = $\underline{72}$ sq. ft.

6. 171 cu. ft. = $\underline{6\frac{1}{3}}$ cu. yd.

7. $3\frac{1}{3}$ cu. yd. = $\underline{90}$ cu. ft.

16



Graphs

Bob is placing a **bar graph** on the bulletin board. The other graph on the bulletin board is called a **line graph.** Why is each name appropriate? Before either graph was drawn, a considerable amount of data had to be collected and organized. The graphs present the data in the picture form you see.

In each graph there is a one-to-one correspondence between two sets. In graph A there is a one-to-one correspondence between the name of each team and a specified number of runs. For example, the Cubs may be paired with 7 runs. May 7 runs also be paired with the name, Cubs?

In graph B there is a one-to-one correspondence between the number of miles per hour and the number of feet per second. For example, 30 miles per hour is paired with 40 feet per second. May 40 feet per second be paired with 30 miles per hour?

Oral Use either graph A or B to help you answer these questions.

1. Which team scored the most runs? The fewest runs? Yankees; Sox

2. Which teams scored more than 4 runs? Fewer than 8 runs? Cubs, Yankees, Braves; Cubs, Sox, Braves

3. Which team scored 8 runs? Yankees

4. At what speed in miles per hour would a car have to travel to cover 60 feet in 1 second? 100 feet in 1 second? 45 mph; 75 mph

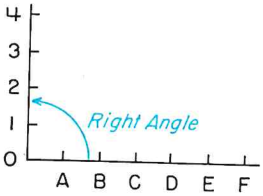
5. At what speed would a car have to travel to cover 30 feet in 1 second? 80 feet in one second? 25 mph; 60 mph

Understanding Bar Graphs

Suppose that you wish to make a graph showing how many parents visited each teacher's classroom on Parents' Night. Suppose also that you wish to make a graph showing the average height in inches of pupils in your class.

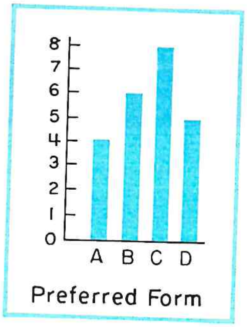
In the first graph you will show the relationship between the number of parents and the teachers' names. In the second graph you will show the relationship between the measures and the pupils' names.

When you use a graph to pair one set of numbers or measures with names of something, you will usually use a bar graph. Two of the lines of the graph, the one at the left and the one across the bottom, are the **reference lines,** or **axes** of the graph. They must be at right angles to each other. It is customary, but not essential, to make one axis vertical and the other horizontal as shown.



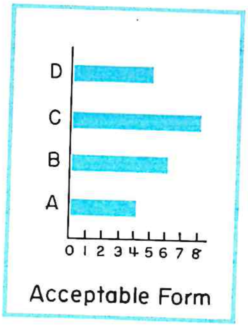
The vertical axis of a bar graph is usually a *number line.* The horizontal axis is usually the *name line.*

However, either line may serve as the number line as shown in the partially completed graphs below.



Preferred Form

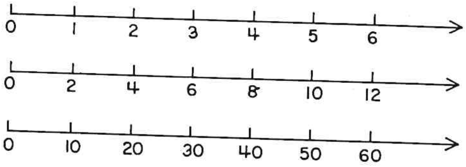
or



Acceptable Form

The person who makes the bar graph must decide which line is to be the name line and which is to be the number line.

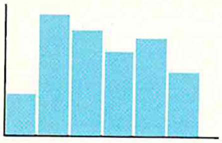
After deciding which line is to be the number line, decide upon suitable units and place marks on it at equal distances from each other. Then label the lines with numerals. Study the number lines below.

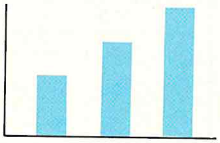


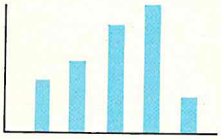
Observe that after a unit, for example 1, 2, or 10, has been selected, that same unit is used along the entire length of each number line.

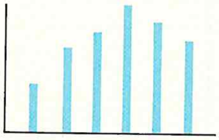
The person who constructs a graph often rounds numbers to multiples of 5, 10, 50, or 100 or even to thousands or millions before placing points on the number line. Why may such rounding off be necessary?

The names on the *name line* of a graph may be spaced in any convenient way. All of the following arrangements are acceptable. Which spacing do you prefer?

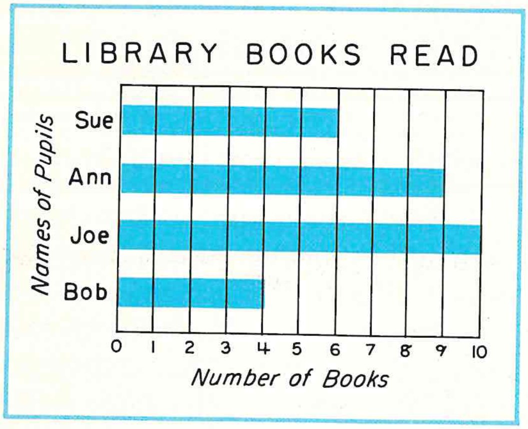








Study the graph below. Find its title and the name given to both its horizontal and its vertical axis.



Oral Answer each question about the graph in column 1.

1. Which numerals do you see on the number line of the graph? 0 through 10

2. Which names do you see on the name line? Sue, Ann, Joe, Bob

3. Which pupil read only 4 books? Bob

4. How many books were read by Joe? By Sue? By Bob? 10; 6; 4

5. How many books were read by all of the children together? 29

Written For each exercise below, draw a number line of any convenient length. On each number line, place marks at equal distance from each other. Label the marks with the appropriate numerals.

1. Use 1 as your unit of measure. Show the numerals through 10.

2. Use 5 as your unit of measure. Show the numerals through 50.

3. Use 100 as your unit of measure. Show the numerals through 1000.

Draw a name line of any convenient length. Use marks at equal distance from each other. Label the marks using the names below.

4. Names of schools: Lincoln, 1st Avenue, Central.

5. Names of years: 1910, 1920, 1930, 1940, 1950, 1960.

Drawing Bar Graphs

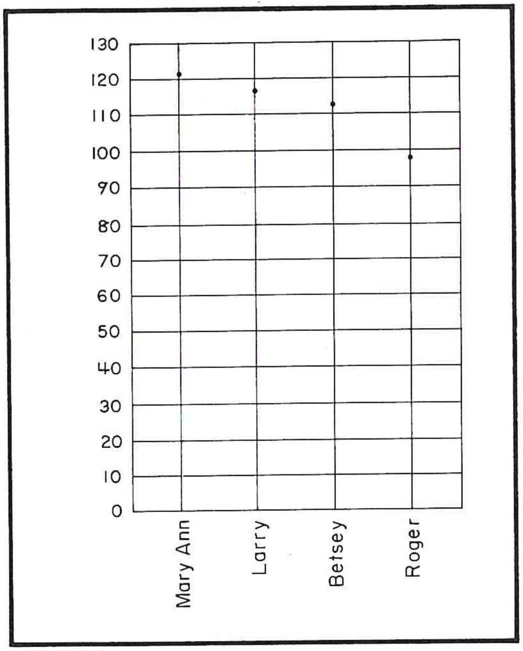
In a school election for president of the student council, the nominees for president received votes as indicated below.

Betsey	112
Mary Ann	121
Roger	98
Larry	117

Suppose you wish to present this information on a bar graph. First, if possible, obtain a piece of squared paper called **graph paper.** Then begin your graph by drawing a horizontal line to use as a name line. You should divide the name line into segments of convenient length. The next step is to construct a vertical number line at right angles to the name line. Then you will need to decide upon a scale for the vertical number line. The largest vote for any candidate was 121. Why is it not practical to separate the number line into 121 segments of equal length and mark each separation line with a numeral?

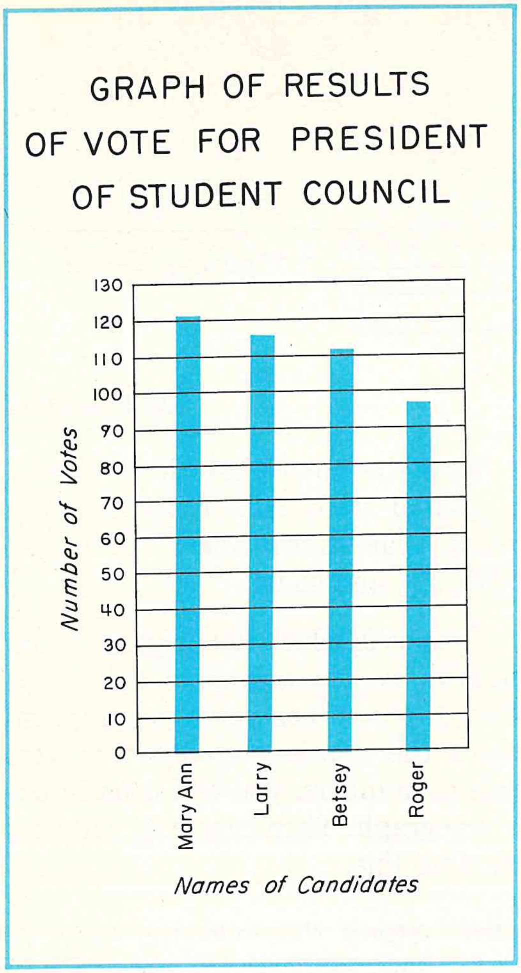
You could let each segment of the number line represent several votes; for example, 10. Since $121 \div 10 > 12$, you need at least 13 equally-spaced segments on the number line. Does this seem like a reasonable number of segments to label?

The partially completed graph is shown below.



What would be a suitable title for the graph? Where should the title be placed? What label would you give to the horizontal name line? To the vertical number line? How many points have been located on the name line? How were they determined?

To make the graph easier to read, bars of reasonable width may be drawn to connect the names on the horizontal reference line to the points above the names. The completed graph is shown on the next page.



Do you see that the bar graph is used to show how the names of candidates may be paired with the number of votes received? The bar which represents Larry's votes ends about $\frac{3}{4}$ of the way between 110 and 120 on the vertical scale. Why? Why does the bar which represents Roger's votes end about $\frac{4}{5}$ of the way between the points for 90 and 100 votes? Each of the bars and each of the spaces between the bars is the same width. In your opinion, does this arrangement give the graph a neat appearance?

Oral Tell the steps you would take in making bar graphs to picture the following data. See page 272 and column 1.

1. Enrollment at Forest View School

Grade	Number Enrolled
1	72
2	65
3	75
4	68
5	64
6	70

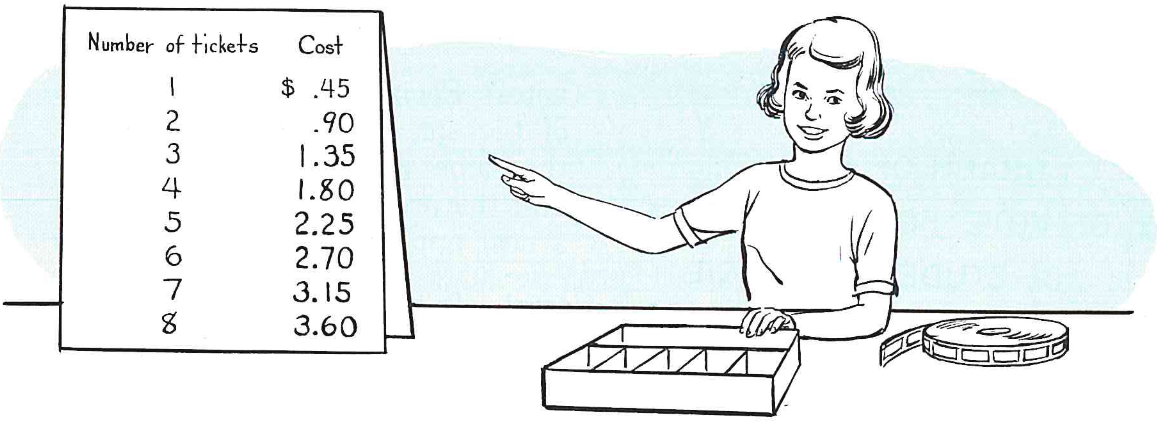
2. Home Run Leaders

1938	Greenberg	58
1947	Kiner	51
1949	Kiner	54
1955	Mays	51
1956	Mantle	52
1961	Maris	61

3. Height of Highest Dams in U.S.

Oroville	730 feet
Hoover	726 feet
Glen Canyon	710 feet
Shasta	602 feet
Hungry Horse	564 feet
Grand Coulee	550 feet

Written Draw bar graphs to illustrate the data presented in 1 through 3 above. Have vertical bars on at least two of the graphs.

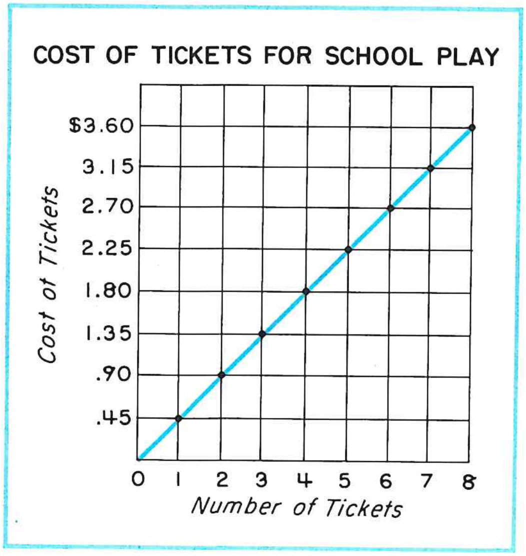


Number of tickets	Cost
1	$ .45
2	.90
3	1.35
4	1.80
5	2.25
6	2.70
7	3.15
8	3.60

Line Graphs

Jane made a table to help her know how much to charge each buyer of tickets. Do you suppose anyone was likely to buy more than 8 tickets? If so, how could Jane use multiplication to find how much to charge?

The data on the table might be more helpful if presented on a graph as shown below.


COST OF TICKETS FOR SCHOOL PLAY

Find the point that shows how 1 is paired with .45. Then find the point that shows how 2 is paired with .90, and so on.

Graphs in which the points are connected by line segments are called line graphs. When you use a graph to show the relationship between two sets of numbers, you will usually use a line graph. Then each axis will be a number line.

Oral Study the following table of data. The Baylors' car averages 13 miles per gallon of gasoline.

Gallons	2	4	6	8	10	12	14
Miles	26	52	78	104	130	156	182

Answer these questions about the table of data.

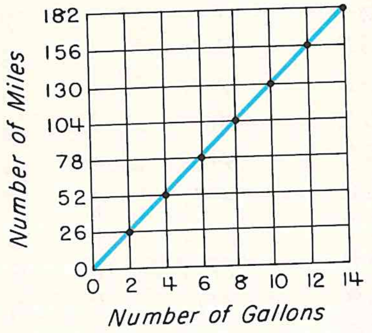
1. The first number pair in the table is $\frac{2}{26}$. What other number pairs are shown in the table?
$\frac{4}{52}$, $\frac{6}{78}$, $\frac{8}{104}$, $\frac{10}{130}$, $\frac{12}{156}$, $\frac{14}{182}$

2. In the table, 26 is 13 times 2. In each of the other number pairs; for example 4 and 52, 6 and 78, and so on, is the second number 13 times the first? Yes

3. What are some other number pairs you could write in an extended table concerning the Baylors' car? $\frac{16}{208}$, $\frac{18}{234}$, $\frac{20}{260}$, etc.

Study the graph below. Then answer questions 4 through 6.

RELATIONSHIP OF GALLONS OF GASOLINE TO MILEAGE



4. On which axis, the horizontal or the vertical axis, do you find the first number of each pair being compared? Horizontal axis

5. Suppose the Baylors had bought 11 gallons of gas. How many miles could they expect to drive? 143

6. On the graph where would you locate a point to show the pairing of 1 and 13? Of 9 and 117? At intersection of vertical line 1 and horizontal line 13; at intersection of vertical line 9 and horizontal line 117

Written Draw line graphs to illustrate the data given below.

1. On May 1 the temperature varied as follows.

Time	Temperature
6 a.m.	45°
8 a.m.	52°
10 a.m.	60°
12 noon	64°
2 p.m.	68°
4 p.m.	75°
6 p.m.	73°
8 p.m.	68°
10 p.m.	64°

2. Betty made the following record of her scores on an achievement test.

Test Number	Score
1	98
2	92
3	87
4	95
5	88
6	90

3. In certain years the railroads of the U.S. carried passengers as follows. Hint: On the vertical axis, make the first segment a little longer than the others. Then begin the vertical scale at 325.

Year	Number of passengers in millions
1940	456
1945	897
1950	488
1955	433
1960	325

Organizing Data

John is 58 inches tall. Is he taller or shorter than most other boys in his grade? One way to answer the question would be to arrange the boys in his grade in order of their size from shortest to tallest or from tallest to shortest. A more accurate way to answer the question would be to measure the height of each boy and to compare the measures. A listing of the measures of 48 sixth-grade boys listed in the alphabetical order of their second names follows.

51, 52, 56, 55, 53, 55, 57, 52
54, 56, 58, 49, 59, 56, 58, 55
52, 57, 54, 56, 52, 62, 54, 61
53, 50, 58, 53, 56, 58, 57, 54
56, 55, 54, 58, 59, 55, 58, 59
53, 51, 62, 55, 63, 56, 48, 53

Is it easy to compare the measures? To simplify the task, you could arrange the measures in order of their length from smallest to largest. You could use tally marks to show how frequently each measure occurred. By counting the tally marks, you could find the **frequency,** or number of times, each measure was recorded. Then you could make a table as shown in column 2.

When you make a summary of data as shown on this page, you are making a **frequency distribution.** From the table you can see that a boy measuring 58 inches tall is one of the taller boys in sixth grade.

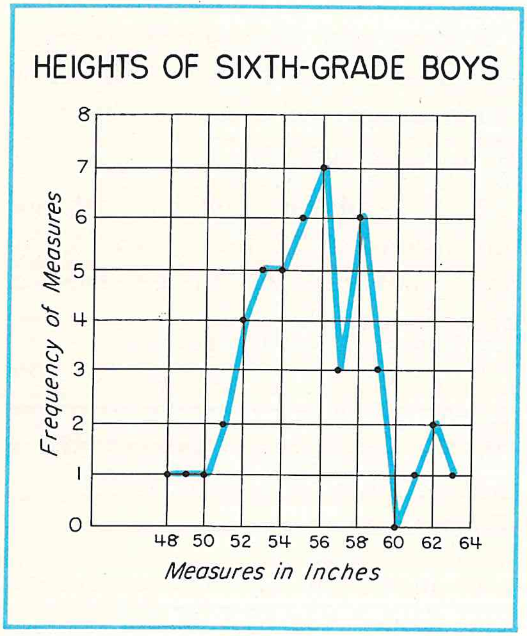
The table of data follows.

HEIGHTS OF SIXTH-GRADE BOYS

Measure	*Tally Marks*	*Frequency*
48	/	1
49	/	1
50	/	1
51	//	2
52	////	4
53	𝍸	5
54	𝍸	5
55	𝍸 /	6
56	𝍸 //	7
57	///	3
58	𝍸 /	6
59	///	3
60		0
61	/	1
62	//	2
63	/	1

Suppose now that you wish to make a graph to show the relationship of each measure to each frequency. Let the horizontal axis of your graph represent the measures. Since the shortest measure is 48 inches, you may begin the horizontal scale at 48. On the graph below, the first segment of the horizontal axis is drawn longer than the other segments to show the omission of measures between 0 and 48. Which axis represents the number of times each measure was recorded? Why does the vertical scale begin at 0 and end at 8?

Study the graph below.



HEIGHTS OF SIXTH-GRADE BOYS

Oral Answer these questions about the graph.

1. What was the measure of the tallest boy in class? Of the shortest boy in class? `63 in.; 48 in.`

2. How many boys measured *more* than 56 inches in height and how many measured *less* than 56 inches in height? What is the average of all of the measures of height?
`16; 25;` $55\frac{3}{8}$

Tell the steps you would take in arranging the following data to show how frequently each test score below was made on an arithmetic test.
`See page 276 and column 1.`

3. 24, 32, 34, 38, 35, 36, 35, 37
33, 39, 33, 37, 36, 34, 34, 39
31, 40, 38, 35, 32, 31, 38, 36
38, 41, 29, 35, 35, 37, 39, 33

Written Follow these directions.

1. Arrange the above data in a frequency distribution table.

2. Make a line graph in which you graph the data in the above frequency distribution table you made. Label the horizontal axis *score* and the vertical axis *frequency of score.*

3. Make a frequency distribution table and then make a line graph to show the data concerning the number of runs scored by players during a baseball season.

Dave	10	Fred	13
Al	9	Joe	8
Roy	12	Tom	12
Bill	7	Paul	7
John	5	Don	13
Roger	13	Harold	6
Art	4	Ralph	10
Bob	9	Harvey	0
Dick	10	Robert	9

Problem Solving

The following suggestions should enable you to solve each problem on this page by yourself.

a. Read the problem carefully at least twice. Know what is given and what you are to find.

b. Decide which operation or operations to use.

c. Translate the entire problem into a suitable equation.

d. Solve the equation. Use the solution in answering the question in the problem.

1. In the 20-year period from 1940 to 1960, the price of fuel oil increased from 6.2 cents a gallon to 15.6 cents a gallon. How much more did 500 gallons of oil cost in 1960 than in 1940?

(500x15.6)-(500x6.2)=n $47.00

2. The price of cars used in police work was $504 in 1940, $944.80 in 1950, and $1,698.45 in 1960. By how many times did the price of a police car increase in the period between 1940 and 1960?

1698.45÷504=n   3.369...

3. David's father plans to use 4 pounds of fertilizer for each 100 square feet of garden. His garden measures 20 feet by 45 feet. How many pounds of fertilizer does he need for his garden?

4x[(20x45)÷100]=n   36 lb.

4. Donald's father can get a new sidewalk for 45¢ a square foot. He needs a sidewalk 4 feet wide and 55 feet long. How much will he have to pay for the sidewalk?

(4x55)x45=n $99.00

5. Mr. Allen can buy a new car for $2750 plus a sales tax of 4%. How much money must he pay as tax on the car if he buys it?

2750x.04=n $110

6. Jane can buy canned peaches priced at 2 cans for 43 cents. How much money will 6 cans of the same kind of peaches cost?

(6÷2)x43=n $1.29

7. A cardboard box had interior dimensions of 12 in. by 15 in. by 8 in. What was the volume of the box in cubic inches?

12x15x8=n   1440 cu.in.

8. Julie's sister planned to save at least $5 of her monthly earnings. On the average she earned $75 a month. What per cent of her earnings did she plan to save?

5÷75=n $6\frac{2}{3}$%

9. Jack spent $2 on a new record. This money was 25% of his savings. How much money did he have in savings before he bought the record?

2=.25xn $8.00

10. On a scale drawing, a measure of $\frac{1}{4}$ inch represented 2 feet and the measure of the line segments representing a rectangular-shaped building were 3 in. by 4 in. What were the dimensions of the building? $\frac{1}{4}$in.=2ft.

3 in.=$\frac{12}{4}$; 12x2=w=24 ft.

4 in.=$\frac{16}{4}$; 16x2=ℓ=32 ft.

Solving Problems

As you study each problem on this page, decide upon the kind of simple closed figure you are to consider. Make a sketch of the figure and write its dimensions. Then translate each problem into an equation. Write the equation to use in answering each question in each problem.

1. Bob wants to put a metal edge around a table top that has the shape of a rectangle. The dimensions of the table top are 19½ in. by 28 in. What is the least possible amount of edging that Bob can use?

$2\times(19\frac{1}{2}+28)=n$ 95 in.

2. Betty has a garden in the shape of a triangle. The sides measure 12 feet, 15 feet, and 12 feet. How many feet of fencing are needed to enclose the garden? $12+15+12=n$ 39 ft.

3. John put a wooden frame around a map. The map had the shape of a square with each side 26 inches long. The frame added 4 inches to the width of the map and also to its length. What was the perimeter of the frame?

$(4\times26)+(4\times4)=n$ 120 in.

4. The floor of a room was rectangular in shape. Its dimensions were 9 ft. 9 in. by 12 ft. 10 in. What was the perimeter of the floor in feet and inches? In inches?

See below.

5. Plastic edging cost 19¢ a foot. Without making allowance for waste, how much would one have to pay for the plastic edging needed to go around the outer edge of a sandwich tray having dimensions of 10 in. by 14 in.? $[2\times(10+14)\div12]\times19=n$ 76¢

6. A door had dimensions of 2 ft. 6 in. and 6 ft. 6 in. What was the perimeter of the door in feet? In yards? In inches? See below.

7. Mr. Johnson had a desk top that measured 30 in. wide and 4 ft. 6 in. long. He put a plastic strip all the way around the edge of the top. How many feet of plastic strip did he use? $2\times(2\frac{1}{2}+4\frac{1}{2})=n$ 14 ft.

8. A city lot had a measure of 51.2 ft. across the front and 75.6 ft. across the back. The measures of the two sides were 176.5 ft. and 189.4 ft. What was the perimeter of the lot in feet? In yards? See below.

9. Mr. Thompson had a garden in the shape of a square. The length of each side was 14 ft. He made a walk 2½ ft. wide all around the garden and built a fence to enclose the garden and walk. How many feet of fencing did he need?

$4\times(14+5)=n$ 76 ft.

Oral Tell your reason for deciding upon the equations you wrote.

Written Solve problems 1 through 9.

4. $2\times(117+154)=y$ 542 in.
$[2\times(117+154)]\div12=n$ 45 ft. 2 in.

6. $2\times(2\frac{1}{2}+6\frac{1}{2})=p$ 18 ft.
$[2\times(2\frac{1}{2}+6\frac{1}{2})]\div3=n$ 6 yd.
$12\times[2\times(2\frac{1}{2}+6\frac{1}{2})]=f$ 216 in.

8. $51.2+75.6+176.5+189.4=n$ 492.7 ft.
$492.7\div3=y$ 164.233... yd.

Because the procedure in using this page parallels that used on page T250, no page T280 is included.

Review and Practice

Part 1 Copy. Perform the operation indicated by each operational sign.

	a	b
1.	4089 $\frac{3}{7}$ 21)85878	324 $\frac{23}{356}$ 356)115367
2.	3 lb.2 oz. 4)12 lb. 8 oz.	15 yd.2 ft. 2)31 yd. 1 ft.
3.	35796.85 148.13 54360.36 +3272.78 93578.12	92013.034 505.2 749.4 +38287.897 131555.531
4.	49275.41 −824.56 48450.85	93214. −7460.975 85753.025
5.	6.4325 ×.07 .450275	9.5705 ×3.10 29.668550
6.	3 lb. 5 oz. +8 lb. 6 oz. 11 lb.11 oz.	9 T. 1000 lb. +8 T. 1500 lb. 18 T.500 lb.
7.	2 lb. 9 oz. ×5 12 lb.13 oz.	4 yd. 9 in. ×6 25 yd.18 in.
8.	11 gal. 3 qt. −5 gal. 2 qt. 6 gal.1 qt.	9 mi. 1100 ft. −3 mi. 2500 ft. 5 mi.3880 ft.

Part 2 Copy. Solve for n.

	a	b
1.	$8\frac{2}{5}+\frac{3}{10}=n$ $8\frac{7}{10}$	$5\frac{17}{20}$ $n=4\frac{3}{5}+1\frac{1}{4}$
2.	$4\frac{5}{6}-\frac{1}{3}=n$ $4\frac{1}{2}$	$2\frac{7}{20}$ $n=9\frac{1}{10}-6\frac{3}{4}$
3.	$1\frac{1}{4}\times\frac{3}{4}=n$ $\frac{15}{16}$	7 $n=2\frac{2}{3}\times2\frac{5}{8}$
4.	$2\frac{2}{3}\div\frac{3}{4}=n$ $3\frac{5}{9}$	$1\frac{1}{8}$ $n=4\frac{1}{2}\div4$

Part 3 Copy. Solve each equation.

	a	b	c
1.	$\frac{12}{20}=\frac{18}{x}$ 30	$\frac{c}{12}=\frac{30}{18}$ 20	$\frac{24}{16}=\frac{b}{192}$ 288
2.	$\frac{9}{15}=\frac{g}{60}$ 36	$\frac{28}{p}=\frac{18}{32}$ $49\frac{7}{9}$	$\frac{k}{27}=\frac{11}{6}$ $49\frac{1}{2}$
3.	$\frac{15}{3}=\frac{e}{4}$ 20	$\frac{9}{20}=\frac{f}{15}$ $6\frac{3}{4}$	$\frac{a}{4}=\frac{6}{3}$ 8
4.	$\frac{25}{n}=\frac{15}{18}$ 30	$\frac{10}{30}=\frac{40}{h}$ 120	$\frac{18}{30}=\frac{27}{d}$ 45
5.	$\frac{24}{2}=\frac{36}{d}$ 3	$\frac{120}{54}=\frac{18}{m}$ $8\frac{1}{10}$	$\frac{c}{64}=\frac{28}{16}$ 112

Part 4 Copy. Write the per cent equivalent of each of the following.

	a	b	c	d
1.	$\frac{1}{2}$ 50%	$\frac{1}{3}$ $33\frac{1}{3}$%	$\frac{2}{3}$ $66\frac{2}{3}$%	.25 25%
2.	$\frac{3}{4}$ 75%	$\frac{1}{5}$ 20%	.40 40%	$\frac{3}{5}$ 60%
3.	$\frac{4}{5}$ 80%	$.16\frac{2}{3}$ $16\frac{2}{3}$%	$\frac{1}{10}$ 10%	$\frac{1}{8}$ $12\frac{1}{2}$%
4.	$\frac{3}{8}$ $37\frac{1}{2}$%	$\frac{5}{8}$ $62\frac{1}{2}$%	$\frac{7}{8}$ $87\frac{1}{2}$%	$\frac{9}{10}$ 90%
5.	.12 12%	$\frac{1}{4}$ 25%	$\frac{2}{5}$ 40%b	$\frac{1}{6}$ $16\frac{2}{3}$%

Part 5 Express each of the following as an equation. Solve.

1. 4 is what per cent of 48?
   4=nx48 $8\frac{1}{3}$%
2. What number is $33\frac{1}{3}$% of 32?
   n=.$33\frac{1}{3}$x32 $10\frac{2}{3}$
3. 25 is 20% of what number?
   25=.20xn 125
4. 84 is what per cent of 336?
   84=nx336 25%
5. 48 is 25% of what number?
   48=.25xn 192
6. 11 is what per cent of 33?
   11=nx33 $33\frac{1}{3}$%

Because the procedure in using this page parallels that of previous pages of *Checkup Time* (see T251), no page T281 is included.

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. A graph is a picture that shows how one set of values may be paired with another. (269, 271, 272)

2. The entire length of the number-line axis of a graph should be divided into shorter segments of equal length. (270)

3. The names on the name line of a graph may be spaced in any convenient way. (271)

4. Each graph must have a title and labels. (272)

Words to Know

1. Bar graph, line graph (269)

2. Reference lines, vertical axis, horizontal axis (270)

3. Graph paper (272)

4. Frequency, frequency distribution table (276)

Questions to Discuss

1. How should you determine the scale to use on the number-line axis of a graph? (270)By determining range of numbers; space allowed for graph

2. How can you decide whether to use a line graph or a bar graph to present data? (270, 274)By determining the purpose of the graph

3. Why might you want to vary the amount of space between the bars of a bar graph? (271) For ease of understanding, clarity, etc.

4. What steps would you take in making a line graph? (274-275) See below.

5. How would you make a frequency distribution table? (276) See below.

6. Why might you wish to make the first segment of an axis longer than the others? (277) See below.

7. What steps should you take when solving problems? (278) See below.

Written Practice

Follow these directions.

1. Draw a bar graph to present the following data, which indicates how Dick Robertson spent his time on a certain day. (270)

meals	2 hr.	school	5 hr.
play	5 hr.	study	2 hr.
sleep	9 hr.	work	1 hr.

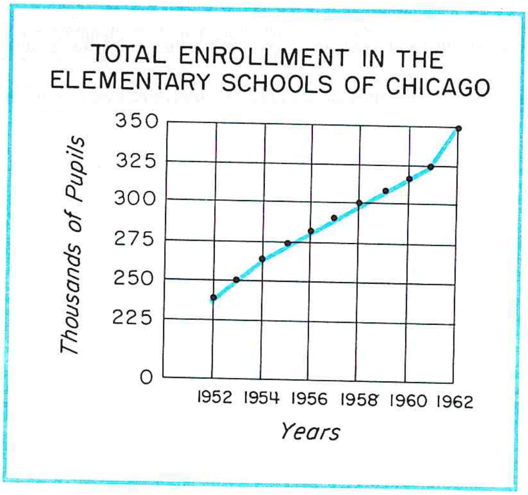
2. Draw a line graph to present the following data, which indicates the sale of new cars on the first six days of May. (274-275)

May 1	12	May 4	14
May 2	8	May 5	7
May 3	9	May 6	10

4. Organize data; represent pairs of numbers by dots; connect dots.
5. List possible scores in order; tally frequency of each.
6. To show omission of numbers between 0 and first number
7. Read problem; decide on operation; write equation; solve equation;

Self-Evaluation

Part 1 Study the graph below and write the answers to the questions which follow it.



1. What is the label of the vertical axis? Of the horizontal axis?
Thousands of pupils; years

2. What do the numerals on the vertical axis tell you? The total enrollment by thousands

3. What is your estimate of the total enrollment in the elementary schools in 1952? 235,000

4. How many pupils were enrolled in the elementary schools in 1953?
250,000

5. In what year did the elementary school enrollment reach 300,000?
1958

6. What is your estimate of the total change in enrollment in the period between 1952 and 1962?
115,000

Part 2 Follow these directions.

1. Make a bar graph of the data concerning Mary's test scores.

Monday	19
Tuesday	24
Wednesday	17
Thursday	25
Friday	23

2. Make a bar graph of the data concerning the number of library books in a room library.

Fiction	85
History	42
Geography	35
Science and Mathematics	30
All others	25

3. Make a line graph of the data concerning the number of persons employed in agriculture.

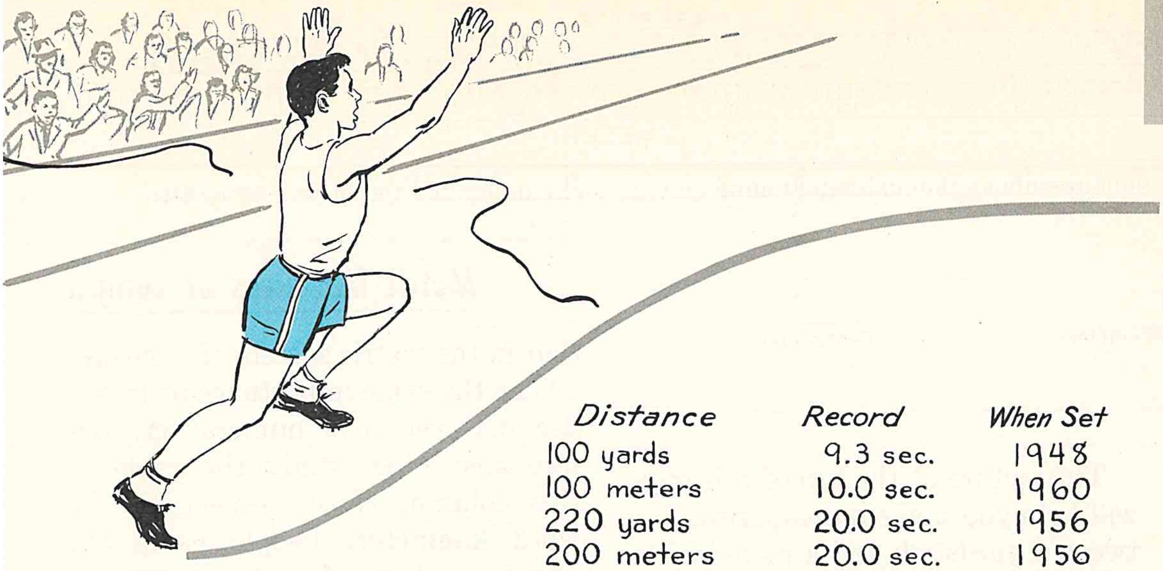
Year	Number in Millions
1960	7.1
1950	9.9
1940	10.9
1930	12.5
1920	13.4

4. Make a frequency distribution table and a line graph of the information given below.

Traffic Count

37	35	31	40	33	42	38	32
44	38	34	40	37	40	34	38
37	42	32	36	44	36	42	45

17



Distance	Record	When Set
100 yards	9.3 sec.	1948
100 meters	10.0 sec.	1960
220 yards	20.0 sec.	1956
200 meters	20.0 sec.	1956

Systems of Measures

Look at the track records shown above. In what two different units is the length of the races expressed? The measure of the length of a **meter** is greater than that of a yard. Could you guess this by reading the table?

The yard is a unit of linear measure in the English system of measures, while the meter is a unit of linear measure in the **metric system** of measures. With the exception of the British Empire, most people living in Europe, in Asia, and in Latin America use the metric system of measures. In this country engineers, scientists, and employees in several branches of our government use it in their work.

Why is it convenient to have several different units of linear measure in the English system? Would you also expect to find such different units in the metric system? Why?

Oral Answer these questions.

1. Why must you know how to convert from one unit of measure to another? To solve problems involving different units

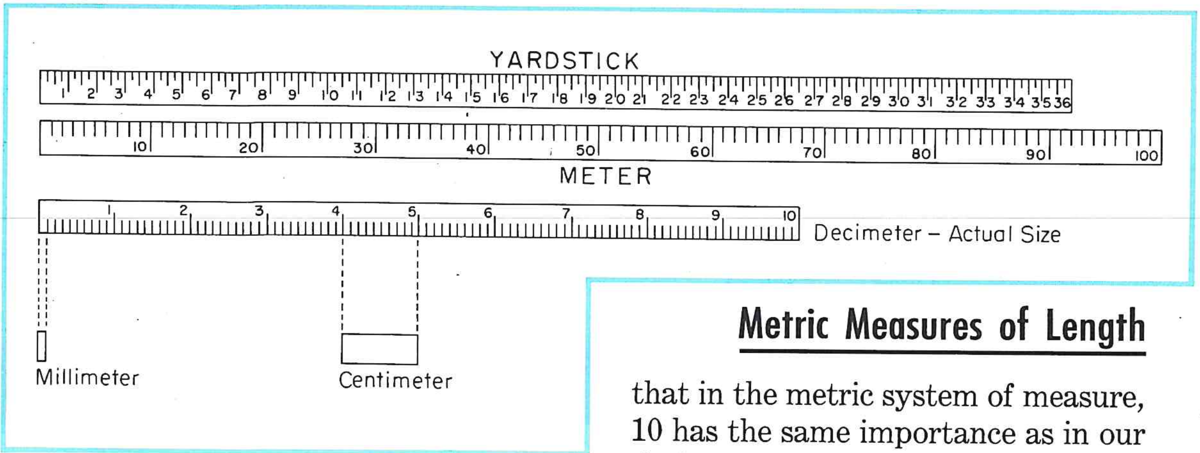
2. Why is it important to have several different units of measure in any system for measuring length? See below.

3. Which system of measures, the English system or the metric system, is the more widely used in the world? Metric system

4. Why would it be difficult to change our system of measures? Many people would have to be re-educated.

5. Why should you know something about the metric system? See below.

2. To measure very large and very small objects
5. Sports, science, and other aspects of life involve the metric system.



Metric Measures of Length

The picture at the top of this page will help you make a comparison between a yardstick and a meter stick. For practical purposes you may think of a meter as being approximately equal in measure to 39 inches. How would you use a yardstick and then a meter stick to measure the length of your room?

In addition to the meter, three other units of metric measure, the **millimeter, centimeter,** and **decimeter,** are shown in actual size on the above illustration. Which would you use to express the measure of the length and width of a library card? To express the measure of the thickness of five sheets of paper in your arithmetic book?

Count the number of millimeters that are equal in measure to 1 centimeter. Your answer should be 10. Count the number of centimeters that are equal in measure to 1 decimeter. Is your answer also 10? Do you think that in the metric system of measure, 10 has the same importance as in our decimal system of numeration? Before answering, study the table in this column. Notice especially the word **kilometer.** People using the metric system of measures use a kilometer in situations where you would use a mile as your unit of measure.

10 millimeters (mm.) = 1 centimeter
10 centimeters (cm.) = 1 decimeter
10 decimeters (dm.) or 100 centimeters = 1 meter (m.)
1000 meters = 1 kilometer (km.)

Oral Answer each question below.

1. What is the correct abbreviation for millimeter? For centimeter? For meter? For kilometer?
mm. ; cm. ; m. ; km.

2. Would you use the millimeter, centimeter, meter, or kilometer as a unit of measure when measuring the length of a playground? The distance to the nearest city? The length of one edge of your textbook?
meter; kilometer; centimeter

3. Does a kilometer have a length equal to, greater than, or less than a mile? Less than

Written Copy. Replace each blank with a numeral so that each statement becomes true. (= means equal in measure.)

1. 1 mm. = $\frac{1}{10}$ cm.

2. 1 cm. = 10 mm.

3. 1 m. = 100 cm.

4. 1 cm. = $\frac{1}{100}$ m.

5. 5 mm. = $\frac{1}{2}$ cm.

6. 6 cm. = 60 mm.

7. 12 m. = 1200 cm.

8. 25 cm. = $\frac{1}{4}$ m.

9. 1 m. = $\frac{1}{1000}$ km.

10. 1 km. = 1000 m.

11. 500 m. = $\frac{1}{2}$ km.

12. 6000 km. = 6,000,000 m.

Solve each problem.

13. A distance runner ran a 1500-meter race in 4 minutes. On the average how many meters did he run in 1 minute?

1500÷4=n 375 m.

14. When in Italy, the Nelsons drove 100 kilometers in $1\frac{3}{4}$ hours. On the average, how many kilometers did they drive in an hour?

$100 \div 1\frac{3}{4}=n$ $57\frac{1}{7}$ km.

15. Jane used all but $1\frac{1}{5}$ meters of material in a piece of material measuring $5\frac{1}{2}$ meters in length. How many meters of material did she use?

$5\frac{1}{2}-1\frac{1}{5}=n$ $4\frac{3}{10}$ m.

16. The level of the mercury in a barometer rose from 755 millimeters to 775 millimeters. What was its rise in millimeters? In centimeters?

775-755=n 20 mm.; $\frac{20}{10}=n$ 2 cm.

17. A piece of material 5 meters long was expected to shrink at least 2% when washed. In meters what was the expected length of the material after being washed?

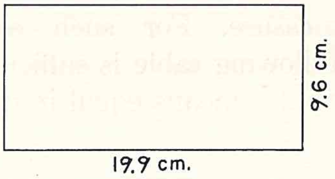
5-(.02x5)=n 4.9 m.

18. Barbara bought a roll of 16-mm. film and a roll of 35-mm. film. By how many millimeters did the two films differ in width?

35-16=n 19 mm.

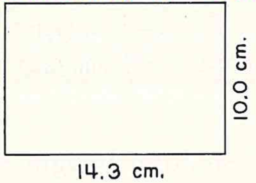
Write the perimeter of each figure below. 2x(19.9+9.6)=n 59 cm.

19.



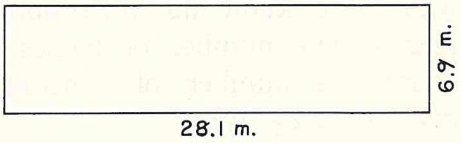
2x(14.3+10.0)=n 48.6 cm.

20.



2x(28.1+6.9)=n 70.0 m.

21.



Converting from Metric to English Units of Length

John Allison visited France with his parents. He saw a sign: *Paris, 80 kilometers*. How many miles were they from Paris?

A meter, you may recall, has a length somewhat greater than that of a yard. A kilometer is 1000 meters. Thus a kilometer has a length somewhat greater than that of 1000 yards. Its length is approximately $\frac{5}{8}$ of the length of a mile.

John can convert 80 km. to mi. by multiplying 80 by $\frac{5}{8}$. Does $\frac{5}{8} \times 80 = 50$? The Allisons were approximately 50 miles from Paris.

World travelers must often make rough estimates of conversions between English and metric units of measure. For such estimates, the following table is sufficiently accurate. (= means equal in measure.)

1 in. = $2\frac{1}{2}$ cm. (approximately)
39 in. = 1 m. (approximately)
$\frac{5}{8}$ or .62 mi. = 1 km. (approximately)

Suppose John bought a picture 18 cm. in width and 25 cm. in depth and wanted to know its dimensions in inches. The number of inches will equal the number of centimeters divided by $2\frac{1}{2}$. Why?

Oral Answer these questions.

1. In inches, what are the dimensions of the picture John bought?
Width: $7\frac{1}{5}$ in.; length: 10 in.

2. In inches and then in yards, what is the approximate length of a piece of silk 6 meters in length?
6x39=x 234 in.; $\frac{234}{36}$=n $6\frac{1}{2}$ yd.

Written Replace each blank with the numeral that makes each statement true. (= means equal in measure.)

1. 1 meter = $\underline{39}$ in. = $\underline{1\frac{1}{12}}$ yd.
2. 500 meters = $\underline{541\frac{2}{3}}$ yd.
3. 50 km. = $\underline{31}$ mi.
4. 9.6 km. = $\underline{5.952}$ mi.
5. 75 cm. = $\underline{30}$ in.

Solve these problems.

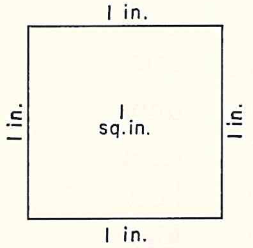
6. The distance between Paris and Marseilles is approximately 675 kilometers. In miles, what is the approximate distance between the cities?
675x.62=n 418.5 mi.

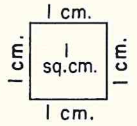
7. A speed limit in a French city read *50 kilometers per hour*. In miles per hour, what was the speed limit in that city?
50x.62=n 31 mph

8. Mrs. Allison bought 4 meters of silk. She used it in making a dress that required $3\frac{2}{3}$ yards of material. In yards, approximately how much of the material was left?
[(4x39)÷36]-$3\frac{2}{3}$=n $\frac{2}{3}$ yd.

Two-Dimensional Measures

If you were using the metric system, you would use the **square centimeter** (sq. cm.) instead of the square inch to measure area. A sq. in. and sq. cm. are shown in actual size below.





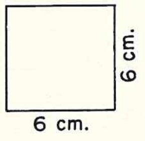
The square inch has two dimensions. Does the square centimeter also have two dimensions? What are they?

Another common unit of two-dimensional measure in the metric system is the **square meter.** (sq. m.)

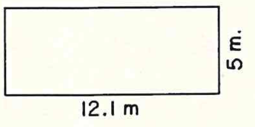
The measure of each side of a square meter is 1 meter. Is the area measure of a square meter equal to, greater than, or less than the area measure of a square yard?

Oral Explain how you would find the area measure of figures 1 and 2.
See below.

1.



2.



1. 6x6=n 36 sq.cm.
2. 12.1x5=n 60.5 sq.m.

Written Find the area of rectangles with the dimensions given below. Reminder: Use the formula $A=lw$.

	a	*b*
1.	15 cm. by 25 cm. 375 sq.cm.	5.2 m. by 6.8 m. 35.36 sq.m.
2.	47 cm. by 60 cm. 2820 sq.cm.	.8 m. by 2.6 m. 2.08 sq.m.
3.	98 cm. by 15 cm. 1470 sq.cm.	7.9 m. by 5.6 m. 44.24 sq.m.
4.	10 cm. by 70 cm. 700 sq.cm.	20 cm. by 36 cm. 720 sq.cm.
5.	24 m. by 54 m. 1296 sq.m.	45 cm. by 59 cm. 2655 sq.cm.
6.	4 m. by 18 m. 72 sq.m.	21 m. by 44 m. 924 sq.m.
7.	2.5 cm. by 8.0 cm. 20 sq.cm.	9.5 m. by 9.0 m. 85.5 sq.m.

Express the area of each figure below in square centimeters.

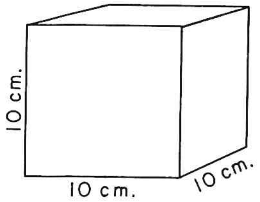
	a	*b*
8.	95 cm. by 1 m. 9500 sq.cm.	2 m. by 85 cm. 17000 sq.cm.
9.	1 m. by 70 cm. 7000 sq.cm.	1.5 m. by 90 cm. 13500 sq.cm.
10.	40 cm. by 1 m. 4000 sq.cm.	1.1 m. by 98 cm. 10780 sq.cm.
11.	65 cm. by 1.4 m. 9100 sq.cm.	1.9 m. by 60 cm. 11400 sq.cm.

Can you do this? Draw a square figure with 1-decimeter sides. Use it to show the relationship between 1 square centimeter and 1 square decimeter. Hint: Divide each side of the figure into 10 segments of equal length. You will need a centimeter ruler to do this.

Measuring Liquids

While touring in Mexico, Bob's father bought gasoline by the **liter** instead of by the gallon. He needed 50 liters of gasoline to fill the tank of his car. He knew that a liter is approximately equivalent to 1.1 quarts. Expressed in quarts and then in gallons, how much gasoline had Bob's father bought?

The liter is a basic unit of the metric system of measures. It is equivalent in measure to the volume of a cube measuring 10 centimeters on each edge as shown below.



Is the measure of a pail of water in liters a number greater than or less than its measure in quarts?

Other metric units used in measuring liquids are related to the liter as shown below.

$\frac{1}{1000}$ liter (l.) = 1 milliliter (ml.)
$\frac{1}{100}$ liter = 1 centiliter (cl.)
$\frac{1}{10}$ liter = 1 deciliter (dl.)
1000 liters = 1 kiloliter (kl.)

Find the prefixes used with *liter*. They are used again and again in the metric system. Study the prefixes given with their meanings below.

$\frac{1}{1,000,000}$	micro
$\frac{1}{1000}$	milli
$\frac{1}{100}$	centi
$\frac{1}{10}$	deci
1	unit
10	deca
100	hecto
1000	kilo
1,000,000	mega

To convert a number of quarts to a number of liters, divide the number of quarts by 1.1. Why? To convert a number of liters to a number of quarts, multiply by 1.1. Why?

Oral Why is it important to know how to use liters in measurement?

Written Make the following conversions. (= means equal in measure.)

	a	*b*
1.	3 kl. = $\underline{3000}$ l.	5 kl. = $\underline{5000}$ l.
2.	2000 l. = $\underline{2}$ kl.	2500 l. = $\underline{2\frac{1}{2}}$ kl.
3.	10 l. = $\underline{11}$ qt.	25 qt. = $\underline{22\frac{8}{11}}$ l.
4.	1 kl. = $\underline{1100}$ qt.	10 gal. = $\underline{36\frac{4}{11}}$ l.
5.	40 l. = $\underline{44}$ qt.	50 l. = $\underline{13\frac{3}{4}}$ gal.

Measuring Weight

Most scientists use the **gram** as the basic unit of weight. A thousand grams, or a **kilogram,** is approximately equal in weight to 2.2 pounds. For weighing very small quantities, the **milligram** is used. A milligram is $\frac{1}{1000}$ of a gram. What would you expect the name of $\frac{1}{100}$ of a gram to be?

Oral What general statement or formula would help you convert a number of pounds to a number of kilograms? To convert a number of kilograms to a number of pounds?

1 lb.=$\frac{1}{2.2}$ kg.; 1 kg.=2.2 lb.

Written Solve each problem.

1. Sue's pen pal, who lives in Germany, weighs 40 kilograms. Sue weighs 79 pounds. Who is the lighter of the two girls and by how much?

See below.

2. An airline in South America had a baggage limit of 25 kilograms on luggage. How much more or less than 40 pounds was the baggage limit?

See below.

3. A ham imported from Holland was marked "net weight, 4.2 kilograms." What was the approximate weight of the ham in pounds?

4.2x2.2=h 9.24 lb.

4. The label on a bottle of medicine read "Contents — 50 grams." What was the approximate measure of the medicine in ounces?

$\left(\frac{16x2.2}{1000}\right)x50=n$ 1.76 oz.

5. A metric ton is equivalent in weight to 1000 kilograms. In English measures what is the approximate weight of a metric ton?

1000x2.2=n 2200 lb.

Copy. Replace each dot with the sign >, <, or =, whichever makes each statement below true.

1. 4 kilograms $\overset{=}{\bullet}$ 4000 grams
2. 500 grams $\overset{<}{\bullet}$ 1 kilogram
3. 50 grams $\overset{<}{\bullet}$ 50 ounces
4. 10 pounds $\overset{<}{\bullet}$ 5 kilograms
5. 20 kilograms $\overset{>}{\bullet}$ 40 pounds
6. 30 ounces $\overset{<}{\bullet}$ 1 kilogram
7. 75 pounds $\overset{>}{\bullet}$ 34 kilograms
8. 100 kilograms $\overset{<}{\bullet}$ 1 ton

MORE PRACTICE PAGE 318

1. 40x2.2=p p=88,
88-79=n Sue: 9 lb.

2. 25x2.2=p p=55,
55-40=n 15 lb. more

Measures of Temperature

Suppose you wish to tell how warm or how cold it is. You will use a number together with the word **degree.** You find the number by reading the scale of a thermometer.

Gabriel Fahrenheit of Amsterdam invented the scale of the thermometer which bears his name. On the Fahrenheit scale the temperature at which water freezes is marked 32°. The temperature at which water boils is marked 212°. The distance between the 32° and 212° marks is subdivided into 180 line segments of equal length. On most thermometer scales, the graduation is extended above 212° and below 32°.

Other numbers could have been used in making a temperature scale. Suppose, for example, that on the scale the point at which water freezes is marked 0 and the point at which it boils is marked 100. There will be 100 divisions of equal length between 0 and 100 on the scale. A thermometer with such a scale is called a **centigrade thermometer.** In your opinion what does the prefix *centi* in centigrade mean? The scale of a centigrade thermometer, too, may be extended in either direction.

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Numbers that indicate temperature below the zero point on either scale are written by numerals like this: −1, −2, −3, and so on. A temperature of −1 degree is read *minus one degree* or *one degree below zero.* Numbers that indicate temperatures above the zero point may be prefixed with the sign +. Use this sign only when it is needed to avoid confusion.

Oral Answer these questions.

1. In what way is the scale of a thermometer similar to a number line? To each point there corresponds one and only one number.

2. What is the advantage, if any, of the centigrade scale over the Fahrenheit scale? Freezing and boiling points more easily remembered

Written Write a numeral to indicate the change in temperature between the pairs of readings below.

	a		*b*	
1.	72° and 81°	9°	+1° and 0°	1°
2.	−1° and 0°	1°	−1° and +1°	2°
3.	−5° and +5°	10°	−10° and 0°	10°
4.	−5° and +15°	20°	61° and 49°	12°
5.	37° and 19°	18°	−8° and +21°	29°
6.	100° and 81°	19°	−10° and +10°	20°
7.	−16° and +5°	21°	−8° and +32°	40°
8.	−21° and +80°	101°	+17° and −5°	22°
9.	67° and 100°	33°	15° and 212°	197°

Practice in Making Conversions

You should be able to complete this page without any help.

Part 1 Copy. Replace each blank with a numeral so that each statement below becomes true. (= means equal in measure.)

1. $\frac{1}{3}$ ft. = 4 in.
2. $\frac{1}{4}$ T. = 500 lb.
3. 6 ft. = 72 in.
4. $\frac{3}{5}$ T. = 1200 lb.
5. $\frac{3}{4}$ yd. = 27 in.
6. $\frac{1}{4}$ mi. = 1320 ft.
7. 5 pt. = $\frac{5}{8}$ gal.
8. 3 pk. = $\frac{3}{4}$ bu.
9. 8 in. = $\frac{2}{3}$ ft.
10. 32 oz. = 2 lb.
11. 16 pt. = 2 gal.
12. 300 yr. = 3 centuries
13. $(2+\frac{3}{8})$ lb. = 38 oz.
14. $(5+\frac{5}{6})$ yd. = 210 in.
15. $(10+\frac{5}{16})$ lb. = 165 oz.
16. $(8+\frac{7}{16})$ lb. = 135 oz.

Part 2 Replace each dot with the sign >, <, or =, whichever makes each statement below true.

1. 1 meter $\overset{>}{\bullet}$ 1 yard
2. 1 kilogram $\overset{>}{\bullet}$ 2 pounds
3. 1 liter $\overset{>}{\bullet}$ 1 quart
4. 100 centimeters $\overset{=}{\bullet}$ 1 meter
5. 1000 grams $\overset{=}{\bullet}$ 1 kilogram
6. 100 ounces $\overset{>}{\bullet}$ 100 grams
7. 100° centigrade $\overset{>}{\bullet}$ 100° Fahrenheit
8. 100 kilograms $\overset{<}{\bullet}$ 1000 pounds

Part 3 Solve these problems.

1. An excavation for the basement of a house was 6 ft. deep, 24 ft. wide, and 38 ft. long. How many cubic feet of dirt were removed in making the excavation?
6x24x38=n 5472 cu.ft.

2. An aquarium was 1 ft. 9 in. long, 1 ft. 6 in. wide, and 1 ft. 1 in. high. What was its volume in cubic inches? In cubic feet? See below.

3. Mr. Paxton needed enough soil to raise the level of his front lawn an average of 2 in. His lawn had dimensions of 24 ft. by 50 ft. How many cubic yards of soil did he need? (Reminder: 2 in. = $\frac{1}{6}$ ft. and 27 cu. ft. = 1 cu. yd.) (24x50x$\frac{1}{6}$)÷27=n
7.407 cu.yd.

2. 21x18x13=n 4914 cu.in.
4914÷1728=n 2.843 cu.ft.

Because the procedure in using this page parallels that used on page T250, no page T292 is included.

Review and Practice

Part 1 Copy. Perform the operation indicated by each operational sign.

	a	*b*
1.	7)16 yd. 1 ft. 2 yd.1 ft.	15)31 hr. 15 min. 2 hr.5 min.
2.	2 min. 38 sec. 5 min. 25 sec. +4 min. 15 sec. 12 min.18 sec.	5 T. 1200 lb. 8 T. 600 lb. +1 T. 1500 lb. 15 T.1300 lb.
3.	15 ft. 7 in. −7 ft. 8 in. 7 ft.11 in.	29 ft. $6\frac{1}{4}$ in. −19 ft. $9\frac{1}{2}$ in. 9 ft.$8\frac{3}{4}$ in.
4.	15 gal. 3 qt. ×6 94 gal.2 qt.	3 qt. $1\frac{1}{2}$ pt. ×6 22 qt.1 pt.

Part 2 Copy. Solve each equation.

	a	*b*	*c*
1.	$\frac{15}{45}=\frac{n}{5}$ $1\frac{2}{3}$	$\frac{25}{a}=\frac{50}{100}$ 50	$\frac{100}{14}=\frac{65}{b}$ 9.1
2.	$\frac{2.5}{n}=\frac{15}{18}$ 3	$\frac{c}{6.4}=\frac{2.8}{16}$ 1.12	$\frac{24}{b}=\frac{16}{19.2}$ 28.8
3.	$\frac{18}{1.20}=\frac{m}{5.4}$ 81	$\frac{11}{k}=\frac{6}{27}$ $49\frac{1}{2}$	$\frac{30}{c}=\frac{18}{12}$ 20
4.	$\frac{7}{14}=\frac{3.12}{x}$ 6.24	$\frac{c}{14}=\frac{4.0}{5.6}$ .1	$\frac{65.17}{1}=\frac{n}{3.50}$ 228.095

Part 3 Find a suitable replacement for n in each statement below.

1. n is $33\frac{1}{3}\%$ of 96 32
2. n is 25% of 96 24
3. 10 is 42% of n $23\frac{17}{21}$
4. n is 63% of 53 33.39
5. n is 40% of 65 26

Part 4 Solve each equation.

1. $37+15+85+96=n$ 233
2. $m=57097-29475$ 27622
3. $795\times876=g$ 696420
4. $115367\div356=x$ $324\frac{23}{356}$
5. $4500\times(3+n)=45000$ 7

Part 5 Solve each equation.

	a	*b*
1.	$a=\frac{1}{3}+\frac{1}{5}$ $\frac{8}{15}$	$c=8\frac{2}{3}+9\frac{7}{12}$ $18\frac{1}{4}$
2.	$r=6\frac{1}{9}-1\frac{2}{3}$ $4\frac{4}{9}$	$s=9\frac{1}{12}-2\frac{1}{9}$ $6\frac{35}{36}$
3.	$4\frac{1}{2}\times1\frac{1}{4}=n$ $5\frac{5}{8}$	$f=7\frac{2}{3}\times6\frac{1}{4}$ $47\frac{11}{12}$
4.	$h=1\frac{1}{6}\div\frac{7}{9}$ $1\frac{1}{2}$	$t=1\frac{7}{9}\div1\frac{1}{15}$ $1\frac{2}{3}$
5.	$n+3\frac{1}{5}=5$ $1\frac{4}{5}$	$6\frac{1}{2}\times n=0$ 0

Part 6 Perform the operation indicated by each operational sign.

	a	*b*
1.	.62)83.70 135.	70.2)112.32 1.6
2.	3987.654 +891.07 4878.724	175478.90 +98975.95 274454.85
3.	97816.27 −39842.85 57973.42	76521.270 −34938.755 41582.515
4.	4.57 ×3.72 17.0004	65.84 ×.493 32.45912

Because the procedure in using this page parallels that of previous pages of *Checkup Time* (see T251), no page T293 is included.

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Several different units of measures of each kind are needed in any system of measures. (283)

2. In making approximations of equivalent English and metric measures, compare the meter to the yard, the liter to the quart, and the kilogram to two pounds. (284, 288, 289)

3. In the metric system of measures, 10 has the same importance as in our numeration system. (284)

4. In the metric system as in any other system of measures, the two-dimensional measures are derived from one-dimensional measures. (287)

Words to Know

1. Meter, metric system (283)

2. Millimeter (mm.), centimeter (cm.), decimeter (dm.), kilometer (km.) (284)

3. Square centimeter (sq. cm.), square meter (sq. m.) (287)

4. Liter (l.), milliliter (ml.), kiloliter (kl.) (288)

5. Gram, kilogram (289)

6. Degree, centigrade (290)

Questions to Discuss

1. What are some common metric measures of length and what is the approximate equivalent measure in the English system? (284, 286) meter($1\frac{1}{12}$yd.); kilometer (.62 mi.); centimeter (.39in.)

2. How do you convert a measure expressed in kilometers to a measure expressed in miles? (286)
.62 x number of kilometers

3. What computation do you do in converting from inches to centimeters? From centimeters to inches? From liters to quarts? From quarts to liters? (286, 288) See below.

4. In what way are the graduations determined for the scale of a Fahrenheit thermometer? Of a centigrade thermometer? (290) See below.

Written Practice

Copy. Replace each dot with the sign >, <, or =, whichever makes each statement true.

1. 35 yards $\overset{<}{\bullet}$ 35 meters (283)

2. 100 meters $\overset{<}{\bullet}$ 1 kilometer (284)

3. 15 inches $\overset{>}{\bullet}$ 30 centimeters (286)

4. 100 sq. in. $\overset{>}{\bullet}$ 100 sq. cm. (287)

5. 75 liters $\overset{>}{\bullet}$ 80 quarts (288)

6. 50 kilograms $\overset{>}{\bullet}$ 100 pounds (289)

3. $2\frac{1}{2}$x no. of inches; no. of centimeters ÷ $2\frac{1}{2}$
1.1 x no. of liters; no. of quarts ÷ 1.1

4. Distance between boiling and freezing points divided into 180 units; into 100 units

Self-Evaluation

Part 1 Write the number of each question below. After each number write the letter or letters that indicate each correct answer.

1. What is the identity number of multiplication?

a. 0
[✓ b. 1]
c. $\frac{1}{0}$
d. n

2. What name is given to the numbers being multiplied?

a. product
b. times
c. difference
[✓ d. factors]

3. Which operation of arithmetic is the inverse operation of division?

a. subtraction
[✓ b. multiplication]
c. addition
d. division

4. In which of the following ways may 1 be renamed?

[✓ a. $\frac{2}{2}$]
[✓ b. $\frac{.1}{.1}$]
[✓ c. 4÷4]
d. 2×2

5. Which of the following statements are true?

a. $\frac{35}{70}=\frac{20}{41}$
b. $\frac{17}{53}=\frac{7}{20}$
[✓ c. $\frac{3.6}{7.2}=\frac{40}{80}$]
[✓ d. $\frac{18}{72}=\frac{42}{168}$]

6. Which of these operations are commutative for whole numbers?

[✓ a. addition]
b. subtraction
[✓ c. multiplication]
d. division

Part 2 Answer each question below.

1. 12 is what per cent of 36?
12=nx36 $33\frac{1}{3}$%

2. What number is 75% of 700?
n=.75x700 525

3. 18 is 40% of what number?
18=.40xn 45

4. What number is 33% of 99?
n=.33x99 32.67

5. 37 is what per cent of 148?
37=nx148 25%

Part 3 Solve each problem.

1. Mr. Ryan drove 172.5 miles on 12.5 gallons of gasoline. What was his average number of miles per gallon of gasoline?
172.5÷12.5=n 13.8 mi.

2. In a class with 32 pupils enrolled, 5 pupils were absent. What per cent of the pupils were present?
(32-5)=nx32 $84\frac{3}{8}$%

3. John saved $12, which was 20% of the money that he earned. How much money had he earned?
12=.20xn $60

4. On a cold day 12% of the pupils were absent in one school. The school enrollment was 625 pupils. How many of the 625 pupils were absent?
n=.12x625 75 pupils

5. A rectangular box had these dimensions: $5\frac{1}{2}$ in., $4\frac{1}{4}$ in., and $3\frac{1}{2}$ in. What was the volume of the box in cubic inches?
$5\frac{1}{2}$x$4\frac{1}{4}$x$3\frac{1}{2}$=n $81\frac{13}{16}$ cu. in.

6. Gasoline was selling at 11.5¢ a liter. At this rate, how much would 35.5 liters cost? 35.5x.115=n $4.08

18

You can review the many important ideas you have learned this year by reading the statements in the tint blocks.

You can get practice in using these ideas by completing the exercises in this chapter. The practice may be oral or written, whichever your teacher thinks will help you most. Before you do the exercises in each practice lesson, read the statement that tells you about it and then study the example.

Numbers and Numerals

> Each place-value position in our Hindu-Arabic numeration system has a value ten times greater than the position to its right.
>
> 111 means
> $(10\times 10\times 1)+(10\times 1)+(1\times 1)$
> $100 \quad + \quad 10 \quad + \quad 1$

Use with page 5.

Write the following numerals, expressing them as the sum of thousands, hundreds, tens, and ones. See below.

	a	*b*	*c*
1.	9402	768	64
2.	4310	5005	1099

Use with page 6.

Copy the following numerals, placing commas correctly. Then write the words to use in reading them.

	a	*b*
1.	301,465,976,010	4,706,352,101
2.	986,431,725	240,093,016,449

> Decimal numerals may represent both whole and fractional numbers.
>
> Read 7.000302 as *seven and three hundred two millionths.*

Use with pages 7 through 10.

Write words to express the following decimal numerals.

	a	*b*	*c*
1.	500.7	21.30	281.08
2.	.0003	2.00001	7.198

> A fractional number can be renamed without changing its value.
>
> $\frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{4}{16}=\frac{5}{20}$
>
> $1=\frac{2}{2}=\frac{5}{5}=\frac{7}{7}=\frac{9}{9}$

Use with pages 14 through 16.

Replace n with the numeral that makes each statement true.

	a	*b*	*c*
1.	$\frac{3}{5}=\frac{n}{15}$ 9	$\frac{6}{8}=\frac{n}{2}$ 1.5	$\frac{10}{12}=\frac{n}{6}$ 5
2.	$\frac{2}{3}=\frac{16}{n}$ 24	$\frac{1}{7}=\frac{n}{49}$ 7	$\frac{6}{8}=\frac{n}{2}$ 1.5

1.a. (10x10x10x9)+(10x10x4)+(2x1)
b. (10x10x7)+(10x6)+(8x1)
c. (10x6)+(4x1)

2.a. (10x10x10x4)+(10x10x3)+(10x1)
b. (10x10x10x5)+(5x1)
c. (10x10x10x1)+(10x9)+(9x1)

Reviewing Addition and Subtraction

Addends can be added in any order or with any grouping.

9+4=13, and 4+9=13
9+(1+3)=(9+1)+3

Use with pages 21 and 22.

Copy the following equations. Add the addends in any order. Use ( ) to show which addends you used first.

1. (5+11)+6+2=n 24

2. 23+(6+4)+12=n 45

You use the addition facts to add in any place-value position, making changes in form when needed.

76+19=n
76+19=95

406
537
+894
1837

Use with pages 23 and 24.

Copy. Write each sum.

	a	*b*	*c*
1.	567 +89 656	915 +180 1095	464 +781 1245
2.	613 20 +726 1359	921 462 +805 2188	449 376 +225 1050
3.	5163 1015 3206 +646 10030	4082 63705 10820 +34123 112730	90841 56732 3496 +26543 177612
4.	7631 5238 7447 +1352 21668	40617 90123 19685 +25069 175494	29836 30448 50322 +17634 128240

You subtract the ones, then the tens, and then the hundreds, making changes in form whenever necessary.

65 −23 42	71 −47 24	593 −297 296

Use with pages 27 and 28.

Copy. Write each difference.

	a	*b*	*c*	*d*
1.	57 −39 18	680 −88 592	603 −195 408	857 −669 188
2.	915 −237 678	457 −329 128	261 −148 113	500 −247 253

Subtract larger numbers as you subtracted ones, tens, or hundreds.

7904 −3985 3919	87654 −8947 78707	90001 −39842 50159

Use with page 29.

	a	*b*	*c*
1.	6009 −431 5578	4832 −2678 2154	9028 −8253 775
2.	58376 −4126 54250	90687 −69475 21212	67342 −44159 23183

In the addition of fractional numbers, the properties of addition apply as with whole numbers.

$\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$ $\qquad$ $\frac{5}{7}-\frac{3}{7}=\frac{2}{7}$

$\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}=\frac{5}{7}$ $\qquad$ $\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$

$\frac{2}{7}+\frac{3}{7}+\frac{5}{7}=\frac{(2+3)+5}{7}=\frac{5+5}{7}=\frac{10}{7}=1\frac{3}{7}$

$\frac{2}{7}+\frac{3}{7}+\frac{5}{7}=\frac{2+(3+5)}{7}=\frac{2+8}{7}=\frac{10}{7}=1\frac{3}{7}$

Use with page 33.

Copy. Solve each equation.

	a	b	c
1.	$\frac{1}{3}+n=\frac{2}{3}$ $\frac{1}{3}$	$\frac{5}{8}+\frac{5}{8}=n$ $1\frac{1}{4}$	$\frac{8}{9}+\frac{1}{9}=n$ 1
2.	$\frac{5}{12}+\frac{11}{12}=n$ $1\frac{1}{3}$	$\frac{7}{10}+n=1$ $\frac{3}{10}$	$n+\frac{5}{8}=\frac{7}{8}$ $\frac{2}{8}$

Use with page 34.

Copy. Write each sum.

	a	b	c	d
1.	$\frac{2}{9}$ $+5\frac{1}{9}$ $5\frac{1}{3}$	$3\frac{1}{7}$ $+8\frac{5}{7}$ $11\frac{6}{7}$	$5\frac{5}{8}$ $+4\frac{1}{8}$ $9\frac{3}{4}$	$8\frac{5}{6}$ $+7\frac{1}{6}$ 16
2.	$7\frac{3}{16}$ $+8\frac{11}{16}$ $15\frac{7}{8}$	$6\frac{9}{10}$ $+3\frac{9}{10}$ $10\frac{4}{5}$	$11\frac{1}{22}$ $+8\frac{11}{22}$ $19\frac{6}{11}$	$8\frac{3}{14}$ $+14\frac{9}{14}$ $22\frac{6}{7}$

Rename fractional numbers with unlike denominators so that they have a common denominator.

$\frac{2}{5}+\frac{5}{6}=\frac{12}{30}+\frac{25}{30}=\frac{12+25}{30}=\frac{37}{30}=1\frac{7}{30}$

Use with pages 35 and 36.

Copy. Solve each equation.

	a	b	c
1.	$\frac{1}{7}+\frac{5}{21}=n$ $\frac{8}{21}$	$\frac{25}{32}+\frac{7}{8}=n$ $1\frac{21}{32}$	$\frac{1}{3}+\frac{1}{10}+\frac{1}{4}=n$ $\frac{41}{60}$
2.	$\frac{8}{9}+\frac{4}{5}=n$ $1\frac{31}{45}$	$\frac{5}{7}+\frac{5}{6}=n$ $1\frac{23}{42}$	$\frac{1}{6}+\frac{3}{8}+\frac{1}{2}=n$ $1\frac{1}{24}$

Copy. Write each sum.

	a	b	c	d
3.	$\frac{4}{15}$ $+\frac{5}{6}$ $1\frac{1}{10}$	$2\frac{1}{8}$ $+7\frac{1}{6}$ $9\frac{7}{24}$	$3\frac{9}{10}$ $+9\frac{1}{5}$ $13\frac{1}{10}$	$11\frac{5}{14}$ $+20\frac{7}{9}$ $32\frac{17}{126}$
4.	$\frac{5}{7}$ $\frac{3}{4}$ $+\frac{2}{3}$ $2\frac{11}{84}$	$\frac{3}{4}$ $12\frac{7}{9}$ $+9\frac{5}{8}$ $23\frac{11}{72}$	$17\frac{1}{11}$ $51\frac{1}{2}$ $+9\frac{3}{4}$ $78\frac{15}{44}$	$90\frac{11}{12}$ $14\frac{1}{6}$ $+27\frac{3}{4}$ $\frac{3}{4}$ $132\frac{5}{6}$

If necessary, rename the minuend as a fractional number with the same denominator as the subtrahend.

$$\begin{array}{rl} 2 & =1\frac{8}{8} \\ -\frac{3}{8} & =\ \ \frac{3}{8} \\ \hline & \ \ 1\frac{5}{8} \end{array} \qquad \begin{array}{rl} 10\frac{2}{9} & =9\frac{11}{9} \\ -7\frac{1}{3} & =7\frac{3}{9} \\ \hline & \ \ 2\frac{8}{9} \end{array}$$

Use with pages 37 and 38.

Copy in column form. Solve.

	a	b
1.	$24\frac{17}{24}-17\frac{7}{24}=n$ $7\frac{5}{12}$	$11\frac{9}{17}-8\frac{9}{17}=n$ 3
2.	$36\frac{1}{6}-14\frac{5}{6}=n$ $21\frac{1}{3}$	$19\frac{3}{7}-8\frac{5}{8}=n$ $10\frac{45}{56}$
3.	$41\frac{5}{9}-3\frac{1}{5}=n$ $38\frac{16}{45}$	$76\frac{7}{10}-47\frac{9}{10}=n$ $28\frac{4}{5}$

Copy. Write each difference.

	a	b	c	d
4.	$7\frac{1}{6}$ $-\frac{5}{6}$ $6\frac{1}{3}$	$10\frac{5}{9}$ $-4\frac{1}{3}$ $6\frac{2}{9}$	$14\frac{1}{7}$ $-3\frac{1}{2}$ $10\frac{9}{14}$	$61\frac{1}{10}$ $-23\frac{3}{5}$ $37\frac{1}{2}$
5.	$8\frac{1}{2}$ $-7\frac{3}{7}$ $1\frac{1}{14}$	$19\frac{1}{6}$ $-7\frac{5}{9}$ $11\frac{11}{18}$	$71\frac{5}{6}$ $-59\frac{8}{15}$ $12\frac{3}{10}$	$29\frac{7}{18}$ $-14\frac{5}{6}$ $14\frac{5}{9}$
6.	$80\frac{1}{7}$ $-9\frac{3}{5}$ $70\frac{19}{35}$	$24\frac{1}{19}$ $-19\frac{2}{3}$ $4\frac{22}{57}$	$51\frac{1}{10}$ $-17\frac{1}{5}$ $33\frac{9}{10}$	$75\frac{3}{25}$ $-38\frac{1}{3}$ $36\frac{59}{75}$

Reviewing Multiplication

Knowing the multiplication facts will help you to multiply by ones.

453 ×4 12 200 1600 1812	431900 ×4 3600 4000 120000 1600000 1727600	431900 ×4 1727600

Use with pages 42 through 44.

Copy. Write each product.

	a	*b*	*c*
1.	73 ×3 219	60 ×9 540	35 ×2 70
2.	967 ×8 7736	604 ×6 3624	570 ×7 3990
3.	5143 ×2 10286	3552 ×4 14208	6071 ×7 42497
4.	46818 ×4 187272	19199 ×5 95995	40312 ×9 362808
5.	21870 ×9 196830	54918 ×4 219672	61324 ×6 367944
6.	384720 ×9 3462480	549010 ×6 3294060	296157 ×7 2073099
7.	122618 ×4 490472	316807 ×8 2534456	412087 ×9 3708783
8.	103612 ×5 518060	212680 ×3 638040	719013 ×6 4314078

You can regroup factors without affecting the product.

$$\begin{aligned} 28 \times 5 &= (14 \times 2) \times 5 \\ &= 14 \times (2 \times 5) \\ &= 14 \times 10 \\ &= 140 \end{aligned}$$

Use with page 45.

Copy. Solve each equation. Regroup the factors whenever possible.

	a	*b*
1.	190 5×38 = n	570 (57×2)×5 = n
2.	170 2×85 = n	290 (29×5)×2 = n
3.	840 168×5 = n	870 2×(5×87) = n

To multiply by a two-digit number, multiply by the ones and then by the tens. Add the partial products.

794 ×32 1588 2382 25408	1263 ×23 3789 2526 29049	12536 ×45 62680 50144 564120

Use with page 47.

Copy in column form. Solve.

	a	*b*
1.	238 17×14 = n	3496 38×92 = n
2.	2204 29×76 = n	2310 42×55 = n
3.	4941 81×61 = n	1764 63×28 = n
4.	3096 43×72 = n	3102 33×94 = n

Use with page 48.

Copy. Write each product.

	a	b	c	d
1.	170 ×70 11900	306 ×49 14994	671 ×28 18788	235 ×84 19740
2.	547 ×61 33367	877 ×19 16663	496 ×53 26288	185 ×92 17020
3.	350 ×13 4550	401 ×26 10426	654 ×25 16350	543 ×32 17376
4.	816 ×37 30192	594 ×48 28512	935 ×65 60775	718 ×94 67492
5.	683 ×27 18441	927 ×81 75087	529 ×73 38617	386 ×48 18528
6.	395 ×41 16195	719 ×70 50330	870 ×92 80040	426 ×63 26838

Use with page 49.

	a	b	c
1.	8600 ×42 361200	7602 ×38 288876	9054 ×65 588510
2.	7189 ×56 402584	6507 ×72 468504	3964 ×48 190272
3.	31400 ×50 1570000	20091 ×28 562548	51263 ×47 2409361
4.	65742 ×79 5193618	81553 ×68 5545604	49719 ×84 4176396
5.	40983 ×62 2540946	71691 ×74 5305134	58945 ×89 5246105
6.	62319 ×72 4486968	48659 ×57 2773563	81293 ×61 4958873
7.	904372 ×63 56975436	139094 ×57 7928358	706723 ×49 34629427

To multiply by a three-digit number, multiply by the ones, then by the tens, and then by the hundreds. Add the partial products.

287 ×163 861 1722 287 46781	13278 ×326 79668 26556 39834 4328628

Use with pages 50 and 52.

Copy. Write each product.

	a	b	c
1.	324 ×212 68688	541 ×322 174202	467 ×501 233967
2.	6061 ×433 2624413	5682 ×713 4051266	8670 ×521 4517070
3.	51068 ×297 15167196	69174 ×584 40397616	38906 ×687 26728422
4.	90167 ×519 46796673	43862 ×604 26492648	76465 ×923 70577195
5.	82304 ×319 26254976	91263 ×287 26192481	49768 ×579 28815672
6.	70841 ×407 28832287	63927 ×910 58173570	38694 ×765 29600910

Reviewing Division

To divide by a one-digit number, divide, multiply, and subtract until the final remainder is 0 or another number less than the divisor.

$$\begin{array}{r} 57 \\ 2\overline{)114} \\ \underline{10} \\ 14 \\ \underline{14} \end{array} \qquad \begin{array}{r} \text{Check} \\ 57 \\ \underline{\times 2} \\ 114 \end{array}$$

Use with page 60.

Copy. Write each quotient. Check.

	a	*b*	*c*
1.	$\begin{array}{r} 55 \\ 3\overline{)165} \end{array}$	$\begin{array}{r} 36r4 \\ 8\overline{)292} \end{array}$	$\begin{array}{r} 409r1 \\ 4\overline{)1637} \end{array}$
2.	$\begin{array}{r} 1031r2 \\ 9\overline{)9281} \end{array}$	$\begin{array}{r} 3665r3 \\ 9\overline{)32988} \end{array}$	$\begin{array}{r} 23575r2 \\ 4\overline{)94302} \end{array}$
3.	$\begin{array}{r} 9902 \\ 6\overline{)59412} \end{array}$	$\begin{array}{r} 12807 \\ 7\overline{)89649} \end{array}$	$\begin{array}{r} 11271r4 \\ 8\overline{)90172} \end{array}$

Use trial quotients to help you divide by large numbers.

$$\begin{array}{r} 2 \\ 42\overline{)84} \\ \underline{84} \end{array} \qquad \begin{array}{l} \text{Trial Quotient} \\ 8\,T \div 4\,T = n \\ 8\,T \div 4\,T = 2 \\ \quad 2 \times 42 = 84 \end{array}$$

Use with page 61.

Find the trial quotient. Solve.

	a	*b*
1.	$56 \div 18 = n$ 3r2	$89 \div 13 = n$ 6r11
2.	$92 \div 23 = n$ 4	$76 \div 40 = n$ 1r36
3.	$210 \div 51 = n$ 4r6	$344 \div 62 = n$ 5r34

To divide a three-digit number by a two-digit number, find the trial quotient and divide, multiply, and subtract as with a one-digit divisor.

$$\begin{array}{r} 5 \\ 23\overline{)115} \\ \underline{115} \end{array} \qquad \begin{array}{r} 20\ r13 \\ 34\overline{)693} \\ \underline{68} \\ 13 \end{array}$$

Use with pages 62 and 63.

Copy. Write each quotient.

	a	*b*	*c*
1.	$\begin{array}{r} 20r5 \\ 21\overline{)425} \end{array}$	$\begin{array}{r} 21r11 \\ 14\overline{)305} \end{array}$	$\begin{array}{r} 21r5 \\ 11\overline{)236} \end{array}$
2.	$\begin{array}{r} 7r24 \\ 35\overline{)269} \end{array}$	$\begin{array}{r} 9r6 \\ 52\overline{)474} \end{array}$	$\begin{array}{r} 9r18 \\ 71\overline{)657} \end{array}$
3.	$\begin{array}{r} 20r19 \\ 39\overline{)799} \end{array}$	$\begin{array}{r} 11r1 \\ 90\overline{)991} \end{array}$	$\begin{array}{r} 11r64 \\ 84\overline{)988} \end{array}$
4.	$\begin{array}{r} 11 \\ 75\overline{)825} \end{array}$	$\begin{array}{r} 13 \\ 63\overline{)819} \end{array}$	$\begin{array}{r} 15r1 \\ 55\overline{)826} \end{array}$

You can rename a dividend as a sum and divide each addend separately without changing the quotient.

$$\begin{aligned} 168 \div 7 &= (140 + 28) \div 7 \\ &= (140 \div 7) + (28 \div 7) \\ &= 20 + 4 \\ &= 24 \end{aligned}$$

Use with page 64.

Rename each dividend. Solve.

	a	*b*
1.	$92 \div 4 = n$ 23	$70 \div 5 = n$ 14
2.	$56 \div 8 = n$ 7	$216 \div 9 = n$ 24

You find an average of two or more numbers by finding their sum and dividing the sum by the number of addends.

$$145+121+202+196=664$$
$$664 \div 4 = n \qquad 664 \div 4 = 166$$

Use with page 65.

Find the average of each set of numbers below.

1. 45, 123, 201, 345, 61 155
2. 209, 110, 89, 602, 501, 367 313
3. 141, 230, 675, 15, 43, 311 $235\frac{5}{6}$
4. 537, 209, 611, 42, 197, 810 401

To divide larger numbers by a two-digit number, you use the same steps as you use in dividing smaller ones. Repeat the steps as often as needed.

$$\begin{array}{r} 660\text{ r}22 \\ 43\overline{)28402} \\ \underline{258} \\ 260 \\ \underline{258} \\ 22 \end{array} \qquad \begin{array}{r} 23310 \\ 34\overline{)792540} \\ \underline{68} \\ 112 \\ \underline{102} \\ 105 \\ \underline{102} \\ 34 \\ \underline{34} \\ 0 \end{array}$$

Use with pages 66 and 67.

	a	b
1.	$\begin{array}{r}53\\37\overline{)1961}\end{array}$	$\begin{array}{r}106\\28\overline{)2968}\end{array}$
2.	$\begin{array}{r}98\text{r}64\\92\overline{)9080}\end{array}$	$\begin{array}{r}20\text{r}12\\63\overline{)1272}\end{array}$
3.	$\begin{array}{r}980\text{r}13\\48\overline{)47053}\end{array}$	$\begin{array}{r}1027\text{r}1\\57\overline{)58540}\end{array}$
4.	$\begin{array}{r}1453\text{r}7\\28\overline{)40691}\end{array}$	$\begin{array}{r}1245\\63\overline{)78435}\end{array}$
5.	$\begin{array}{r}292\text{r}25\\55\overline{)16085}\end{array}$	$\begin{array}{r}988\text{r}5\\13\overline{)12849}\end{array}$
6.	$\begin{array}{r}19751\text{r}3\\46\overline{)908549}\end{array}$	$\begin{array}{r}21713\text{r}10\\39\overline{)846817}\end{array}$
7.	$\begin{array}{r}7582\text{r}24\\72\overline{)545928}\end{array}$	$\begin{array}{r}31560\\26\overline{)820560}\end{array}$

You use the same steps in dividing by a three-digit number as in dividing by a two-digit number.

$$\begin{array}{r} 15\text{ r}23 \\ 242\overline{)3653} \\ \underline{242} \\ 1233 \\ \underline{1210} \\ 23 \end{array} \qquad \begin{array}{r} 562 \\ 145\overline{)81490} \\ \underline{725} \\ 899 \\ \underline{870} \\ 290 \\ \underline{290} \end{array}$$

Use with pages 68 and 69.

Copy. Write each quotient.

	a	b
1.	$\begin{array}{r}9\\140\overline{)1260}\end{array}$	$\begin{array}{r}8\\376\overline{)3008}\end{array}$
2.	$\begin{array}{r}17\\295\overline{)5015}\end{array}$	$\begin{array}{r}23\text{r}1\\421\overline{)9684}\end{array}$
3.	$\begin{array}{r}24\\517\overline{)12408}\end{array}$	$\begin{array}{r}37\\409\overline{)15133}\end{array}$
4.	$\begin{array}{r}136\text{r}32\\624\overline{)84896}\end{array}$	$\begin{array}{r}137\text{r}608\\718\overline{)98974}\end{array}$
5.	$\begin{array}{r}865\\266\overline{)230090}\end{array}$	$\begin{array}{r}514\text{r}4\\429\overline{)220510}\end{array}$
6.	$\begin{array}{r}2043\text{r}259\\357\overline{)729610}\end{array}$	$\begin{array}{r}1539\text{r}387\\581\overline{)894546}\end{array}$
7.	$\begin{array}{r}1098\text{r}382\\823\overline{)904036}\end{array}$	$\begin{array}{r}536\text{r}365\\724\overline{)388429}\end{array}$
8.	$\begin{array}{r}1203\text{r}240\\728\overline{)876024}\end{array}$	$\begin{array}{r}1943\text{r}352\\468\overline{)909676}\end{array}$

Multiplying Fractional Numbers

To multiply a whole number by a fractional number, multiply by the number of the numerator and divide by the number of the denominator.

$$4\times\frac{2}{5}=\frac{4\times2}{5}=\frac{8}{5}=\frac{5+3}{5}=1+\frac{3}{5}=1\frac{3}{5}$$

$$10\times\frac{5}{8}=\frac{10\times5}{8}=\frac{50}{8}=50\div8=6\frac{2}{8}=6\frac{1}{4}$$

Use with pages 73 through 75.

	a	*b*
1.	$3\times\frac{3}{5}=n$ $1\frac{4}{5}$	$3\times\frac{5}{6}=n$ $2\frac{1}{2}$
2.	$\frac{4}{7}\times4=n$ $2\frac{2}{7}$	$6\times\frac{3}{21}=n$ $\frac{6}{7}$
3.	$5\times\frac{2}{9}=n$ $1\frac{1}{9}$	$\frac{1}{3}\times3=n$ 1
4.	$8\times\frac{3}{14}=n$ $1\frac{5}{7}$	$18\times\frac{4}{9}=n$ 8

If one factor is named by a mixed fraction and the other is a whole number, multiply the fractional number and the whole number separately and add the partial products.

A

$8\times5\frac{1}{4}=n$

$8\times(5+\frac{1}{4})=n$

$(8\times5)+(8\times\frac{1}{4})=n$

$40+2=n$

$42=n$

B

$$\begin{array}{r} 31\frac{5}{9} \\ \times27 \\ \hline 15 \\ 217 \\ 620 \\ \hline 852 \end{array}$$

Use with page 77.

Solve for n, using either the equation or the column form.

	a	*b*
1.	$8\times2\frac{3}{4}=n$ 22	$72\times23\frac{5}{12}=n$ 1686
2.	$9\times8\frac{1}{3}=n$ 75	$15\times81\frac{4}{5}=n$ 1227
3.	$2\times7\frac{1}{2}=n$ 15	$24\times17\frac{5}{8}=n$ 423
4.	$5\times9\frac{4}{5}=n$ 49	$96\times84\frac{7}{12}=n$ 8120

If one factor is named by a mixed fraction and the other is a whole number, rename the mixed fraction as an improper fraction before multiplying.

$$6\times2\frac{2}{3}=6\times\frac{8}{3}=\frac{6\times8}{3}=\frac{48}{3}=48\div3=16$$

$$8\times2\frac{1}{7}=8\times\frac{15}{7}=\frac{8\times15}{7}=\frac{120}{7}=120\div7=17\frac{1}{7}$$

Use with pages 78 and 79.

	a	*b*
1.	$6\times9\frac{2}{3}=n$ 58	$5\frac{1}{8}\times10=n$ $51\frac{1}{4}$
2.	$7\times4\frac{1}{9}=n$ $28\frac{7}{9}$	$4\frac{3}{5}\times8=n$ $36\frac{4}{5}$
3.	$18\times3\frac{2}{5}=n$ $61\frac{1}{5}$	$5\times17\frac{3}{4}=n$ $88\frac{3}{4}$
4.	$6\frac{2}{7}\times17=n$ $106\frac{6}{7}$	$14\times11\frac{1}{6}=n$ $156\frac{1}{3}$

Multiply two fractional numbers by multiplying the numbers named by the numerators, and by multiplying the numbers named by the denominators.

$$\frac{1}{3}\times\frac{1}{6}=\frac{1\times1}{3\times6}=\frac{1}{18}$$

$$\frac{2}{5}\times\frac{3}{10}=\frac{2\times3}{5\times10}=\frac{6}{50}=\frac{3}{25}$$

Use with pages 80 through 83.

	a	*b*
1.	$\frac{1}{2}\times\frac{1}{12}=n$ $\frac{1}{24}$	$\frac{1}{9}\times\frac{1}{8}=n$ $\frac{1}{72}$
2.	$\frac{1}{10}\times\frac{5}{7}=n$ $\frac{1}{14}$	$\frac{5}{6}\times\frac{1}{5}=n$ $\frac{1}{6}$

	a	*b*
3.	$\frac{1}{8}\times\frac{2}{3}=n$ $\frac{1}{12}$	$\frac{1}{2}\times\frac{10}{11}=n$ $\frac{5}{11}$
4.	$\frac{2}{3}\times\frac{13}{14}=n$ $\frac{13}{21}$	$\frac{3}{7}\times\frac{6}{8}=n$ $\frac{9}{28}$
5.	$\frac{5}{9}\times\frac{3}{5}=n$ $\frac{1}{3}$	$\frac{7}{8}\times\frac{4}{5}=n$ $\frac{7}{10}$
6.	$\frac{1}{11}\times\frac{3}{5}=n$ $\frac{3}{55}$	$\frac{91}{120}$ $n=\frac{7}{8}\times\frac{13}{15}$
7.	$\frac{9}{14}\times\frac{4}{5}=n$ $\frac{18}{35}$	$\frac{7}{18}\times\frac{5}{6}=n$ $\frac{35}{108}$
8.	$\frac{21}{22}\times\frac{3}{4}=n$ $\frac{63}{88}$	$\frac{17}{87}$ $n=\frac{10}{29}\times\frac{17}{30}$

If the factors are named by a fraction and a mixed fraction, or by two mixed fractions, you can rename the mixed fractions as improper fractions and multiply as you multiply fractional numbers.

$$2\tfrac{1}{2}\times4\tfrac{3}{4}=\tfrac{5}{2}\times\tfrac{19}{4}=\tfrac{5\times19}{2\times4}=\tfrac{95}{8}=95\div8=11\tfrac{7}{8}$$

$$\tfrac{3}{4}\times2\tfrac{1}{4}=\tfrac{3}{4}\times\tfrac{9}{4}=\tfrac{3\times9}{16}=\tfrac{27}{16}=27\div16=1\tfrac{11}{16}$$

Use with pages 84 and 85.

Copy. Solve for n.

	a	*b*
1.	$\frac{2}{3}\times3\frac{5}{6}=n$ $2\frac{5}{9}$	$1\frac{5}{8}\times\frac{3}{4}=n$ $1\frac{7}{32}$
2.	$8\frac{9}{10}\times\frac{7}{8}=n$ $7\frac{63}{80}$	$\frac{3}{5}\times7\frac{5}{9}=n$ $4\frac{8}{15}$
3.	$1\frac{3}{7}\times2\frac{1}{14}=n$ $2\frac{47}{49}$	$34\frac{19}{32}$ $n=5\frac{1}{8}\times6\frac{3}{4}$
4.	$1\frac{8}{9}\times2\frac{3}{8}=n$ $4\frac{35}{72}$	$10\frac{4}{5}\times\frac{15}{16}=n$ $10\frac{1}{8}$
5.	$7\frac{8}{11}\times4\frac{4}{5}=n$ $37\frac{1}{11}$	$81\frac{1}{30}$ $n=20\frac{7}{9}\times3\frac{9}{10}$
6.	$41\frac{19}{21}$ $n=15\frac{5}{7}\times2\frac{2}{3}$	$5\frac{1}{2}\times18\frac{5}{6}=n$ $103\frac{7}{12}$
7.	$20\frac{1}{3}\times2\frac{1}{4}=n$ $45\frac{3}{4}$	$74\frac{13}{16}$ $n=7\frac{1}{8}\times10\frac{1}{2}$
8.	$10\frac{1}{5}\times30\frac{2}{3}=n$ $312\frac{4}{5}$	$8\frac{7}{8}\times6\frac{4}{7}=n$ $58\frac{9}{28}$

You may be able to reduce fractions to lowest terms before, rather than after, you multiply the numbers.

$$n=\tfrac{3}{4}\times56=\tfrac{3\times56}{1\times4}=\tfrac{3}{1}\times\tfrac{14}{1}=3\times14=42$$

$$n=2\tfrac{2}{3}\times3\tfrac{1}{2}=\tfrac{8\times7}{3\times2}=\tfrac{8\times7}{2\times3}=\tfrac{4\times7}{1\times3}=\tfrac{28}{3}=9\tfrac{1}{3}$$

Use with pages 86 and 87.

Copy. Solve for n.

	a	*b*
1.	$\frac{3}{8}\times\frac{16}{17}=n$ $\frac{6}{17}$	$\frac{5}{12}\times\frac{3}{5}=n$ $\frac{1}{4}$
2.	$\frac{1}{2}\times3\frac{1}{3}=n$ $1\frac{2}{3}$	$2\frac{1}{2}$ $n=2\frac{7}{9}\times\frac{9}{10}$
3.	$\frac{11}{12}\times\frac{6}{33}=n$ $\frac{1}{6}$	$\frac{3}{7}\times10\frac{11}{15}=n$ $4\frac{3}{5}$
4.	$4\frac{2}{3}\times3\frac{6}{7}=n$ 18	$7\frac{1}{3}\times3\frac{1}{2}=n$ $25\frac{2}{3}$
5.	$2\frac{1}{7}\times4\frac{9}{10}=n$ $10\frac{1}{2}$	$10\frac{1}{5}\times1\frac{2}{3}=n$ 17
6.	$18\frac{1}{2}\times3\frac{1}{3}=n$ $61\frac{2}{3}$	$4\frac{5}{6}\times4\frac{4}{5}=n$ $23\frac{1}{5}$

To find the product of three fractional numbers, you can regroup the factors without changing the product.

$$\begin{aligned}n&=\tfrac{4}{5}\times\tfrac{3}{4}\times\tfrac{2}{3}\\&=\tfrac{(4\times3)\times2}{(4\times3)\times5}\\&=\tfrac{12\times2}{12\times5}=1\times\tfrac{2}{5}=\tfrac{2}{5}\end{aligned}$$

Use with pages 88 and 89.

Copy. Solve for n.

	a	*b*
1.	$\frac{50}{231}$ $n=\frac{2}{8}\times\frac{10}{11}\times\frac{20}{21}$	$\frac{7}{12}\times\frac{4}{5}\times\frac{3}{8}=n$ $\frac{7}{40}$
2.	$\frac{7}{8}\times\frac{2}{3}\times\frac{9}{14}=n$ $\frac{3}{8}$	$\frac{4}{15}$ $n=\frac{1}{2}\times\frac{2}{3}\times\frac{4}{5}$
3.	$\frac{3}{8}\times\frac{5}{9}\times\frac{3}{5}=n$ $\frac{1}{8}$	$\frac{19}{20}\times\frac{10}{13}\times\frac{2}{3}=n$ $\frac{19}{39}$
4.	$\frac{5}{7}\times\frac{7}{15}\times\frac{3}{4}=n$ $\frac{1}{4}$	$\frac{5}{6}\times\frac{18}{19}\times\frac{2}{3}=n$ $\frac{10}{19}$

Dividing Fractional Numbers

To divide fractional numbers with equal denominators, divide the numbers named by the numerators. The denominator of the quotient will be 1.

$$\frac{8}{15} \div \frac{4}{15} = \frac{8 \div 4}{15 \div 15} = \frac{2}{1} = 2$$

$$\frac{9}{10} \div \frac{4}{10} = \frac{9 \div 4}{10 \div 10} = \frac{9 \div 4}{1} = 2\frac{1}{4}$$

Use with pages 93 and 94.

	a	*b*
1.	$n \div \frac{3}{8} = \frac{9}{8}$ $\frac{27}{64}$	$\frac{8}{9} \div \frac{2}{9} = n$ 4
2.	$\frac{49}{50} \div \frac{7}{50} = n$ 7	$\frac{35}{72} \div \frac{7}{72} = n$ 5
3.	$\frac{21}{25} \div \frac{8}{25} = n$ $2\frac{5}{8}$	$\frac{13}{15} \div \frac{2}{15} = n$ $6\frac{1}{2}$
4.	$\frac{18}{19} \div \frac{9}{19} = n$ 2	$1\frac{4}{5}$ $n = \frac{27}{28} \div \frac{15}{28}$
5.	$1\frac{13}{15}$ $n = \frac{28}{39} \div \frac{15}{39}$	$\frac{14}{26} \div \frac{3}{26} = n$ $4\frac{2}{3}$

If the dividend is named by a mixed fraction and the divisor by a fraction, first rename the mixed fractions as improper fractions. Then proceed as with fractional numbers.

$$\begin{aligned} n &= 3\tfrac{1}{5} \div \tfrac{3}{5} \\ &= \tfrac{16}{5} \div \tfrac{3}{5} \\ &= \tfrac{16 \div 3}{5 \div 5} \\ &= (16 \div 3) \div 1 \\ &= 5\tfrac{1}{3} \end{aligned}$$

$$\begin{aligned} n &= 4\tfrac{3}{4} \div 2\tfrac{1}{4} \\ &= \tfrac{19}{4} \div \tfrac{9}{4} \\ &= \tfrac{19 \div 9}{4 \div 4} \\ &= (19 \div 9) \div 1 \\ &= 2\tfrac{1}{9} \end{aligned}$$

Use with pages 95 and 96.

	a	*b*
1.	$2\frac{7}{8} \div \frac{3}{8} = n$ $7\frac{2}{3}$	$4\frac{3}{7} \div \frac{4}{7} = n$ $7\frac{3}{4}$
2.	$3\frac{1}{6} \div \frac{5}{6} = n$ $3\frac{4}{5}$	$5\frac{2}{9} \div \frac{8}{9} = n$ $5\frac{7}{8}$
3.	$2\frac{1}{4} \div 1\frac{3}{4} = n$ $1\frac{2}{7}$	$3\frac{1}{9} \div 2\frac{8}{9} = n$ $1\frac{1}{13}$
4.	$4\frac{1}{7} \div 3\frac{5}{7} = n$ $1\frac{3}{26}$	$1\frac{7}{8} \div 1\frac{1}{8} = n$ $1\frac{2}{3}$
5.	$8\frac{2}{5} \div 2\frac{4}{5} = n$ 3	$3\frac{11}{13}$ $n = 14\frac{2}{7} \div 3\frac{5}{7}$
6.	$15\frac{2}{3} \div 12\frac{1}{3} = n$ $1\frac{10}{37}$	$1\frac{39}{65}$ $n = 11\frac{5}{9} \div 7\frac{2}{9}$

In a division involving whole numbers and fractional numbers, rename the whole numbers and the fractional numbers as fractions (may be improper fractions) with the same denominator. Then proceed as with fractional numbers.

$$\begin{aligned} n &= 8 \div \tfrac{3}{5} = \tfrac{40}{5} \div \tfrac{3}{5} = \tfrac{40 \div 3}{5 \div 5} \\ &= (40 \div 3) \div 1 = \tfrac{40}{3} \text{ or } 13\tfrac{1}{3} \end{aligned}$$

$$\begin{aligned} n &= 14 \div 3\tfrac{3}{5} = \tfrac{70}{5} \div \tfrac{18}{5} = \tfrac{70 \div 18}{5 \div 5} \\ &= (70 \div 18) \div 1 = \tfrac{70}{18} = \tfrac{35}{9} \text{ or } 3\tfrac{8}{9} \end{aligned}$$

$$\begin{aligned} n &= 8\tfrac{2}{3} \div 5 = \tfrac{26}{3} \div \tfrac{15}{3} = \tfrac{26 \div 15}{3 \div 3} \\ &= (26 \div 15) \div 1 = \tfrac{26}{15} \text{ or } 1\tfrac{11}{15} \end{aligned}$$

Use with pages 97 through 99.

	a	*b*
1.	$8 \div \frac{2}{5} = n$ 20	27 $n = 9 \div \frac{1}{3}$
2.	$7 \div \frac{5}{6} = n$ $8\frac{2}{5}$	10 $n = 3 \div \frac{3}{10}$
3.	$12 \div 1\frac{1}{4} = n$ $9\frac{3}{5}$	$2\frac{8}{11}$ $n = 6 \div 2\frac{1}{5}$
4.	$5 \div 2\frac{2}{3} = n$ $1\frac{7}{8}$	$3\frac{3}{19}$ $n = 15 \div 4\frac{3}{4}$
5.	$9\frac{1}{5} \div 5 = n$ $1\frac{21}{25}$	$3\frac{5}{6}$ $n = 7\frac{2}{3} \div 2$
6.	$10\frac{5}{7} \div 3 = n$ $3\frac{4}{7}$	$2\frac{19}{28}$ $n = 18\frac{3}{4} \div 7$

To divide fractional numbers with different denominators, first rename the fractional numbers so that they have like denominators.

$$
\begin{aligned}
n &= \tfrac{4}{5}\div\tfrac{2}{3}\\
&= (\tfrac{4}{5}\times 1)\div(1\times\tfrac{2}{3})\\
&= (\tfrac{4}{5}\times\tfrac{3}{3})\div(\tfrac{5}{5}\times\tfrac{2}{3})\\
&= \tfrac{4\times 3}{15}\div\tfrac{5\times 2}{15}\\
&= \tfrac{(4\times 3)\div(5\times 2)}{1}\\
&= \tfrac{4\times 3}{5\times 2}=\tfrac{12}{10}=\tfrac{6}{5}\text{ or }1\tfrac{1}{5}
\end{aligned}
$$

Use with pages 100 and 101.

	a	*b*
1.	$\frac{7}{8}\div\frac{3}{5}=n$ $1\frac{11}{24}$	$1\frac{1}{5}$ $n=\frac{2}{5}\div\frac{1}{3}$
2.	$\frac{5}{6}\div\frac{5}{7}=n$ $1\frac{1}{6}$	$3\frac{1}{4}$ $n=\frac{13}{14}\div\frac{2}{7}$
3.	$\frac{9}{10}\div\frac{3}{4}=n$ $1\frac{1}{5}$	3 $n=\frac{1}{5}\div\frac{1}{15}$
4.	$\frac{5}{6}\div\frac{7}{12}=n$ $1\frac{3}{7}$	$2\frac{10}{27}$ $n=\frac{8}{9}\div\frac{3}{8}$
5.	$\frac{9}{15}\div\frac{1}{2}=n$ $1\frac{1}{5}$	$1\frac{1}{11}$ $n=\frac{8}{11}\div\frac{2}{3}$

When you divide by a fractional number, you can find the quotient by multiplying the dividend by the reciprocal of the divisor.

$$
\begin{aligned}
n &= \tfrac{3}{5}\div\tfrac{3}{8}\\
n &= \tfrac{3}{5}\times\tfrac{8}{3}\\
n &= \tfrac{24}{15}=\tfrac{8}{5}\text{ or }1\tfrac{3}{5}
\end{aligned}
$$

Use with pages 102 and 103.

	a	*b*
1.	$1\frac{3}{4}$ $n=\frac{1}{2}\div\frac{2}{7}$	$\frac{25}{32}$ $n=\frac{5}{16}\div\frac{2}{5}$
2.	$\frac{5}{8}\div\frac{2}{3}=n$ $\frac{15}{16}$	$\frac{7}{9}\div\frac{2}{5}=n$ $1\frac{17}{18}$
3.	$\frac{7}{10}\div\frac{1}{3}=n$ $2\frac{1}{10}$	$\frac{5}{6}\div\frac{1}{8}=n$ $6\frac{2}{3}$
4.	$\frac{3}{4}\div\frac{3}{7}=n$ $1\frac{3}{4}$	$1\frac{1}{2}$ $n=\frac{5}{12}\div\frac{5}{18}$
5.	$3\frac{3}{7}$ $n=\frac{4}{7}\div\frac{1}{6}$	$\frac{9}{16}\div\frac{1}{4}=n$ $2\frac{1}{4}$
6.	$\frac{2}{9}\div\frac{2}{3}=n$ $\frac{1}{3}$	$\frac{4}{15}\div\frac{3}{8}=n$ $\frac{32}{45}$
7.	$\frac{11}{12}\div\frac{3}{7}=n$ $2\frac{5}{36}$	$\frac{15}{16}\div\frac{1}{2}=n$ $1\frac{7}{8}$
8.	$\frac{12}{13}\div\frac{3}{8}=n$ $2\frac{6}{13}$	$1\frac{1}{20}$ $n=\frac{9}{10}\div\frac{6}{7}$

When the divisor is greater than the dividend, the quotient will always be a fractional number.

$$
\begin{aligned}
n &= \tfrac{5}{6}\div 1\tfrac{1}{3}\\
&= \tfrac{5}{6}\div\tfrac{4}{3}\\
&= \tfrac{5}{6}\times\tfrac{3}{4}\\
&= \tfrac{5\times 3}{6\times 4}=\tfrac{5\times 3}{4\times 6}\text{ or }\tfrac{5\times 1}{4\times 2}\text{ or }\tfrac{5}{8}
\end{aligned}
$$

Use with page 107.

	a	*b*
1.	$\frac{1}{5}\div\frac{2}{3}=n$ $\frac{3}{10}$	$\frac{7}{16}\div\frac{9}{10}=n$ $\frac{35}{72}$
2.	$\frac{7}{8}$ $n=\frac{4}{8}\div\frac{4}{7}$	$\frac{9}{16}$ $n=\frac{9}{10}\div 1\frac{3}{5}$
3.	$\frac{1}{8}\div 3\frac{3}{4}=n$ $\frac{1}{30}$	$\frac{1}{12}\div 3\frac{2}{3}=n$ $\frac{1}{44}$
4.	$\frac{7}{18}\div 9\frac{1}{2}=n$ $\frac{7}{171}$	$1\frac{2}{3}\div 2\frac{1}{2}=n$ $\frac{2}{3}$
5.	$2\frac{1}{2}\div 4\frac{1}{4}=n$ $\frac{10}{17}$	$\frac{5}{6}$ $n=5\frac{1}{3}\div 6\frac{2}{5}$
6.	$1\frac{2}{3}\div 1\frac{5}{6}=n$ $\frac{10}{11}$	$\frac{153}{182}$ $n=6\frac{3}{8}\div 7\frac{7}{12}$
7.	$7\frac{1}{8}\div 8\frac{1}{8}=n$ $\frac{57}{65}$	$\frac{57}{110}$ $n=\frac{19}{20}\div 1\frac{5}{6}$
8.	$14\frac{1}{2}\div 14\frac{7}{8}=n$ $\frac{116}{119}$	$3\frac{8}{9}\div 5\frac{5}{8}=n$ $\frac{56}{81}$
9.	$11\frac{1}{9}\div 12\frac{1}{8}=n$ $\frac{800}{873}$	$9\frac{5}{6}\div 10\frac{5}{6}=n$ $\frac{59}{65}$

Measures

Measures may be expressed in more than one form. In all cases, the = sign is used to mean *equal in measure.* See page 332 for a table of measures.

Use with pages 116 through 120.

1. 7 mi. = 2240 rd. = 36960 ft.

2. 2 qt. = 8 c. = 64 fl. oz.

Add measures in either equation or column form.

$8\frac{1}{2}+5\frac{1}{2}=n$

$14=n$

8 qt. 1 pt.
+5 qt. 1 pt.
13 qt. 2 pt. = 14 qt.

Use with page 124.

	a	b
1.	15 hr. 18 min. +27 hr. 47 min. 43 hr. 5 min.	11 bu. 3 pk. +19 bu. 1 pk. 31 bu.
2.	15 rd. 120 yd. +31 rd. 91 yd. 84 rd. 2 yd.	5 yr. 7 mo. +18 yr. 11 mo. 24 yr. 6 mo.

In subtracting measures, you may need first to rename the minuend.

4 yd. − 2 yd. 1 ft. = 3 yd. 3 ft. − 2 yd. 1 ft.
= 1 yd. 2 ft.

$4-2\frac{1}{3}=3\frac{3}{3}-2\frac{1}{3}=1\frac{2}{3}$

$1\frac{2}{3}$ yd., or 1 yd. 2 ft.

Use with page 125.

	a	b
1.	17 gal. 1 qt. −5 gal. 3 qt. 11 gal. 2 qt.	51 wk. 0 da. −25 wk. 6 da. 25 wk. 1 da.
2.	6 cwt. 17 lb. −1 cwt. 28 lb. 4 cwt. 89 lb.	10 bu. 1 pk. −7 bu. 3 pk. 2 bu. 2 pk.

In multiplying measures, multiply each unit separately.

6 ft. 8 in.
×6
36 ft. 48 in. = 40 ft.

Use with page 126.

	a	b
1.	6 yr. 5 mo. ×7 44 yr. 11 mo.	15 pk. 6 qt. ×9 141 pk. 6 qt.
2.	1 pt. 15 fl. oz. ×3 5 pt. 13 fl. oz.	15 yd. 2 ft. ×10 156 yd. 2 ft.

In dividing measures, use the equation form with fractional numbers, or use the sign) and divide each unit separately.

4 da. 6 hr. ÷ 3 = __da.__hr.

$4\frac{1}{4}\div 3=n$

$\frac{17}{4}\times\frac{1}{3}=n$

$\frac{17}{12}=n$

$1\frac{5}{12}=n$

1 da. 10 hr.

```
     1 da. 10 hr.
3) 4 da.  6 hr.
   3 da.
   1 da.  6 hr.
         30 hr.
         30 hr.
```

Use with page 127.

Find each answer by both methods.

1. 1.5 qt. 1 pt. ÷ 2 = n 1 qt. or 2 pt.

2. 8 T. 976 lb. ÷ 4 = n 2 T. 244 lb.

Problem Solving

You can express every problem as an equation. Read the problem carefully to decide whether you should add, subtract, multiply, or divide in order to solve the equation.

A jet plane flew for 4 hours at an average speed of 525 miles an hour. How many miles did it fly?

$4 \times 525 = n$ $2100 = n$

The plane flew 2100 miles.

Use with pages 131 through 137.

Write an equation for each problem. Solve it. Write the answer.

1. Jack spent $1.85 on Monday, $.77 on Tuesday, $.90 on Wednesday, and $1.28 on Thursday. What was his expenditure on the four days?

185+77+90+128=n $4.80

2. The distance by car from Chicago to San Francisco is 2199 miles; from Chicago to New York it is 824 miles. How much farther is it to San Francisco than to New York?

2199-824=n 1375 mi.

3. How far is it from San Francisco to New York by way of Chicago?

2199+824=n 3023 mi.

4. Mr. Phillips averaged 975 miles of driving a month. About how far did he drive in 3 years?

(3x12)x975=n 35100 mi.

5. The boys arranged 258 chairs in 3 equal sections. How many chairs did they put in each section?

258÷3=n 86 chairs

You can use one equation to express a problem that is solved by using more than one operation.

Peter earned $2.35 mowing lawns and $1.65 washing windows. He bought a book and had $2.73 left. How much was the book?

$(235+165) - n = 273$ $n = 127$

The book cost $1.27.

Use with pages 138 through 142.

Solve each problem below.

1. Mrs. Clark made 6 dozen doughnuts. She sold $\frac{1}{3}$ of them and put the rest in bags, with 2 in a bag. How many bags did she need?

[(6x12)x$\frac{2}{3}$]÷2=n 24 bags

2. John and his 3 brothers weighed 83 lb., 72 lb., 70 lb., and 69 lb. What was their average weight?

(83+72+70+69)÷4=n 73$\frac{1}{2}$ lb.

3. June bought 3 yards of material at $.69 a yard and a spool of thread for $.10. How much change did she get from a 10-dollar bill?

1000-[(3x69)+10] $7.83

4. The dairy packed 300 eggs in cartons of 1 dozen each and sold all the eggs for $.55 a dozen. How much did the dairy receive for them?

(300÷12)x55=n $13.75

5. Judy bought 3 pounds of steak at $.98 a pound and 18 oranges at $.60 a dozen. If 4 girls shared the cost, what did each girl pay?

[(3x98)+(1$\frac{1}{2}$x60)] ÷4=n $.96 each

Computation with Decimal Numbers

You add decimal numbers as you add whole numbers. When you add in column form, write the numerals so that the decimal points lie on a vertical line.

2.3+4.5 = n 6.8 = n	2.3 +4.5 6.8

Use with pages 147 through 151.

Copy in column form. Solve.

	a	b
1.	18.3+19.8 = n 38.1	9.3+21.8 = n 31.1
2.	31+47.9 = n 78.9	25.6+52 = n 77.6
3.	760.4+.7 = n 761.1	308.1+7.7 = n 315.8

Copy. Write each sum.

	a	b	c
4.	84.52 +49.17 133.69	90.07 +6.8 96.87	67.71 +12.89 80.60
5.	80.01 .37 +53.8 134.18	61.7 79.06 +5.32 146.08	76.53 42.17 +.09 118.79
6.	73.007 .925 46.28 +88.416 208.628	66.804 804.109 36.01 +7.548 914.471	9.452 27.3 56.293 +618.224 711.269
7.	.007 46.201 5.862 +51.99 104.060	300.605 77.911 199.02 +1.004 578.540	402.5 28.769 372.98 +100.321 904.570
8.	713.18 21.004 419.9 +1.29 1155.374	68.401 984.05 360.4 +1.083 1413.934	484.038 52.201 386.48 +5.642 928.361

You subtract decimal numbers as you subtract whole numbers. Write the numerals in column form so that the decimal points lie on a vertical line.

8.7−6.8 = 1.9 9.16−4.27 = 4.89	67.309 −57.419 9.890

Use with pages 152 and 153.

Copy in column form. Solve.

	a	b
1.	31.08−26.8 = n 4.28	91.3−87.4 = n 3.9
2.	65.3−26.17 = n 39.13	71.328−.419 = n 70.909

Copy. Write each difference.

	a	b	c
3.	623.117 −514.09 109.027	504.368 −225.798 278.570	420.006 −298.947 121.059
4.	900.008 −99.103 800.905	569.030 −508.211 60.819	141.299 −100.109 41.190
5.	237.9 −136.213 101.687	360.804 −209.175 151.629	711.067 −66.193 644.874
6.	717.354 −594.012 123.342	623.514 −387.604 235.910	404.671 −213.672 190.999

You multiply a decimal number and a whole number as you multiply whole numbers. The number of decimal places in the product and in the decimal factor is the same.

15×.09 = n
1.35 = n

321.
×.273
.963
22.47
64.2
87.633

Use with pages 154-156, 158, 159.

Copy in column form. Solve.

	a	*b*
1.	18×.29 = n 5.22	43×.707 = n 30.401
2.	25×3.469 = n 86.725	.17×98 = n 16.66
3.	.035×48 = n 1.680	.763×653 = n 498.239

When you multiply two decimal numbers, the number of decimal places in the product is the same as the sum of the decimal places in both factors.

.49×5.167 = n
2.53183 = n

8.745
×4.06
.52470
34.9800
35.50470

Use with pages 160 and 162.

Copy. Find each product.

	a	*b*	*c*	*d*
1.	.8 ×.71 .568	2.68 ×.9 2.412	.63 ×.04 .0252	.75 ×.25 .1875
2.	5.08 ×.07 .3556	.773 ×.29 .22417	6.317 ×.03 .18951	18.8 ×1.93 36.284
3.	70.19 ×63.2 4436.008	31.77 ×5.42 172.1934	129.3 ×2.75 355.575	36.99 ×7.68 284.0832
4.	1.329 ×4.63 6..15327	209.7 ×.315 66.0555	79.09 ×52.8 4175.952	5.004 ×.638 3.192552
5.	100.7 ×.864 87.0048	36.58 ×4.77 174.4866	9.998 ×7.06 70.58588	4.007 ×.003 .012021
6.	209.8 ×40.7 8538.86	6.534 ×20.9 136.5606	50.57 ×.404 20.43028	181.1 ×.675 122.2425

When you multiply a decimal number by 10, 100, or 1000, the place-value position of the digits in the product is determined by the number of zeros in the multiplier.

8.1 ×10 81.0	8.1 ×100 810.0	8.1 ×1000 8100.0

Use with page 164.

Copy in column form. Solve.

	a	*b*
1.	10×.7 = n 7.0	10×.04 = n .4
2.	100×.5 = n 50.0	10×.802 = n 8.02
3.	100×.218 = n 21.8	1000×7.6 = n 7600
4.	10×31.02 = n 310.2	100×9.06 = n 906
5.	1000×4.009 = n 4009	100×57.08 = n 5708
6.	100×73.9 = n 7390	1000×52.39 = n 52390

Using Decimal Numbers in Division

You divide tenths in the same way that you divide ones.

$.8 \div 4 = .2$
$1.6 \div .4 = 4$

Use with page 171.

Copy. Solve for n.

	a	*b*
1.	$.8 \div 2 = n$ 4	$6 \div .3 = n$ 20
2.	$2.4 \div .6 = n$ 4	$3.0 \div .3 = n$ 10
3.	$.9 \div 3 = n$ .3	$1.8 \div .9 = n$ 2
4.	$2.7 \div 9 = n$ .3	$1.4 \div .7 = n$ 2
5.	$n \times .4 = 1.2$ 3	$n \times .3 = 2.1$ 7

You can find an exact decimal equivalent of a fractional number when the denominator of the fractional number in reduced form divides 10, 100, 1000, and so on.

$$\frac{2}{5} = \frac{2 \times 2}{2 \times 5} = \frac{4}{10} = .4 \qquad \begin{array}{r} .4 \\ 5\overline{)2.0} \\ \underline{2\,0} \end{array}$$

Use with pages 172 through 174.

	a	*b*	*c*	*d*	*e*
1.	$\frac{6}{25}$ .24	$\frac{4}{125}$ .032	$\frac{7}{200}$ .035	$\frac{3}{50}$ .06	$\frac{9}{20}$ .45
2.	$\frac{7}{40}$ .175	$\frac{119}{250}$ .476	$\frac{8}{25}$ .32	$\frac{9}{250}$ .036	$\frac{11}{500}$ .022
3.	$\frac{7}{10}$ .70	$\frac{13}{20}$ .65	$\frac{17}{200}$ .085	$\frac{19}{50}$ .38	$\frac{7}{60}$ .116...
4.	$\frac{9}{75}$ .12	$\frac{12}{40}$ .3	$\frac{18}{150}$ .12	$\frac{3}{500}$ .006	$\frac{60}{75}$ .8
5.	$\frac{10}{250}$ .04	$\frac{375}{500}$ .75	$\frac{3}{5}$ .6	$\frac{17}{50}$ .34	$\frac{21}{25}$ .84
6.	$\frac{38}{80}$ .475	$\frac{75}{90}$ .833...	$\frac{17}{40}$ .425	$\frac{91}{250}$ .364	$\frac{199}{500}$ .398

When the denominator of a fractional number in reduced form does not divide 10, 100, 1000, and so on, the decimal equivalent of the fractional number will be approximate.

$$\frac{7}{15} = 7 \div 15 = .466\cdots \qquad \begin{array}{r} .466 \\ 15\overline{)7.000} \\ \underline{6\,0} \\ 1\,00 \\ \underline{90} \\ 100 \\ \underline{90} \\ 10 \end{array}$$

Use with pages 175 and 176.

Find the approximate decimal equivalent of each fraction, expressed in thousandths.

	a	*b*	*c*	*d*	*e*
1.	$\frac{19}{45}$ .422...	$\frac{17}{37}$ .459	$\frac{5}{19}$ .263	$\frac{11}{13}$ .846	$\frac{25}{59}$ .424
2.	$\frac{9}{11}$ .818...	$\frac{2}{3}$ .666...	$\frac{4}{9}$ .444...	$\frac{18}{21}$ .857	$\frac{14}{23}$ .608
3.	$\frac{59}{61}$ .967	$\frac{17}{90}$ .188...	$\frac{79}{80}$ .988	$\frac{42}{43}$ .977	$\frac{31}{60}$ .516...
4.	$\frac{5}{7}$ .714	$\frac{20}{21}$ .952	$\frac{67}{71}$ .945	$\frac{8}{63}$ .127	$\frac{5}{56}$ .089
5.	$\frac{17}{39}$ .436	$\frac{8}{35}$ .229	$\frac{7}{81}$ .086	$\frac{3}{14}$ .214	$\frac{11}{15}$ .733...
6.	$\frac{11}{18}$ .611...	$\frac{17}{54}$ .315	$\frac{42}{43}$ .977	$\frac{75}{76}$ .987	$\frac{2}{17}$ .118
7.	$\frac{7}{12}$ .583	$\frac{2}{51}$ .039	$\frac{17}{29}$ .586	$\frac{8}{27}$ .296	$\frac{11}{39}$ .282

Divide decimal numbers by whole numbers as you divide whole numbers.

$17.4 \div 6 = n$
$2.9 = n$

$$\begin{array}{r} 2.9 \\ 6\overline{)17.4} \\ \underline{12} \\ 5\ 4 \\ \underline{5\ 4} \end{array}$$

Check $\begin{array}{r} 2.9 \\ \underline{\times 6} \\ 17.4 \end{array}$

Use with page 177.

Solve, using the sign $\overline{)}$. Check.

	a	*b*
1.	$44.8 \div 7 = \overset{6.4}{n}$	$89.4 \div 6 = \overset{14.9}{n}$
2.	$228.6 \div 9 = \overset{25.4}{n}$	$79.32 \div 12 = \overset{6.61}{n}$
3.	$804.0 \div 24 = \overset{33.5}{n}$	$637.2 \div 64 = \overset{9.956}{n}$

You can continue division to rename a remainder as a decimal number.

$$\begin{array}{r} 3.5 \\ 4\overline{)14.0} \\ \underline{12} \\ 2\ 0 \\ \underline{2\ 0} \end{array} = 3\tfrac{5}{10} \text{ or } 3\tfrac{1}{2}$$

Check $\begin{array}{r} 3.5 \\ \underline{\times 4} \\ 14.0 \end{array}$

Use with page 178.

Solve. Write each remainder in decimal form.

	a	*b*	*c*
1.	$\overset{5.875}{8\overline{)47}}$	$\overset{13.5}{6\overline{)81}}$	$\overset{6.125}{8\overline{)49}}$
2.	$\overset{16.5}{32\overline{)528}}$	$\overset{6.75}{28\overline{)189}}$	$\overset{4.625}{64\overline{)296}}$
3.	$\overset{2.125}{56\overline{)119}}$	$\overset{32.8}{75\overline{)2460}}$	$\overset{57.2}{45\overline{)2574}}$
4.	$\overset{52.5}{32\overline{)1680}}$	$\overset{171.25}{48\overline{)8220}}$	$\overset{109.5}{14\overline{)1533}}$

When the divisor is a decimal number, change it to its whole number form by multiplying by 1 in its most convenient form.

$$.96 \div .08 = \tfrac{100}{100} \times \tfrac{.96}{.08} = \tfrac{96}{8} = 12$$

$$1.44 \div 1.2 = \tfrac{10}{10} \times \tfrac{1.44}{1.2} = \tfrac{14.4}{12} = 1.2$$

Use with pages 179 through 181.

Copy. Write each quotient.

	a	*b*
1.	$\overset{18.1}{.46\overline{)8.326}}$	$\overset{315.}{.54\overline{)170.1}}$
2.	$\overset{210.}{3.98\overline{)835.8}}$	$\overset{455.}{1.8\overline{)819}}$
3.	$\overset{6.1}{.125\overline{).7625}}$	$\overset{2.375}{3.20\overline{)7.60}}$
4.	$\overset{11.4}{.012\overline{).1368}}$	$\overset{210.}{.199\overline{)41.79}}$

When you divide a decimal number by 10, 100, or 1000, you determine the place-value position of the digits in the quotient by counting the number of zeros in the divisor.

$12 \div 10 = 1.2$ $\qquad$ $12 \div 1000 = .012$

Use with page 182.

Solve for n.

	a	*b*
1.	$673 \div 10 = \overset{67.3}{n}$	$376 \div 100 = \overset{3.76}{n}$
2.	$42.8 \div 10 = \overset{4.28}{n}$	$9 \div 1000 = \overset{.009}{n}$
3.	$12.60 \div 100 = \overset{.1260}{n}$	$4.28 \div 1000 = \overset{.00428}{n}$
4.	$2675 \div 100 = \overset{26.75}{n}$	$.125 \div 10 = \overset{.0125}{n}$

Use a fraction to express a ratio, or correspondence between the numbers of 2 sets.

1 boat for 4 men	1 to 4	$\frac{1}{4}$
3 boats for 12 men	3 to 12	$\frac{3}{12}$

Use with pages 187 through 189.

Express the following as ratios in fraction form.

1. 3 pencils for 10 cents $\frac{3}{10}$

2. 5 pies for 30 people $\frac{5}{30}$

3. 1 star for 3 perfect scores $\frac{1}{3}$

4. 5 gallons for 90 miles $\frac{5}{90}$

5. 60 eggs for 5 cartons $\frac{60}{5}$

You can express equality between 2 ratios by using a proportion.

3 × 12

$\frac{3}{7}=\frac{\overset{36}{n}}{84}$ = 12 $\frac{3}{7}=\frac{36}{84}$ 5/ $\frac{.35}{.60}=\frac{7}{12}$

Use with pages 190 through 195.

Solve each proportion below.

	a	*b*	*c*
1.	$\frac{3}{8}=\frac{21}{n}$ 56	$\frac{2}{9}=\frac{n}{54}$ 12	$\frac{n}{112}=\frac{8}{14}$ 64
2.	$\frac{12}{1}=\frac{36}{n}$ 3	$\frac{52}{13}=\frac{n}{1}$ 4	$\frac{27}{n}=\frac{3}{4}$ 36
3.	$\frac{7}{21}=\frac{n}{168}$ 56	$\frac{38}{3}=\frac{n}{15}$ 190	$\frac{15}{90}=\frac{5}{n}$ 30
4.	$\frac{88}{n}=\frac{44}{51}$ 102	$\frac{n}{27}=\frac{76}{81}$ $25\frac{1}{3}$	$\frac{49}{147}=\frac{n}{3}$ 1
5.	$\frac{308}{154}=\frac{n}{22}$ 44	$\frac{7}{8}=\frac{28}{n}$ 32	$\frac{144}{36}=\frac{n}{4}$ 16

	a	*b*	*c*
6.	$\frac{n}{100}=\frac{3}{5}$ 60	$\frac{64}{n}=\frac{1}{4}$ 256	$\frac{225}{297}=\frac{n}{9}$ $6\frac{9}{11}$
7.	$\frac{1}{8}=\frac{11}{n}$ 88	$\frac{14}{n}=\frac{2}{3}$ 21	$\frac{42}{98}=\frac{n}{7}$ 3

Express each ratio below in simplest fractional form.

8. \$.75 to \$1.95 $\frac{5}{13}$ 15/ $\frac{.75}{1.95}=\frac{5}{13}$

9. \$.98 to \$5.46 $\frac{7}{39}$

10. \$10.00 for 8 hours $\frac{5}{4}$ 4/ $\frac{10.00}{8}=\frac{5}{4}$

Write each ratio below as a common fraction in lowest terms.

	a	*b*
11.	$\frac{.06}{.96}=\frac{n}{1.92}$ .12	$\frac{.12}{.84}=\frac{n}{2.52}$ .36
12.	$\frac{16}{.25}=\frac{n}{200}$ 12800	$\frac{.08}{12}=\frac{n}{288}$ 1.92
13.	$\frac{1.08}{9.72}=\frac{2.16}{n}$ 19.44	$\frac{10.00}{1.25}=\frac{n}{3.75}$ 30
14.	$\frac{.77}{1.32}=\frac{1.54}{n}$ 2.64	$\frac{60}{7.20}=\frac{120}{n}$ 14.40

To discover whether 2 ratios are equivalent, reduce both fractions or give them a common denominator.

$\frac{10}{12}=\frac{5}{6}$ and $\frac{15}{18}=\frac{5}{6}$, or

$\frac{10}{12}=\frac{30}{36}$ and $\frac{15}{18}=\frac{30}{36}$

Then $\frac{10}{12}=\frac{15}{18}$

Use with page 196.

Using either method, determine whether each pair of ratios on page 313 is equivalent. Then write each pair of ratios, replacing the comma with = or ≠, whichever is correct.

	a	*b*	*c*
1.	$\frac{10}{18}, = \frac{15}{27}$	$\frac{21}{96}, \neq \frac{3}{8}$	$\frac{2}{3}, \neq \frac{5}{9}$
2.	$\frac{25}{50}, = \frac{17}{34}$	$\frac{3}{5}, \neq \frac{4}{7}$	$\frac{11}{13}, = \frac{66}{78}$
3.	$\frac{57}{228}, = \frac{56}{224}$	$\frac{1}{6}, = \frac{18}{108}$	$\frac{50}{8}, \neq \frac{75}{24}$
4.	$\frac{98}{21}, = \frac{154}{33}$	$\frac{2}{3}, \neq \frac{3}{5}$	$\frac{15}{16}, = \frac{240}{256}$
5.	$\frac{34}{28}, \neq \frac{69}{56}$	$\frac{23}{25}, = \frac{115}{125}$	$\frac{6}{1}, \neq \frac{90}{14}$
6.	$\frac{15}{98}, = \frac{45}{294}$	$\frac{7}{8}, \neq \frac{81}{96}$	$\frac{21}{1}, \neq \frac{105}{6}$
7.	$\frac{39}{13}, = \frac{27}{9}$	$\frac{72}{6}, = \frac{12}{1}$	$\frac{48}{50}, \neq \frac{24}{26}$
8.	$\frac{25}{625}, = \frac{5}{125}$	$\frac{17}{34}, = \frac{96}{192}$	$\frac{9}{216}, = \frac{3}{72}$

When ratios are expressed in equation form, you solve the equation by multiplying each numerator by the denominator of the other fraction.

$$\frac{7}{15} = \frac{n}{75}$$

$$15 \times n = 75 \times 7 = 525$$

$$n = 525 \div 15 = 35$$

Therefore $\frac{7}{15} = \frac{35}{75}$

Use with page 197.

Copy each equation below. Solve.

	a	*b*	*c*
1.	$\frac{5}{2} = \frac{60}{n}$ 24	$\frac{80}{n} = \frac{2}{3}$ 120	$\frac{n}{18} = \frac{4}{9}$ 8
2.	$\frac{8}{3} = \frac{n}{57}$ 152	$\frac{n}{95} = \frac{2}{38}$ 5	$\frac{5}{72} = \frac{n}{36}$ 2.5
3.	$\frac{n}{32} = \frac{8}{4}$ 64	$\frac{48}{12} = \frac{36}{n}$ 9	$\frac{15}{21} = \frac{80}{n}$ 112
4.	$\frac{2}{7} = \frac{36}{n}$ 126	$\frac{9}{24} = \frac{n}{64}$ 24	$\frac{25}{n} = \frac{70}{98}$ 35
5.	$\frac{n}{8} = \frac{15}{4}$ 30	$\frac{12}{26} = \frac{72}{n}$ 156	$\frac{49}{84} = \frac{n}{168}$ 98
6.	$\frac{5}{2} = \frac{n}{112}$ 280	$\frac{n}{21} = \frac{133}{147}$ 19	$\frac{6}{9} = \frac{40}{n}$ 60
7.	$\frac{51}{n} = \frac{17}{44}$ 132	$\frac{7}{8} = \frac{n}{128}$ 112	$\frac{12}{17} = \frac{132}{n}$ 187
8.	$\frac{13}{n} = \frac{4}{14}$ 45.5	$\frac{49}{50} = \frac{n}{600}$ 588	$\frac{280}{70} = \frac{4}{n}$ 1

When you know the measure of a scale drawing, you may use ratio and proportion to find the measure of the real object.

Where $\frac{1}{2}$ inch on the drawing represents 1 foot, how long is the object in inches if the drawing is 5 inches long?

$$\frac{\frac{1}{2}}{12} = \frac{5}{n}$$

$$n \times \frac{1}{2} = 5 \times 12$$

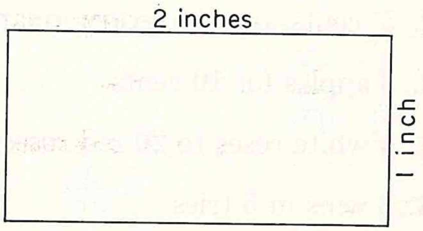
$$n \times \frac{1}{2} = 60$$

$$n = 60 \div \frac{1}{2} = 120$$

The object is 120 inches long.

Use with pages 200 through 202.

In the scale drawing of a garden plot below, each $\frac{1}{4}$ inch represents 1 yard. Write the equation you would use to find the length and the width of the garden. Solve the equation. See below.



$$\frac{\frac{1}{4}}{1} = \frac{2}{n}$$ 8 yd. long

$$\frac{\frac{1}{4}}{1} = \frac{1}{n}$$ 4 yd. wide

Fractional or decimal numbers expressed in hundredths can be read as ratios or as per cents (%).

$\frac{3}{100}$ or $.03 = 3$ to $100 = 3\%$

$\frac{131}{100}$ or $1.31 = 131$ to $100 = 131\%$

Use with pages 207 through 209.

Write each of the following as a ratio, and then as per cent using the % sign. See below.

	a	*b*	*c*	*d*
1.	.28	1.97	.01	2.00
2.	.15	.07	3.00	1.01
3.	.01	.99	.30	.77

To use per cent in comparison, rename each ratio so that it has a denominator of 100.

$\frac{1}{2} = \frac{50}{100} = 50\%$ $\quad$ $\frac{3}{5} = \frac{60}{100} = 60\%$

Use with pages 210 and 211.

Write the following as ratios. For each one, find an equivalent ratio with a denominator of 100.

1. 1 book for every two pupils $\frac{1}{2} = \frac{50}{100}$

2. 5 cents out of every quarter $\frac{5}{25} = \frac{20}{100}$

3. 3 apples for 10 cents $\frac{3}{10} = \frac{30}{100}$

4. 4 white roses to 20 red roses $\frac{4}{20} = \frac{20}{100}$

5. 3 wins in 5 tries $\frac{3}{5} = \frac{60}{100}$

To use a ratio to find per cent, find the equivalent ratio and translate the problem into a proportion.

$\frac{72}{180} = \frac{n}{100}$	$\frac{75}{120} = \frac{n}{100}$
$180n = 7200$	$120n = 7500$
$n = \frac{7200}{180} = 40$	$n = \frac{7500}{120} = 62.5$
$\frac{40}{100} = 40\%$	$\frac{62.5}{100} = 62.5\%$
	or $62\frac{1}{2}\%$

Use with pages 214 and 215.

Solve for n.

	a	*b*	*c*
1.	$\frac{8}{5} = \frac{n}{100}$ 160	$\frac{25}{40} = \frac{n}{100}$ 62.5	$\frac{16}{125} = \frac{n}{100}$ 12.8
2.	$\frac{112}{28} = \frac{n}{100}$ 400	$\frac{57}{171} = \frac{n}{100}$ $33\frac{1}{3}$	$\frac{150}{60} = \frac{n}{100}$ 250

When you know the per cent, use proportion to find the number to which another number is compared.

Sue misspelled 20% of 90 words. How many words did she misspell?

$$\frac{20}{100} = \frac{n}{90}$$

$$100n = 20 \times 90 = 1800$$

$$n = \frac{1800}{100} = 18$$

Sue misspelled 18 words.

Use with pages 216 through 218.

	a	*b*	*c*
1.	$\frac{40}{100} = \frac{36}{n}$ 90	$\frac{15}{100} = \frac{n}{900}$ 135	$\frac{21}{100} = \frac{63}{n}$ 300
2.	$\frac{28}{n} = \frac{14}{100}$ 200	$\frac{115}{n} = \frac{46}{100}$ 250	$\frac{n}{70} = \frac{30}{100}$ 21
3.	$\frac{n}{210} = \frac{70}{100}$ 147	$\frac{8}{100} = \frac{n}{450}$ 36	$\frac{36}{100} = \frac{108}{n}$ 300

1. $\frac{28}{100}$, 28%; $\frac{197}{100}$, 197%; $\frac{1}{100}$, 1%; $\frac{200}{100}$, 200%
2. $\frac{15}{100}$, 15%; $\frac{7}{100}$, 7%; $\frac{300}{100}$, 300%; $\frac{101}{100}$, 101%
3. $\frac{1}{100}$, 1%; $\frac{99}{100}$, 99%; $\frac{30}{100}$, 30%; $\frac{77}{100}$, 77%

Equivalent fractions and decimals can be written as per cents.

$\frac{1}{5} = \frac{n}{100} = \frac{20}{100} = .20 = 20\%$

$75\% = .75 = \frac{75}{100} = \frac{3}{4}$

Use with pages 223 through 225.

Write an equivalent decimal and per cent for each fraction below.

	a	*b*	*c*	*d*
1.	$\frac{3}{4}$ .75 75%	$\frac{1}{9}$ .1111 11.11%	$\frac{7}{10}$ .7 70%	$\frac{50}{50}$ .1 100%
2.	$\frac{5}{12}$ $.41\frac{2}{3}$ $41\frac{2}{3}\%$	$\frac{4}{5}$ .80 80%	$\frac{1}{3}$ .333... $33\frac{1}{3}\%$	$\frac{7}{20}$ .35 35%

Write the decimal and the fractional form of each per cent below.
See below.

	a	*b*	*c*	*d*
3.	125%	$12\frac{1}{2}\%$	$33\frac{1}{3}\%$	20%
4.	7.5%	$8\frac{1}{3}\%$	8%	50%

In computation, you can use either the decimal or the fractional form of the per cent.

$n = 50\%$ of 120
$n = .50 \times 120$
$n = 60$

80% of $n = 24$
$\frac{4}{5}$ of $n = 24$
$n \times \frac{4}{5} = 24$
$n = 24 \div \frac{4}{5}$
$n = 30$

Use with page 230.

Solve, using either form.

	a	*b*
1.	$12\frac{1}{2}\%$ of $40 = n$ 5	35% of $121 = n$ 42.35
2.	40% of $250 = n$ 100	19% of $300 = n$ 57
3.	$33\frac{1}{3}\%$ of $915 = n$ 305	75% of $48 = n$ 36
4.	20% of $95 = n$ 19	66% of $132 = n$ 87.12
5.	7% of $96 = n$ 6.72	23% of $260 = n$ 59.8

Use with page 232.

Solve for n.

	a	*b*
1.	50% of $n = 78$ 156	30% of $n = 150$ 500
2.	$33\frac{1}{3}\%$ of $n = 208$ 624	11% of $n = 57$ $518\frac{2}{11}$
3.	$16\frac{2}{3}\%$ of $n = 48$ 288	92% of $n = 575$ 625
4.	$12\frac{1}{2}\%$ of $n = 77$ 616	10% of $n = 79$ 790
5.	$24\frac{1}{2}\%$ of $n = 375$ 1530.61...	19% of $n = 323$ 1700

Round off a per cent either as a decimal or as a fraction to estimate an answer.

$n = 18\%$ of 60
Estimate: $n = .20 \times 60 = 12$
Answer: $n = .18 \times 60 = 10.8$

Use with page 233.

Write an equation to use in estimating an answer to each of the following. Then write the equation to compute the answer. Solve both equations.

.24x84=n 20.16
1. What number is 24% of 84?
54÷.72=n 75
2. 54 is 72% of what number?
.48x916=n 439.68
3. What number is 48% of 916?
.09x90=n 8.1
4. What number is 9% of 90?

3. 1.25, $\frac{5}{4}$; .125, $\frac{1}{8}$; .333... , $\frac{1}{3}$; .20, $\frac{1}{5}$

4. .075, $\frac{3}{40}$; .083... , $\frac{8\frac{1}{3}}{100}$; .08, $\frac{2}{25}$; .50, $\frac{1}{2}$

Find the perimeter of a rectangle or other polygon by finding the sum of the measures of its sides.

Find the perimeter of a rectangle 4 ft. long and 2 ft. wide.

$$p=2\times(l+w)$$
$$p=2\times(4+2)$$
$$p=2\times6=12$$

The perimeter is 12 feet.

Use with pages 253 through 255 and page 257.

Write an equation to use in finding the perimeter of each of the following. Solve each equation.

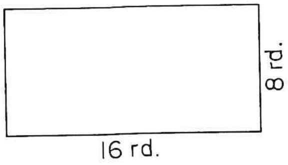
1. Rectangle, 17 mi. by 19 mi.
   2x(19+17)=n 72 mi.
2. Square, 51.5 rd. wide
   4x51.5=n 206 rd.
3. Triangle, 6 ft. by 8 ft. by 4 ft.
   6+8+4=n 18 ft.
4. Equilateral triangle, 9 ft. wide
   3x9=n 27 ft.

Find each perimeter.

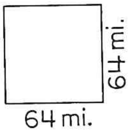
See below.

5.

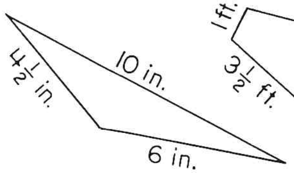
a

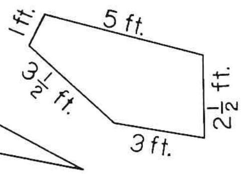


b



6.





5. a. 2x(16+8)=n 48 rd.
   b. 4x64=n 256 mi.
6. a. $10+4\frac{1}{2}+6=n$ $20\frac{1}{2}$ in.
   b. $5+2\frac{1}{2}+3+3\frac{1}{2}+1=n$ 15 ft.

You can use the formula for finding the perimeter to find an unknown dimension of a rectangle.

The perimeter of a rectangle is 24 ft. and the width is 5 ft. What is the length?

$$p=2\times(l+w)$$
$$24=2\times(l+5)$$
$$\frac{24}{2}=l+5$$
$$12=l+5$$
$$12-5=l$$
$$7=l$$

The length is 7 feet.

Use with page 256.

Solve for l (length), w (width), or s (side) in each equation below.

1. $2\times(17+l)=52$ 9
2. $68=4\times s$ 17
3. $364=2\times(w+96)$ 86
4. $4s=1752$ 438
5. $936=4s$ 234
6. $5280=2\times(320+l)$ 2320
7. $2\times(w+91.7)=403$ 109.8
8. $670.8=2\times(w+284)$ 51.4
9. $2\times(11.5+l)=860$ 418.5
10. $484=2\times(112+l)$ 130

Find the area of a rectangle or a square by multiplying the measures of the length and the width. Express area in square measure.

What is the area of a rectangle 4 ft. long and 2 ft. wide?

$$A = lw$$
$$A = 4 \times 2$$
$$A = 8$$

The area is 8 square feet.

Use with pages 258 through 261.

Find the area of rectangles with the following dimensions.

	a	b
1.	240 sq. yd. 15 yd. by 16 yd.	3380 sq. in. 52 in. by 65 in.
2.	$43\frac{5}{16}$ sq. mi. $4\frac{1}{2}$ mi. by $9\frac{5}{8}$ mi.	76.5 sq. ft. 8.5 ft. by 9 ft.
3.	52 sq. yd. $6\frac{1}{2}$ yd. by 8 yd.	8 sq. mi. $1\frac{1}{2}$ mi. by $5\frac{1}{3}$ mi.
4.	$44\frac{7}{9}$ sq. yd. 31 ft. by $4\frac{1}{3}$ yd.	77.6 sq. mi. 8 mi. by 9.7 mi.
5.	14.7 sq. yd. 6.3 ft. by 7 yd.	3.6 sq. mi. 1.8 mi. by 2 mi.

Find the area of squares with these dimensions.

	a	b
6.	5184 sq. in. 72 inches	1600 sq. rd. 40 rods
7.	324 sq. ft. 18 feet	24336 sq. yd. 156 yards
8.	48841 sq. mi. 221 miles	342.25 sq. mi. 18.5 miles
9.	391500.49 sq. yd. 625.7 yards	270.920.25 sq. ft. $520\frac{1}{2}$ feet
10.	$184\frac{24}{25}$ sq. mi. $13\frac{3}{5}$ miles	$833\frac{49}{64}$ sq. yd. $28\frac{7}{8}$ yards

Find the volume of a rectangular solid by multiplying the measures of its length, width, and height. Express volume in cubic measure.

What is the volume of a box that is 10 in. long, 9 in. wide, and 4 in. high?

$$V = lwh$$
$$V = 10 \times 9 \times 4$$
$$V = 360$$

The volume of the box is 360 cubic inches.

Use with pages 264 and 265.

Find the volume of rectangular solids having these dimensions.

1. 16 ft., 10 ft., 8 ft. 1280 cu. ft.

2. 21 yd., $15\frac{1}{2}$ yd., 8 yd. 2604 cu. yd.

3. $18\frac{2}{3}$ yd., $16\frac{1}{6}$ yd., 14 yd. $4224\frac{8}{9}$ cu. yd.

4. 59.2 in., 27.7 in., 13.3 in. 21809.872 cu. in.

5. 65 ft., 31.6 ft., 7.3 ft. 14994.2 cu. ft.

Copy. Replace each blank with the numeral that makes each statement below true.

6. $7\frac{1}{2}$ cu. ft. = 12960 cu. in.

7. 21 cu. yd. = 567 cu. ft.

8. 12096 cu. in. = 7 cu. ft.

9. 3 cu. yd. = 81 cu. ft.

10. 6 cu. ft. = 10368 cu. in.

The Metric System

Learn to use metric units to measure length and area, liquids, and weight.

100 centimeters = 1 meter
10 deciliters = 1 liter
1000 milligrams = 1 gram

Use with pages 283 through 289.

Copy. Replace each blank with a numeral so that each statement becomes true. (Remember that = means equal in measure.)

1. 30 mm. = 3 cm.
2. 5 dm. = 50 cm.
3. 4 km. = 4000 m.
4. 2 sq. m. = 200 sq. dm.
5. 13.5 m. × 8.0 m. = 108 sq. m.
6. 15 km. × 14.5 km. = 217.5 sq. km.
7. 1 l. = 100 cl.
8. 7 kl. = 7000 l.
9. 100 l. = 1000 dl.
10. 2000 g. = 2 kg.
11. $\frac{7}{1000}$ g. = 7 mg.
12. $\frac{16}{100}$ g. = 16 cg.
13. $\frac{2}{10}$ l. = 2 dl.
14. $\frac{3}{100}$ g. = 3 cg.

Copy. Convert each metric unit to its approximate equivalent in the English system of measures, as indicated below.

15. 1000 m. = $1083\frac{1}{3}$ yd.
16. 800 km. = 500 mi.
17. 23 m. = 897 in.
18. 2 l. = 2.2 qt.
19. 21 l. = 23.1 qt.
20. 1 kl. = 2200 pt.
21. 10 kg. = 22 lb.
22. 2 metric T. = 4400 lb.
23. 100 g. = 3.52 oz.

Learn to read the temperature scale on both a Fahrenheit and a centigrade thermometer.

97°F. means 97 degrees Fahrenheit

4°C. means 4 degrees centigrade

Use with page 290.

Write a numeral to indicate the change in temperature between each pair of readings below.

1. −10° and 16°C. 26°
2. 13° and 100°C. 87°
3. −18° and 212°F. 230°

Diagnostic Self-Tests

The tests on pages 319 through 322 of your book will help you and your teacher see how much you have learned in arithmetic. Do each test carefully, and check your work before you give your paper to your teacher.

Self-Test 1—Whole Numbers

Copy the following expressions and blanks. Replace each dot with either = or ≠, whichever sign will make the sentence true. On each blank write *commutative, associative, distributive, not commutative,* or *not associative,* as appropriate.

1. $56+78 \overset{=}{\bullet} 78+56$ Comm.
2. $116-97 \overset{\neq}{\bullet} 97-116$ Not comm.
3. $(9+8)+12 \overset{=}{\bullet} 9+(8+12)$ Assoc.
4. $18\times33 \overset{=}{\bullet} 33\times18$ Comm.
5. $(4\times9)\times10 \overset{=}{\bullet} 4\times(9\times10)$ Assoc.
6. $150\div25 \overset{\neq}{\bullet} 25\div150$ Not comm.

Copy. Perform the indicated operation. Check.

	a	*b*	*c*
7.	51 $7\overline{)357}$	106.5 $6\overline{)639}$	925.8 $5\overline{)4629}$
8.	9422r1 $9\overline{)84799}$	5 $19\overline{)95}$	6 $42\overline{)252}$
9.	23 $23\overline{)529}$	80 $32\overline{)2560}$	$2976\frac{5}{17}$ $17\overline{)50597}$
10.	52 $26\overline{)1352}$	518 $19\overline{)9842}$	97r328 $357\overline{)34957}$

	a	*b*	*c*
11.	9100 646 +231 9977	2324 1139 +5227 8690	3132 2486 +1164 6782
12.	1357 3215 1496 +2987 9055	2897 5986 3754 +2978 15615	96321 37405 29617 +38429 201772
13.	83214 2156 314 +1798 87482	76154 4371 2189 +1576 84290	97201 48253 59368 +60897 265719
14.	5268 −243 5025	7815 −903 6912	6932 −3481 3451
15.	14625 −1890 12735	85432 −1987 83445	87806 −49987 37819
16.	242 ×2 484	273 ×3 819	890 ×5 4450
17.	2578 ×3 7734	65092 ×9 585828	13 ×15 195
18.	467 ×24 11208	1321 ×32 42272	7867 ×45 354015
19.	122 ×412 50264	8300 ×232 1925600	76198 ×765 58291470

Self-Test 2—Fractional Numbers

Copy. Solve for n. The replacement for n should be in lowest terms.

	a	*b*
1.	$4\frac{1}{5}\div3\frac{3}{5}=n$ $1\frac{1}{6}$	$n=7\frac{4}{9}+2\frac{2}{9}$ $9\frac{2}{3}$
2.	$n=1\frac{1}{2}+4\frac{3}{4}$ $6\frac{1}{4}$	$n=9\frac{2}{3}+7\frac{1}{2}$ $17\frac{1}{6}$
3.	$6\frac{9}{10}-3\frac{7}{10}=n$ $3\frac{1}{5}$	$9\frac{5}{6}-3\frac{1}{12}=n$ $6\frac{3}{4}$
4.	$67-29\frac{5}{16}=n$ $37\frac{11}{16}$	$n=19\frac{1}{8}-12\frac{3}{5}$ $6\frac{21}{40}$
5.	$9\times\frac{3}{4}=n$ $6\frac{3}{4}$	$\frac{2}{3}\times17=n$ $11\frac{1}{3}$
6.	$5\frac{1}{4}\times2=n$ $10\frac{1}{2}$	$\frac{1}{6}\times\frac{3}{5}=n$ $\frac{1}{10}$
7.	$\frac{3}{4}\times\frac{2}{3}=n$ $\frac{1}{2}$	$9\frac{1}{4}\times\frac{5}{6}=n$ $7\frac{17}{24}$
8.	$n=6\frac{1}{8}\times2\frac{1}{3}$ $14\frac{7}{24}$	$n=\frac{1}{2}\times\frac{5}{6}\times\frac{7}{8}$ $\frac{35}{96}$
9.	$n=\frac{6}{8}\div\frac{3}{8}$ 2	$n=4\frac{5}{6}\div\frac{5}{6}$ $5\frac{4}{5}$
10.	$6\frac{1}{3}\div2\frac{1}{3}=n$ $2\frac{5}{7}$	$9\div\frac{4}{9}=n$ $20\frac{1}{4}$
11.	$n=12\div3\frac{1}{4}$ $3\frac{9}{13}$	$n=7\frac{1}{2}\div5$ $1\frac{1}{2}$
12.	$\frac{3}{4}\div\frac{3}{8}=n$ 2	$n=\frac{2}{3}\div\frac{2}{5}$ $1\frac{2}{3}$
13.	$\frac{1}{3}\div1\frac{1}{4}=n$ $\frac{4}{15}$	$n=9\frac{5}{6}\div2\frac{3}{4}$ $3\frac{19}{33}$
14.	$35\frac{1}{6}\div7\frac{1}{4}=n$ $4\frac{74}{87}$	$64\frac{8}{9}\div8\frac{66}{7}=n$ $3\frac{397}{549}$

Self-Test 3—Decimal Numbers

Copy. Write each sum or difference as indicated by the operational sign.

	a	*b*	*c*
1.	$\begin{array}{r}461.1\\31.3\\+701.5\\\hline 1193.9\end{array}$	$\begin{array}{r}8.08\\.26\\+1.34\\\hline 9.68\end{array}$	$\begin{array}{r}28.96\\59.04\\+30.41\\\hline 118.41\end{array}$
2.	$\begin{array}{r}19.8\\-7.6\\\hline 12.2\end{array}$	$\begin{array}{r}86.4\\-48.5\\\hline 37.9\end{array}$	$\begin{array}{r}90.08\\-63.59\\\hline 26.49\end{array}$
3.	$\begin{array}{r}47.205\\-8.225\\\hline 38.980\end{array}$	$\begin{array}{r}716.071\\-127.172\\\hline 588.899\end{array}$	$\begin{array}{r}238.411\\-139.522\\\hline 98.889\end{array}$

Copy. Solve for n.

	a	*b*
4.	$n=4\times2.21$ 8.84	$6\times2.206=n$ 13.236
5.	$22\times9.2=n$ 202.4	$n=.2\times74$ 14.8
6.	$.005\times75=n$ .375	$n=.135\times625$ 84.375
7.	$n=1.5\times2.5$ 3.75	$1.35\times2.07=n$ 2.7945
8.	$n=1.8\div.9$ 2	$47.97\div3=n$ 15.99
9.	$123.2\div14=n$ 8.8	$860.0\div40=n$ 21.5
10.	$n=1.322\div.18$ 7.344...	$n=10\div2.5$ 4
11.	$n=8.40\div3.50$ 2.4	$n=.273\div.003$ 91
12.	$.026\div.104=n$ .25	$n=5.852\div.209$ 28

Self-Test 4—Measures

Copy. Replace each blank with the numeral that makes the statement true. (= means *is equivalent to.*)

1. 5300 ft. = 321 rd. and ~~3½~~ ft. 7 ft.
2. 725 yd. = 2175 ft. = 26100 in. 725×12 725×36
3. 7 gal. = 28 qt. = 56 pt. ×4 7×4 or 7×8
4. 7762 lb. = 3 T. and 1762 lb. 2000)7762
5. 75 hr. = 3 da. and 3 hr. 24)75

$\frac{7}{33}\div\frac{2}{2}=\frac{3\frac{1}{2}}{16\frac{1}{2}}$

$5300\div16\frac{1}{2}$

$5300\times\frac{2}{33}=\frac{10600}{33}$

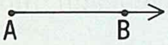
321 rd. 33)10600 99 — 70 66 40 33 7

Self-Test 5—Geometry

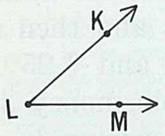
Match the drawings in column 1 with the words in column 2 by using the numerals and letters. You will not use all of the words.

e. **1.** .P

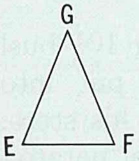
a. **2.**



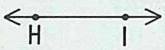
g. **3.**



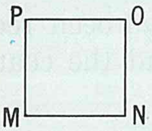
b. **4.**



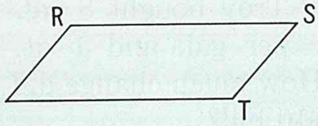
c. **5.**



d. **6.**

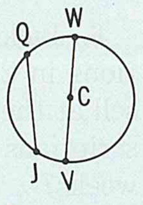


i. **7.**



a. Ray
b. Triangle
c. Line
d. Square
e. Point
f. Vertex A
g. Angle KLM
h. Circle
i. Parallelogram
j. Chord HI

In the circle at the right, find examples of the following and identify them by letters.



	a	*b*
8.	center C	chord $\overline{QJ}$, $\overline{VW}$
9.	radius $\overline{CW}$, $\overline{CV}$	diameter $\overline{VW}$

Self-Test 6—Perimeter, Area, Volume

Solve for p, l, w, or s.

76
1. $304=4s$

29
2. $2\times(58+w)=174$

98
3. $2\times(l+77)=350$

1796
4. $p=2\times(640+258)$

5280
5. $p=3\times1760$

1765
6. $9050=2\times(2760+w)$

Write the area of each rectangle or square.

1566 sq.in.
7. 29 in. by 54 in.

284.8 sq.yd.
8. 32 yd. by 8.9 yd.

6707.61 sq.ft.
9. 4 sides of 81.9 ft.

$53\frac{1}{28}$ sq.mi.
10. $6\frac{7}{8}$ mi. by $7\frac{5}{7}$ mi.

$228\frac{4}{9}$ sq.f
11. $28\frac{5}{9}$ ft. by 8 ft.

9960.04 sq.yd.
12. 4 sides of 99.8 yd.

Write the volume of each rectangular solid.

157.95 cu.ft.
13. 8.1 ft., 3.9 ft., and 5 ft.

3525 cu.in.
14. $23\frac{1}{2}$ in., 16 in., and $9\frac{3}{8}$ in.

7.83 cu.yd.
15. 2.9 yd., 1.8 yd., and 1.5 yd.

$349\frac{1}{4}$ cu.in.
16. 18 in., $6\frac{3}{4}$ in., and $2\frac{7}{8}$ in.

1728 cu.ft.
17. 12 ft., 12 ft., and 12 ft.

Self-Test 7—Ratio, Proportion, and Per Cent

Solve each equation below.

	a	*b*	*c*
1.	$\frac{1}{4}=\frac{n}{16}$ 4	$\frac{9}{12}=\frac{n}{4}$ 3	$\frac{3}{2}=\frac{n}{6}$ 9
2.	$\frac{4}{5}=\frac{8}{n}$ 10	$\frac{6}{8}=\frac{24}{n}$ 32	$\frac{12}{15}=\frac{48}{n}$ 60
3.	$\frac{n}{9}=\frac{5}{20}$ 2.25	$\frac{6}{n}=\frac{8}{15}$ 11.25	$\frac{n}{.48}=\frac{.06}{.32}$ .09
4.	$\frac{2}{5}=\frac{n}{100}$ 40	$\frac{12}{50}=\frac{n}{100}$ 24	$\frac{17}{25}=\frac{n}{100}$ 68
5.	$\frac{21}{84}=\frac{n}{100}$ 25	$\frac{n}{100}=\frac{18}{90}$ 20	$\frac{75}{100}=\frac{15}{n}$ 20

In per cent form write the equivalent number for each number in fractional or in decimal form below.

	a	*b*	*c*	*d*
6.	$\frac{3}{4}$ 75%	$\frac{9}{100}$ 9%	$\frac{1}{2}$ 50%	$\frac{9}{10}$ 90%
7.	$\frac{5}{8}$ $62\frac{1}{2}$%	$\frac{9}{9}$ 100%	$\frac{4}{10}$ 40%	$\frac{5}{6}$ $83\frac{1}{3}$%
8.	.05 5%	.50 50%	.005 .5%	$.12\frac{1}{2}$ $12\frac{1}{2}$%
9.	.75 75%	1.00 100%	1.005 100.5%	4.70 470%

Answer each question below.

10. What number is 25% of 60? 15

11. 9 is what per cent of 72? 12.5%

12. 17% of 96 is what number? 16.32

13. 80 is 5% of what number? 1600

14. What number is 30% of 920? 276

15. $33\frac{1}{3}$% of 200 is what number? $66\frac{2}{3}$

16. 15 is 25% of what number? 60

Self-Test 8—Problem Solving

Solve each problem below.

1. Ray's father drove 352.5 miles in May, 604.3 miles in June, 576.7 miles in July, and 810.9 miles in August. What was the average number of miles he drove each month?

(352.5+604.3+576.7+810.9)÷4=n   586.1

2. Ted earned \$.85 an hour. He worked for 7 hours and then spent \$1.98 for a record and \$.25 for a magazine. How much money did he have left from his earnings?

(7x85)-(198+25)=n   \$3.72

3. Mr. Moore dug $10\frac{1}{2}$ bushels of potatoes, which he put into peck measures for sale in his store. If he sold all but 3 pecks of potatoes, how many pecks did he sell?

$(10\frac{1}{2}x4)-3=n$ 39 pk.

4. Sally bought a coat on sale for \$29.60. The coat had been reduced 20%. How much had the coat sold for before the sale?

$\frac{2960}{n}=\frac{20}{100}$ \$37.00

5. Mr. Troy bought 8 gal. of gas at 34.5¢ per gal. and 1 qt. of oil at 69¢. How much change did he get from a \$10 bill?

1000-[(8x34.5)+69]=n   \$6.55

6. Jack sold 15 magazine subscriptions in 2 weeks. If he continues to sell at the same rate, how many subscriptions will he have sold in 6 weeks? $\frac{15}{2}=\frac{n}{6}$ 45 subscriptions

7. Joan spelled 85 words correctly out of 90 words. What per cent of the words did she misspell?

$\frac{5}{90}=\frac{n}{100}$ $5\frac{5}{9}$%

HANDBOOK

Many of your questions about arithmetic can be answered by using this handbook. Form the habit of turning to it when you need to review an idea you have studied.

The handbook is easy to use. Its eight sections are arranged in alphabetical order. Each section is divided into parts. The numerals in ( ) tell the pages in the book that give more information on the topic.

Addition

The Main Ideas

1. Properties of addition The same properties hold for addition of whole numbers, fractional numbers, and decimal numbers. (21, 22, 33, 150)

a. Closure property When two numbers of a set are added, their sum is also a number of that set.

$6+3=9 \qquad \frac{6}{10}+\frac{3}{10}=\frac{9}{10} \qquad .6+.3=.9$

b. Commutative property The order of addition of two numbers does not change the sum.

$8+6=14 \qquad 6+8=14 \qquad 8+6=6+8$

c. Associative property Regrouping does not change the sum.

$$7+3+6=n \qquad \begin{cases}(7+3)+6=n\\ 7+(3+6)=n\end{cases}$$

2. Zero is the identity number of addition A number remains the same, whether you add 0 to the number or add the number to 0. (22)

$29+0=29 \qquad 0+29=29$

3. Checking addition Add the numbers in a different order to check each sum. (24)

$5+6+7=18 \qquad 7+6+5=18$

How to Add

1. Use the basic addition combinations Add the digits in each place-value position. (23, 25)

$$\begin{array}{r} 52\\ +26\\ \hline 78 \end{array} \qquad \begin{array}{r} 683\\ +215\\ \hline 898 \end{array} \qquad \begin{array}{r} 73465\\ +29879\\ \hline 103344 \end{array}$$

2. Use combinations of 10, 100, and so on Regroup numbers and add in the most convenient way. (24)

$3+6+4+7=(3+7)+(6+4)=20$

$82+27=(80+20)+(2+7)=109$

Decimals

The Main Ideas

1. Meanings, words, and signs Fractional numbers may be expressed in decimal form. (7-9)

$\frac{9}{10}$ as .9 $\qquad$ $\frac{54}{1000}$ as .054

$\frac{5}{100}$ as .05 $\qquad$ $\frac{54}{10000}$ as .0054

2. Place-value To show that a digit is in ones place, put a decimal point after it. (7)

325.7 $\qquad$ 32.57 $\qquad$ 3.257

How to Add and Subtract

1. Use the arithmetic of fractional numbers Write the numerals so that the decimal points lie on a vertical line. (148, 149, 152)

$$\frac{54}{100}+\frac{60}{100}=n \qquad \begin{array}{r} .54+.60=n\\ 1.14=n\end{array} \qquad \begin{array}{r} .54\\ +.60\\ \hline 1.14\end{array}$$

$$\begin{array}{r} \frac{8}{10}\\ -\frac{3}{10}\\ \hline \frac{5}{10}\end{array} \qquad \begin{array}{r} 8\text{ tenths}\\ -3\text{ tenths}\\ \hline 5\text{ tenths}\end{array} \qquad \begin{array}{r} .8\\ -.3\\ \hline .5\end{array}$$

2. Use the properties of addition The commutative and associative properties apply. (150)

$.7+.5=1.2$ $\qquad$ $.5+.7=1.2$

$$.82+.18+7=n \left\{\begin{array}{l}(.82+.18)+7=n\\ .82+(.18+7)=n\end{array}\right.$$

How to Multiply

1. Use the arithmetic of fractional numbers The number of decimal places in the product is the same as the number of decimal places in the decimal factor. (154, 155, 160)

$$\begin{array}{l} 6\times\frac{4}{100}=\frac{6\times4}{100}=\frac{24}{100}\\ 6\times.04=.24\end{array} \qquad \begin{array}{r} .04\\ \times6.\\ \hline .24\end{array} \qquad \begin{array}{r} .04\\ \times6\\ \hline .24\end{array}$$

2. Use the arithmetic of whole numbers Multiply the numbers. Count the number of decimal places in both factors. (155, 158-159, 160, 162)

$$\begin{array}{r} .735\\ \times15.\\ \hline 3.675\\ 7.35\\ \hline 11.025\end{array} \qquad \begin{array}{l} n=.007\times9\\ \quad=\frac{7}{1000}\times9\\ \quad=\frac{63}{1000}\\ \quad=\frac{63\times1}{1000}\\ \quad=.063\end{array} \qquad \begin{array}{r} 3.125\\ \times.25\\ \hline .15625\\ .6250\\ \hline .78125\end{array}$$

3. Use the properties of multiplication The commutative property of multiplication applies. (158)

$3\times.1=.3$ $\qquad$ $.1\times3=.3$

How to Divide

1. Use the arithmetic of fractional numbers Division is the inverse operation of multiplication. (171)

$\frac{8}{10}\div\frac{2}{10}=n$ $\qquad$ $\frac{8}{10}=n\times\frac{2}{10}$ $\qquad$ $n=4$

$.8\div.2=n$ $\qquad$ $.8=n\times.2$ $\qquad$ $n=4$

2. Find decimal equivalents by dividing Rename the numerator as a number in decimal form. (174)

$$\frac{7}{20} \qquad \begin{array}{r} .35\\ 20\overline{)7.00}\end{array} \qquad \frac{7}{20}=.35$$

3. Divide tenths, hundredths, and so on as you divide ones and tens The decimal points in the dividend and quotient must be on the same vertical line. (177, 178)

$$\begin{array}{r} 15.3 \\ 4\overline{)61.2} \end{array} \qquad \begin{array}{r} 6.25 \\ 4\overline{)25.00} \end{array}$$

4. Rename the number that is the divisor to make it a whole number Multiply by 1 in the most convenient form. Divide as by a whole number. (179-180)

$$\frac{5.92}{3.2} = \frac{5.92}{3.2}\times 1$$
$$= \frac{5.92}{3.2}\times\frac{10}{10}$$
$$= \frac{59.2}{32.}$$

$$\begin{array}{r} 1.85 \\ 32\overline{)59.20} \\ \underline{32} \\ 27\ 2 \\ \underline{25\ 6} \\ 1\ 60 \\ \underline{1\ 60} \end{array}$$

Division

The Main Ideas

1. Division is the inverse operation of multiplication Use multiplication to find a quotient. (57-59)

$56 \div 8 = n \qquad 56 = n \times 8 \qquad n = 7$

2. One in division Any number, other than 0, when divided by itself produces 1. Dividing a number by 1 leaves the number unchanged. (58)

$7 \div 7 = 1 \qquad 7 \div 1 = 7$

3. Zero in division Zero divided by a number other than 0 always gives a quotient of 0. Division by 0 is meaningless. (59)

$0 \div 5 = n \qquad 0 = n \times 5 \qquad n = 0$

4. Checking division Multiply the divisor and quotient; add the remainder. The result should be the dividend. (61)

$98 \div 24 = 4\ \text{r}2 \qquad (4 \times 24) + 2 = 98$

How to Divide

1. Use the basic division combinations Then you can take these steps in division: Divide, multiply, and subtract. (58, 60)

$136 \div 9 = n$

$$\begin{array}{r} 15\ \text{r}1 \\ 9\overline{)136}\phantom{\ \text{r}1} \\ \underline{9}\phantom{\ \text{r}1} \\ 46\phantom{\ \text{r}1} \\ \underline{45}\phantom{\ \text{r}1} \\ 1\phantom{\ \text{r}1} \end{array}$$

2. Find a trial quotient Estimate the number which you think most nearly tells how many times the divisor is contained in the dividend. Check by using multiplication. (61)

$53 \div 17 = n \qquad 5\ \text{T} \div 1\ \text{T} = n$

$5 \times 17 = 85 \qquad 4 \times 17 = 68 \qquad 3 \times 17 = 51$

$85 > 53 \qquad 68 > 53 \qquad 51 < 53$

3. Multiply the divisor by the trial quotient You should obtain a number equal to or less than the dividend. (62)

$$210 \div 42 = n$$

$21\text{ T} \div 4\text{ T} = n$ $\qquad 5 \times 42 = 210$

4. Subtract Write the difference in the appropriate place-value position. (62)

$$\begin{array}{r} 20 \\ 32\overline{)653} \\ \underline{64} \\ 13 \end{array}$$

Trial Quotients

$65\text{ T} \div 3\text{ T} = n$

$13\ 0 \div 3\text{ T} = n$

5. Repeat the steps The final remainder must be 0 or some other number less than the divisor. (66)

$$\begin{array}{r} 52\ \text{r}1 \\ 34\overline{)1769\phantom{\ \text{r}1}} \\ \underline{170}\phantom{\ \text{r}1} \\ 69\phantom{\ \text{r}1} \\ \underline{68}\phantom{\ \text{r}1} \\ 1\phantom{\ \text{r}1} \end{array}$$

Trial Quotients

$17\text{ H} \div 3\text{ T} = n$

$6\text{ T} \div 3\text{ T} = n$

6. Estimate the number of digits in the quotient Then you will know where to write the first digit in the quotient. (66-67)

$$\begin{array}{r} 3420 \\ 60\overline{)205200} \\ \underline{180} \\ 252 \\ \underline{240} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

$2052 \div 60 = n$

$1000 \times 60 = 60000$

$60000 < 205200$

$10000 \times 60 = 600000$

$600000 > 205200$

n must have 4 digits

7. Divide by three-digit divisors as you divide by smaller divisors Find a trial quotient. Estimate the result. Then divide, multiply, and subtract. (68)

$$\begin{array}{r} 35 \\ 212\overline{)7420} \\ \underline{636} \\ 1060 \\ \underline{1060} \end{array}$$

Trial Quotients

$7\text{ Th} \div 2\text{ H} = n$

$10\text{ H} \div 2\text{ H} = n$

Fractions

The Main Ideas

1. Meanings, words, and signs A fraction has a denominator that tells you the number of parts of equal size into which a given quantity has been divided and a numerator that tells how many of these parts are being considered. (12)

3 numerator
5 denominator

2. Different fractions may name the same fractional number Such fractions are equivalent. (14)

$$\tfrac{1}{2}=\tfrac{2}{4}=\tfrac{3}{6}=\tfrac{4}{8}=\tfrac{5}{10}\cdots$$

3. Fractions may be changed to higher terms Multiply by 1 in a convenient form. (15)

$\frac{1}{2}=\frac{n}{6}$ $\qquad 6 \div 2 = 3$ $\qquad$ Choose $\frac{3}{3}$ for 1

$1\times\frac{1}{2}=\frac{3}{3}\times\frac{1}{2}=\frac{3\times1}{3\times2}=\frac{3}{6}$ $\qquad \frac{1}{2}=\frac{3}{6}$

4. Fractions may be changed to lower terms Divide by 1 in a convenient form. (16)

$\frac{3}{6}=\frac{n}{2}$ 3 is greatest common factor
Choose $\frac{3}{3}$ for 1

$\frac{3}{6}\div 1=\frac{3}{6}\div\frac{3}{3}=\frac{3\div 3}{6\div 3}=\frac{1}{2}$ $\qquad \frac{3}{6}=\frac{1}{2}$

How to Add and Subtract

1. Use the arithmetic of whole numbers The same properties apply to the addition of fractional numbers as to the addition of whole numbers. (33, 34)

$\frac{2}{5}+\frac{1}{5}=\frac{2+1}{5}=\frac{3}{5} \qquad \frac{1}{5}+\frac{2}{5}=\frac{1+2}{5}=\frac{3}{5}$

$\frac{2}{5}+\frac{3}{5}+\frac{4}{5}=n \qquad \begin{cases}(\frac{2}{5}+\frac{3}{5})+\frac{4}{5}=n\\ \frac{2}{5}+(\frac{3}{5}+\frac{4}{5})=n\end{cases}$

$$\begin{array}{r} 4\frac{3}{8}\\ +7\frac{7}{8}\\ \hline 11\frac{10}{8}=11+1\frac{2}{8}=12\frac{2}{8}=12\frac{1}{4}\end{array} \qquad \begin{array}{r} 11\frac{10}{8}\\ -7\frac{7}{8}\\ \hline 4\frac{3}{8}\end{array}$$

2. Use the least common denominator Rename the fractional numbers to give them a denominator that is the smallest number divisible by all of the denominators. (36)

$$\begin{array}{rr} 1\frac{1}{3}= & 1\frac{2}{6}\\ \underline{+4\frac{1}{2}}= & \underline{4\frac{3}{6}}\\ & 5\frac{5}{6}\end{array} \qquad \begin{array}{rrr} 8\frac{1}{2}= & 8\frac{3}{6}= & 7\frac{9}{6}\\ \underline{-4\frac{5}{6}}= & \underline{4\frac{5}{6}}= & \underline{4\frac{5}{6}}\\ & & 3\frac{4}{6}=3\frac{2}{3}\end{array}$$

How to Multiply

1. Use repeated addition Multiply a fractional number by a whole number as you multiply whole numbers. (73, 74)

$3\times\frac{2}{5}$ means $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$

$3\times\frac{2}{5}=\frac{3\times 2}{5}=\frac{6}{5}=\frac{5+1}{5}=1+\frac{1}{5}=1\frac{1}{5}$

2. Use the arithmetic of whole numbers The same properties apply to the multiplication of fractional and whole numbers. (75, 77, 88)

$\frac{3}{4}\times 1=\frac{3\times 1}{4}=\frac{3}{4} \qquad \frac{1}{4}\times 3=\frac{1\times 3}{4}=\frac{3}{4}$

$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=n \qquad \begin{cases}(\frac{1}{2}\times\frac{2}{3})\times\frac{3}{4}=n\\ \frac{1}{2}\times(\frac{2}{3}\times\frac{3}{4})=n\end{cases}$

$$\begin{aligned}4\times 5\tfrac{1}{4}=4\times(5+\tfrac{1}{4})&=(4\times 5)+(4\times\tfrac{1}{4})\\ &=20+1=21\end{aligned}$$

3. Use multiplication of whole numbers To find the product of fractional numbers, multiply the numbers named by the numerators and multiply the numbers named by the denominators. Express the result as a fraction. (80, 82)

$\frac{1}{4}\times\frac{1}{2}=\frac{1\times 1}{4\times 2}=\frac{1}{8} \qquad \frac{2}{3}\times\frac{4}{5}=\frac{2\times 4}{3\times 5}=\frac{8}{15}$

4. Rename the mixed fractions as improper fractions Then multiply the fractional numbers. (79, 84, 85)

$7\times\frac{17}{5}=\frac{7\times 17}{5}=\frac{119}{5}=23\frac{4}{5}$

$2\frac{1}{2}\times\frac{1}{3}=\frac{5}{2}\times\frac{1}{3}=\frac{5\times 1}{2\times 3}=\frac{5}{6}$

$3\frac{1}{2}\times 2\frac{3}{4}=\frac{7}{2}\times\frac{11}{4}=\frac{7\times 11}{2\times 4}=\frac{77}{8}=9\frac{5}{8}$

How to Divide

1. Use the inverse operation of multiplication Divide by a fractional number as you divide by a whole number. (93)

$\frac{15}{16}\div\frac{3}{16}=n \qquad \frac{15}{16}=n\times\frac{3}{16} \qquad n=5$

$\frac{15}{16}\div\frac{3}{16}=\frac{15\div 3}{16\div 16}=\frac{5}{1}=5$

2. Rename the dividend and the divisor as fractional numbers with a common denominator Then divide the numbers in the numerators and in the denominators. (95, 96, 97)

$$2\tfrac{1}{4}\div\tfrac{3}{4}=\tfrac{9}{4}\div\tfrac{3}{4}=\tfrac{9\div3}{4\div4}=(9\div3)\div1=3$$

$$2\tfrac{1}{4}\div1\tfrac{3}{4}=\tfrac{9}{4}\div\tfrac{7}{4}=\tfrac{9\div7}{4\div4}=(9\div7)\div1=1\tfrac{2}{7}$$

$$4\div\tfrac{1}{2}=\tfrac{8}{2}\div\tfrac{1}{2}=\tfrac{8\div1}{1}=8$$

3. Multiply by the reciprocal of the divisor Dividing by a fractional number is the same as multiplying by the reciprocal of the divisor. (102, 103, 106)

$n=\tfrac{3}{5}\div\tfrac{2}{3}$ The reciprocal of $\tfrac{2}{3}$ is $\tfrac{3}{2}$

$$n=\tfrac{3}{5}\times\tfrac{3}{2}=\tfrac{3\times3}{5\times2}=\tfrac{9}{10}$$

$n=3\tfrac{1}{2}\div1\tfrac{1}{8}$ and $n=\tfrac{7}{2}\div\tfrac{9}{8}$ and $n=\tfrac{7}{2}\times\tfrac{8}{9}$

$$n=\tfrac{7}{2}\times\tfrac{8}{9}=\tfrac{7\times8}{2\times9}=\tfrac{8\times7}{2\times9}=\tfrac{4\times7}{1\times9}=\tfrac{28}{9}=3\tfrac{1}{9}$$

Multiplication

The Main Ideas

1. Meanings, words, and signs To add a number a specified number of times, you multiply. (41)

$$6+6+6 \qquad \begin{array}{r} \left.\begin{array}{r} 6 \\ \times 3 \end{array}\right\} \\ \hline 18 \end{array} \begin{array}{l} \text{factors} \\ \\ \text{product} \end{array} \qquad 6\times3=18$$

2. Properties of multiplication The same properties hold for multiplying whole, fractional, and decimal numbers. (41, 75, 158)

a. Commutative property Changing the order of the factors does not change the product. (41)

$3\times6=18 \qquad 6\times3=18 \qquad 3\times6=6\times3$

$\tfrac{3}{4}\times1=\tfrac{3}{4} \qquad 1\times\tfrac{3}{4}=\tfrac{3}{4} \qquad \tfrac{3}{4}\times1=1\times\tfrac{3}{4}$

$.1\times3=.3 \qquad 3\times.1=.3 \qquad .1\times3=3\times.1$

b. Associative property Regrouping the factors does not change the product. (45, 88)

$$2\times3\times4=n \qquad \left\{\begin{array}{l}(2\times3)\times4=n\\ 2\times(3\times4)=n\end{array}\right.$$

3. The identity number is 1 Multiplication by 1 in any form leaves a number unchanged. (42, 78)

$1\times3=3 \qquad 3\times1=3 \qquad 2\times\tfrac{3}{3}=2$

How to Multiply

1. Use the basic multiplication combinations Multiply the digits separately. (41, 42, 44)

	323	875
	×3	×6
3×3 =9	9	30
3×20 =60	60	420
3×300=900	900	4800
Add	969	5250

2. Multiply by tens, hundreds, and so on as you multiply by ones Multiply each digit separately. (47, 50)

	18	144
	×53	×175
3 O×18=54 O	54	720
5 T×18=90 T	90	1008
	954	144
		25200

Problem Solving

The Main Ideas

1. Story problems can be translated into equations Use numerals, letters, operational signs, and relations signs. (132)

Betty needs 15 pictures to fill her photograph album. The album has spaces for 90 pictures. How many pictures are in the album?

$$n+15=90$$
$$n=90-15$$
$$n=75$$

Betty has 75 pictures in her album.

2. More than one solution may be correct Plan a solution carefully before you begin. (138)

Bob had 75 stamps. He gave 18 to Dan and 23 to John. How many stamps did Bob have left?

$75-(18+23)=s$ $(75-18)-23=s$

Bob had 34 stamps left.

How to Solve Story Problems

1. Read carefully Be sure you understand the situation. (131)

Sue had 50 pages to read. She read 29 pages in $\frac{1}{2}$ hour. How many pages did she still have to read?

2. Determine the operation Translate the problem into an equation. The statements that follow may help you decide which operation to use in solving the equation. (133)

a. To find the sum or total of two different numbers, you *add.*

b. To find the total of two or more equal numbers, you may either *add* or *multiply.*

c. To find the difference between two numbers, or by how much one number is more or less than the other, you *subtract.*

d. To find the number in each set or group of equal size, you *divide.*

e. To find the number of sets or groups of equal size, you *divide.*

Sue had 50 pages to read. She read 29 pages in $\frac{1}{2}$ hour. How many pages did she still have to read?

$$n=50-29$$
$$n=21$$

Sue had 21 pages to read.

3. Use a formula The same plan, or formula, may be used for specific types of problems. (136)

I can buy records at \$.79 each. How much will 6 of the records cost?

$$c=n\times p$$
$$=6\times .79=4.74$$

The records will cost \$4.74.

Ratio and Proportion

The Main Ideas

1. Sets may be matched in many ways The matching establishes a correspondence. (187)

1 to 1	**A**	0 0 0 0
	B	X X X X
1 to 2	**A**	0 0 0
	B	X X X X X X
2 to 3	**A**	0 0 0 0
	B	X X X X X X

2. A correspondence is a ratio A ratio may be expressed as a number in fractional form. (188)

A	0 0 0 0 0 0	$\frac{6}{8}$
B	X X X X X X X X	

3. Ratios may have different forms Different forms of the same ratio may be equivalent. (188-189)

A / **B**: $\frac{0\,0\,0\,0\,0\,0}{X\,X\,X\,X\,X\,X\,X\,X} = \frac{6}{8}$

A / **B**: $\frac{0\,0\,0}{X\,X\,X\,X} \quad \frac{0\,0\,0}{X\,X\,X\,X} = \frac{2\times3}{2\times4} = \frac{3}{4}$

$\frac{6}{8}=\frac{3}{4} \qquad \frac{8}{12}=\frac{2}{3} \qquad \frac{8}{4}=\frac{2}{1}$

4. Equivalent ratios make a proportion Use your knowledge of fractions to determine equivalence. (191)

$\frac{2}{3}=\frac{24}{m}$

$m=36$

$\frac{2}{3}=\frac{24}{36}$

$\frac{2}{3} = 1\times\frac{2}{3}$

$\frac{2}{3} = \frac{12}{12}\times\frac{2}{3}$

$\frac{2}{3} = \frac{12\times2}{12\times3}=\frac{24}{36}$

5. Problems can be translated into proportions The number you are seeking may be any of the terms of the proportion. (192)

a. Bob made 3 baskets in 5 attempts. At this rate, how many baskets can he make in 20 attempts?

$\frac{3}{5}=\frac{b}{20} \qquad \frac{3}{5}=\frac{12}{20} \qquad b=12$

Bob can make 12 baskets.

b. Bob made 3 baskets in 5 attempts. At this rate, how many shots must he attempt before he can expect to make 12 baskets?

$\frac{3}{5}=\frac{12}{a} \qquad \frac{3}{5}=\frac{12}{20} \qquad a=20$

Bob must attempt 20 shots.

6. Equivalence between ratios can be determined Multiply the numerator of each ratio by the denominator of the other. The ratios are equivalent if both multiplications produce the same result. (196)

$\frac{4}{6}=\frac{10}{15}$ Think: $15\times4=60$

$6\times10=60$

7. Ratios can be renamed as per cent Any ratio or fraction with a denominator of 100 may be renamed as per cent. (207, 208, 224)

$\frac{1}{100}=1\%$ $\qquad \frac{110}{100}=110\%$

$.05=5\%$ $\qquad .99=99\%$

$\frac{1}{4}=\frac{25}{100}=25\%$ $\qquad \frac{3}{10}=\frac{30}{100}=30\%$

8. Per cents can be renamed Use numbers in fractional or in decimal form. (225)

$50\% = \frac{50}{100} = .50 \qquad 75\% = \frac{75}{100} = .75$

9. Problems with per cents can be translated into proportions One denominator will be 100. (212, 214, 216)

Andy sold 12 rabbits out of 48 rabbits he raised. What per cent of his rabbits did he sell?

$$\begin{aligned} \frac{n}{100} &= \frac{12}{48} \\ n \times 48 &= 12 \times 100 \\ n \times 48 &= 1200 \\ n &= 1200 \div 48 \quad \text{or} \quad 25 \end{aligned}$$

Andy sold 25% of his rabbits.

How to Solve a Proportion

1. Use the arithmetic of fractional numbers Find the equivalent fraction. (190-191)

$\frac{2}{3} = \frac{n}{6} \qquad \frac{2}{3} = \frac{4}{n} \qquad \frac{n}{3} = \frac{4}{6} \qquad \frac{2}{n} = \frac{4}{6}$

$\frac{2}{3} = \frac{4}{6} \qquad \frac{2}{3} = \frac{4}{6} \qquad \frac{2}{3} = \frac{4}{6} \qquad \frac{2}{3} = \frac{4}{6}$

2. Use the numerator test Multiply each numerator by the denominator of the other fraction. (197)

$$\begin{aligned} \frac{n}{3} &= \frac{20}{5} \\ n \times 5 &= 20 \times 3 \\ n \times 5 &= 60 \\ n &= 60 \div 5 \\ n &= 12 \end{aligned} \qquad \begin{aligned} \frac{25}{100} &= \frac{12}{r} \\ 25 \times r &= 12 \times 100 \\ 25 \times r &= 1200 \\ r \times 25 &= 1200 \\ r &= 1200 \div 25 \\ r &= 48 \end{aligned}$$

Subtraction

The Main Ideas

1. Subtraction is the inverse operation of addition Use addition to help you find a difference. (27)

$8 + 5 = 13 \qquad 13 - 5 = 8$

2. Zero in subtraction Subtracting 0 from a number does not change the number, and subtracting a number from itself produces 0. (27)

$9 - 0 = 9 \qquad 9 - 9 = 0$

3. Checking subtraction Add the subtrahend and the difference. You should obtain the minuend. (28)

$97 - 9 = 88 \qquad 88 + 9 = 97$

How to Subtract

1. Use the basic subtraction combinations Subtract the digits in each place-value position. (28)

$$\begin{array}{r} 37 \\ -9 \\ \hline 28 \end{array} \qquad \begin{array}{r} 95 \\ -28 \\ \hline 67 \end{array} \qquad \begin{array}{r} 835 \\ -567 \\ \hline 268 \end{array}$$

2. Use multiples of 10, 100, and so on Subtract numbers in the most convenient form. (30)

$$\begin{aligned} 45 - 18 &= n \\ 45 - (15 + 3) &= n \\ (45 - 15) - 3 &= n \\ 30 - 3 &= n \\ 27 &= n \end{aligned} \qquad \begin{aligned} 100 - 47 &= n \\ 100 - (40 + 7) &= n \\ (100 - 40) - 7 &= n \\ 60 - 7 &= n \\ 53 &= n \end{aligned}$$

Tables of Measure

In the tables of measure the sign = means *equal in measure* or *is equivalent to.*

Liquid Measure

2 tablespoonfuls = 1 fluid ounce (fl. oz.)
16 fluid ounces = 1 pint (pt.)
2 cups (c.) = 1 pt.
2 pt. = 1 quart (qt.)
4 qt. = 1 gallon (gal.)
10 milliliters (ml.) = 1 centiliter
10 centiliters (cl.) = 1 deciliter
10 deciliters (dl.) = 1 liter (l.)
1000 liters = 1 kiloliter (kl.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cu. ft.
27 cu. ft. = 1 cu. yd.

Weight

16 ounces (oz.) = 1 pound (lb.)
100 lb. = 1 hundredweight (cwt.)
2000 lb. = 1 ton (T.)
10 milligrams (mg.) = 1 centigram
10 centigrams (cg.) = 1 decigram
10 decigrams (dg.) = 1 gram (g.)
1000 grams = 1 kilogram (kg.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 qt. = 1 peck (pk.)
4 pk. = 1 bushel (bu.)

Linear Measure

12 inches (in.) = 1 foot (ft.)
36 in. = 1 yard (yd.)
3 ft. = 1 yd.
$16\frac{1}{2}$ ft. = 1 rod (rd.)
$5\frac{1}{2}$ yd. = 1 rd.
5280 ft. = 1 mile (mi.)
1760 yd. = 1 mi.
320 rd. = 1 mi.
10 millimeters (mm.) = 1 centimeter
10 centimeters (cm.) = 1 decimeter
10 decimeters (dm.) = 1 meter (m.)
100 cm. = 1 m.
1000 m. = 1 kilometer (km.)

Time

60 seconds (sec.) = 1 minute (min.)
60 min. = 1 hour (hr.)
24 hr. = 1 day (da.)
7 da. = 1 week (wk.)
12 months (mo.) = 1 year (yr.)
10 yr. = 1 decade
100 yr. = 1 century

Square Measure

144 square inches (sq. in.) = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
160 sq. rd. = 1 acre
640 acres = 1 sq. mi.

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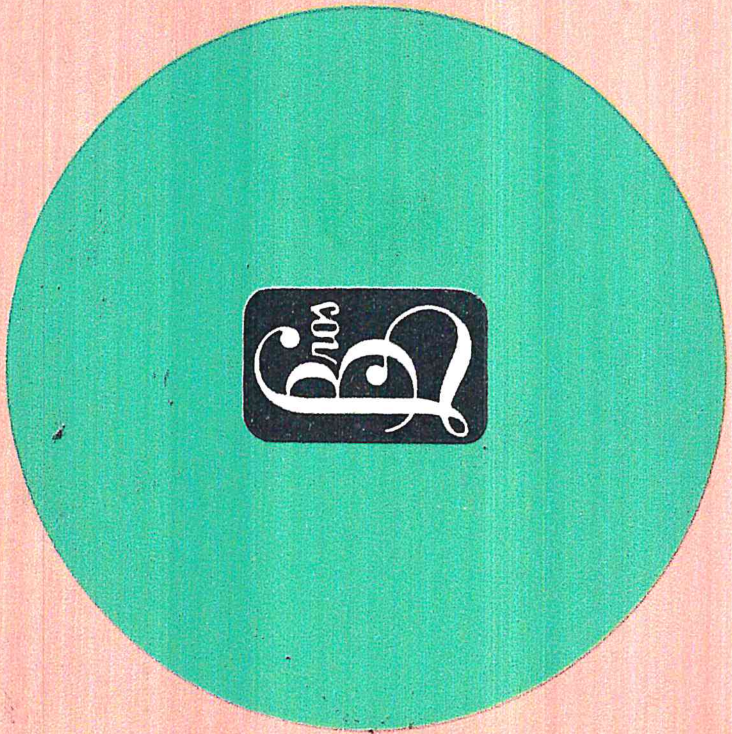

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